Order Reduction of LTIV Continuous MIMO System using Stability Preserving Approximation Method

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ABSTRACT
In modeling physical systems, the order of the system gives an idea of the measure of accuracy of the modeling of the system. The higher the order, the more accurate the model can be in describing the physical system. But in several cases, the amount of information contained in a complex model may obfuscate simple, insightful behaviors, which can be better captured and explored by a model with a much lesser order. In this paper, stability preserving method is proposed for the Multiple Input Multiple Output linear time invariant system to obtain the stable reduced order system. The genetic algorithm is used at the tail end of the proposed scenarios to get error minimized reduced model.

Keywords
Model order reduction, Integral Square Error (ISE), Genetic Algorithm (GA), Transient gain, Steady state gain

1. INTRODUCTION
In the analysis of many systems for which the physical laws are well known, one is frequently confronted with problems arising from the high dimensions of descriptive state model, the famous curse of dimensionality. The reduction of such high order systems (also termed as large scale systems) into low order models is one of the important problems in control and system theory system and is considered important in analysis, synthesis and simulation of practical systems. The exact analysis of high order systems is both tedious and costly.

The concept of retaining the dominant dynamical characteristics of the original system in the reduced model is intuitive and has two appealing advantages: the reduced-order model retains the basic physical features (such as time constants) of the original system; and the stability of the simplified model is guaranteed. These characteristics confer upon the reduced-order models a greater physical meaning. The mode retention methods produce the reduced model such that it matches a certain number of coefficients computed from the original system. T.N.Lucas [4] proposed a method which is an alternative approach for linear system reduction by Pade approximation [5-6] to allow retention of dominant modes. It avoids calculation of system time moments and the solution of Pade equations by simply dividing out the unwanted pole factors. This method adjusts the numerator polynomial coefficients based on the original systems pole values. The Routh approximation, stability equation method are used to guarantee the stability of reduced model. Research works proposed by Shamash [7], Chen et al [8-9], Pal et al [10] and Gupta et al [11] proves the quality of the reduction process. Most of the model order reduction techniques are concerned with preserving stability and matching initial time moments between the full and reduced systems. The stability of the system is preserved by obtaining the reduced order denominator polynomial based on selecting stable poles or using the properties of Routh table. To preserve the steady state characteristics it is usual either to solve the pade equations or invert a continued fraction, which yields the reduced numerator. Instead of using single method to derive the reduced model, now-a-days researchers prepare some mixed methods of model simplification for continuous time systems.

In S.K.Bhagat et al [12] method stability preserving methods namely, (γ-δ) canonical expansion [13], Gutman's differentiation method [14] and stability equation method [15] have been used to obtain the denominator of original system and the Lucas factor division method[4] is used to yield the numerator. The mihailov criterion has been combined with pade and factor division method to obtain the better approximation. In R.Prasad, Mihailov criterion is combined with the Cauer second form for reducing the order of the large scale SISO systems.

Recently evolutionary techniques such as Genetic algorithm, Particle Swarm Optimization are applied to obtain the better approximation. Few methods [16-23] uses the ISE as performance parameter which produces reduced order closer to given higher order system behavior. In these methods, denominator polynomial is obtaining by using any of the stability preserving criterions like stability equation method, mihailov stability criterion [1-3], routh approximation and etc.

In this paper, a mixed method is proposed to obtain the reduced model for the given stable higher order model. The proposed method retains the important characteristics of the original system in its reduced model. The parameters those related to the dynamic behavior of the system such as damping ratio and undamped natural frequency are adjusted with the help of genetic algorithm. In this proposed scenario, the genetic algorithm is used at the tail end of the approximation method to get the error minimized reduced model.

2. PROBLEM STATEMENT
Consider an nth order linear time invariant dynamic multivariable system with q inputs and p outputs described in time domain by state space equations given as

$$\left.\begin{aligned} \overset{*}{X} &= Ax(t) + Bu(t) \\ y &= Cx(t) + Du(t) \end{aligned}\right\} \qquad (1)$$

Where,

x is n dimensional state vector,

u is Input control vector,

y is output vector with,

A is n × n system matrix,

B is n × 1 input matrix and

C is 1 × n output matrix.

Alternatively, corresponding transfer function representation of an nth order system is given by,

$$G(s)=\frac{N(s)}{D(s)}=\frac{\sum_{i=0}^{n-1}a_i s^i}{\sum_{i=0}^{n}b_i s^i} \quad (2)$$

Irrespective of the statement in equation 1 and 2, the problem is to find the rth order reduced model, where r<n the following form represented in equation 3. Resultant approximation model will retain the important characteristics of given higher order system. With the common denominator D(s) of the given higher order system, its transfer function in matrix form is given by,

$$G(s)=\frac{\begin{bmatrix} N_{11}(s) & N_{22}(S) & \ldots & N_{1q}(s) \\ N_{21}(s) & N_{22}(s) & \ldots & N_{2q}(s) \\ \ldots & \ldots & \ldots & \ldots \\ N_{p1}(s) & N_{p2}(s) & \ldots & N_{pq}(s) \end{bmatrix}}{D(s)}$$

Or

$$\mathrm{G_{ij}}(\mathrm{s})=\frac{\mathrm{N_{ij}}(s)}{D_{ij}(s)} \quad (3)$$

Where, i=1,2…p and j=1,2…q

$$[G_r(s)]=\begin{bmatrix} R_{11}(s) & R_{12}(s) \\ R_{21}(s) & R_{22}(s) \end{bmatrix} \quad (4)$$

The resultant order transfer coefficients are adjusted using genetic algorithm concept to achieve the good approximation.

3. PROPOSED METHODOLOGY

The proposed scenario consists of following steps to obtain the reduced order model.

Step-1: Obtain the denominator and numerator polynomial constant terms in the reduced model through pade approximation.

Consider, the transfer function of higher order (nth) as

$$G(s)=\frac{a_0+a_1 s+a_2 s^2+\ldots+a_{n-1}s^{n-1}}{(s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$=\frac{a_0+a_1 s+a_2 s^2+\ldots+a_{n-1}s^{n-1}}{b_0+b_1 s+b_2 s^2+\ldots+b_{n-1}s^{n-1}+b_n s^n} \quad (5)$$

$G(s)$ Can be expanded into a power series about $s=0$ of the form,

$$G(s)=c_0+c_1 s+c_2 s^2+\cdots \quad (6)$$

Where, $c_0=\frac{a_0}{b_0}$ (7)

and $c_k=\frac{1}{b_0}\left[a_k-\sum_{j=1}^{k}b_j c_{k-j}\right], \quad k>0$ (8)

with $d_k=0 \quad \forall\, k>n-1$

The b_i are directly proportional to the time moments of the system, assuming the system is stable.

$$G_r(s)=\frac{d_0+d_1 s+d_2 s^2+\ldots+d_{r-1}s^{r-1}}{e_0+e_1 s+e_2 s^2+\ldots+e_{r-1}s^{r-1}+e_r s^r} \quad (9)$$

Then for $R(s)$ to be pade approximant of $G(s)$, we have equations,

$$d_0=e_0\cdot c_0 \quad (10)$$

$$\begin{aligned} d_1&=e_0\cdot c_1+e_1\cdot c_0 \\ &\vdots \\ 0&=e_0\cdot c_{2r-2}+e_1\cdot c_{2r-1}+\cdots c_{r-1} \\ 0&=e_0\cdot c_{2r-1}+\cdots c_r \end{aligned} \quad (11)$$

From the equations (7) and (10),

$$c_0=\frac{a_0}{b_0}=\frac{d_0}{e_0} \quad (12)$$

From the equation (12), let

$$\left.\begin{aligned} a_0&=d_0 \\ b_0&=e_0 \end{aligned}\right\} \quad (13)$$

Step-2: Determine the unknown coefficients of‘s’ remaining in reduced model

The given higher order system transfer function is equated and cross multiplied with k^{th} order general transfer function.

$$\frac{a_0+a_1 s+a_2 s^2+\ldots+a_{n-1}s^{n-1}}{b_0+b_1 s+b_2 s^2+\ldots+b_{n-1}s^{n-1}+b_n s^n}=\frac{d_0+d_1 s+d_2 s^2+\ldots+d_{r-1}s^{r-1}}{e_0+e_1 s+e_2 s^2+\ldots+e_{r-1}s^{r-1}+e_r s^r} \quad (14)$$

$$\begin{aligned} &(a_0+a_1 s+a_2 s^2+\ldots+a_{n-1}s^{n-1})(e_0+e_1 s+e_2 s^2+\ldots+e_{r-1}s^{r-1}+e_r s^r) \\ &=(b_0+b_1 s+b_2 s^2+\ldots+b_{n-1}s^{n-1}+b_n s^n)(d_0+d_1 s+d_2 s^2+\ldots+d_{r-1}s^{r-1}) \end{aligned} \quad (15)$$

The co-efficients of same power of ‘s’ on both sides of the equation (15) equated with each other and is given by,

$$\begin{aligned} a_{n-1}\cdot e_r&=b_n\cdot d_{r-1} \\ a_{n-1}\cdot e_{r-1}+a_{n-2}\cdot e_r&=b_{n-1}\cdot d_{r-1}+b_n\cdot d_{r-2} \\ &\vdots \\ a_2\cdot e_0+a_1\cdot e_1+a_0\cdot e_2&=b_0\cdot d_2+b_1\cdot d_1+b_2\cdot d_0 \\ a_1\cdot e_0+a_0\cdot e_1&=b_1\cdot d_0+b_0\cdot d_1 \\ a_0\cdot e_0&=b_0\cdot d_0 \end{aligned} \quad (16)$$

On solving the set of equations in (16) with the values of d_0, e_0 obtained in (13), the resultant reduced model is obtained as,

$$G_r(s)=\frac{d_0+d_1s+d_2s^2+...+d_{r-1}s^{r-1}}{e_0+e_1s+e_2s^2+...+e_{r-1}s^{r-1}+e_rs^r} \quad (17)$$

If $r=2\Rightarrow$

$$G_2(s)=\frac{d_0+d_1s}{e_2s^2+e_1s+e_0} \quad (18)$$

Sometimes the numerator part obtained in this step leads to an unstable reduced model. To overcome the same, the proposed method may be extended to the step-3 to obtain the stable reduced model.

Step-3: Determine the reduced order numerator polynomial for stable condition

Equating the given higher order transfer function in to the general reduced order model of r^{th} order and where the denominator polynomial obtained in step 1 is used to obtain the reduced order numerator polynomial.

$$\frac{a_0+a_1s+a_2s^2+...+a_{n-1}s^{n-1}}{b_0+b_1s+b_2s^2+...+b_ns^n}=\frac{d_0+d_1s+d_2s^2+...+d_{r-1}s^{r-1}}{e_0+e_1s+e_2s^2+...+e_rs^r} \quad (19)$$

Consider,

$$G(s)=\frac{a_0+a_1s+a_2s^2+...+a_{n-1}s^{n-1}}{b_0+b_1s+b_2s^2+...+b_ns^n}$$

$$=k\cdot\frac{\left(1+A_1s+A_2s^2+...+A_{n-1}s^{n-1}\right)}{\left(1+B_1s+B_2s^2+...+B_ns^n\right)};n\geq m \quad (20)$$

where,

$$k=\frac{a_0}{b_0}\,;A_i=\frac{a_i}{a_0}\,;B_j=\frac{b_j}{b_0}$$

$i=0,1,2,...m$ and $j=0,1,2,...n$

Let the transfer function of the approximated system be,

$$G_r(s)=k\cdot\frac{\left(1+D_1s+D_2s^2+...+D_{r-1}s^{r-1}\right)}{\left(1+E_1s+E_2s^2+...+E_rs^r\right)} \quad (21)$$

The following relation should be satisfied as closely as possible,

$$\frac{|G(j\omega)|^2}{|G_r(j\omega)|^2}=1 \qquad for\ 0\leq\omega\leq\infty \quad (22)$$

$$\frac{\mathrm{G(s)}}{\mathrm{G_r(s)}}=\frac{(1+A_1s+A_2s^2+...+A_{n-1}s^{n-1})}{(1+B_1s+B_2s^2+...+B_ns^n)}\cdot\frac{\left(1+E_1s+E_2s^2+...+E_rs^r\right)}{\left(1+D_1s+D_2s^2+...+D_{r-1}s^{r-1}\right)} \quad (23)$$

$$\frac{\mathrm{G(s)}}{\mathrm{G_r(s)}}=\frac{1+m_1s+m_2s^2+...+m_ns^n}{1+l_1s+l_2s^2+...+l_{v-1}s^{v-1}+b_vs^v} \quad (24)$$

From the equations (23) and (24), we can calculate the values of $m_1,m_2,...m_n$ and $l_1,l_2,...l_v$

$$\frac{\left|G(j\omega)\right|^2}{\left|G_r(j\omega)\right|^2}=\left.\frac{G(s)G(-s)}{G_r(s)G_r(-s)}\right|_{s=j\omega} \quad (25)$$

$$\frac{|G(j\omega)|^2}{|G_r(j\omega)|^2}=1+\frac{\left(L_2-M_2\right)s^2+\left(L_4-M_4\right)s^4+...+\left(L_{2u}-M_{2u}\right)s^{2u}}{1+M_2s^2+M_4s^4+...+M_{2v}s^{2v}} \quad (26)$$

To satisfy the condition stated in (22),

$$L_2=M_2$$
$$L_4=M_4$$
$$\vdots$$
$$L_{2u}=M_{2v}$$

From the above relations,

$$L_{2x}=\sum_{i=0}^{x-1}(-1)^i\cdot 2m_i\cdot m_{2x-i}+(-1)^x m_x^2 \quad (27)$$

for $x=1,2,3,...n$ and $m_0=1$ and

$$M_{2y}=\sum_{i=0}^{y-1}(-1)^i\cdot 2l_i\cdot l_{2y-i}+(-1)^x l_y^2 \quad (28)$$

for $y=1,2,3,...v$ and $l_0=1$

On solving the above equation, we can calculate the co-efficient of numerator polynomial.

$$G_r(s)=k\cdot\frac{\left(1+D_1s+D_2s^2+...+D_{r-1}s^{r-1}\right)}{D_r(s)} \quad (29)$$

Step-4: Calculate the cumulative error index (J) for initial reduced model

Consider, the transfer function of higher order (n^{th}) as,

$$G(s)=\frac{a_0+a_1s+a_2s^2+...+a_{n-1}s^{n-1}}{b_0+b_1s+b_2s^2+...+b_{n-1}s^{n-1}+b_ns^n}$$

The general form of the transfer function of a second order system in the *s*-domain can be represented as,

$$G_{ri}(s)=\frac{T_1+T_2S}{s^2+2\zeta\omega_nS+\omega_n{}^2} \quad (30)$$

Where, ζ is the damping ratio and ω_n is the undamped natural frequency of oscillation in rad/sec. The values of T_1 and T_2 corresponding to Equation (30) can be computed as $T_1 = T_g$ and $T_2=S_g/\omega_n^2$. Where, the transient gain (T_g) and Steady state gain (S_g) are computed as,

$$\mathrm{Tg}=\frac{a_{n-1}}{b_n}\ \text{and}\ \ \mathrm{Sg}=\frac{a_0}{b_0}$$

By using proposed scenario-1, the reduced model obtained in step-2/step-3 is modified in to an initial form as,

$$G_{ri}(s)=\frac{\frac{d_0}{e_2}+\frac{d_1}{e_2}s}{\frac{e_0}{e_2}+\frac{e_1}{e_2}s+s^2}=\frac{A_0+A_1s}{B_0+B_1s+s^2} \quad (31)$$

where,

$$A_0=\frac{d_0}{e_2}=T_1,\ A_1=\frac{d_1}{e_2}=T_2,\ B_0=\frac{e_0}{e_2}\ and\ \ B_1=\frac{e_1}{e_2}$$

The unit step time response of the initial second order approximant $G_{ri}(s)$ is analyzed with a computer program and its characteristics are noted. The cumulative error index *J* using the Integral square error of the unit step time responses of the given higher order system *G(s)* represented by Equation (3) and the initial second order approximant $G_{ri}(s)$ represented by Equation (4) is calculated. The cumulative error index *J* is calculated using the formula,

$$\mathrm{J} = \sum_{t=0}^{N}[y(t) - y_r(t)]^2$$

(Or)

$$J = \int_0^{t_\infty}[y(t) - y_r(t)]^2\,dt \qquad (32)$$

Where, *y(t) is* the output response of the higher order system at the N^{th} instant of time, $y_r(t)$ is the output response of the second order model at the N^{th} instant of time and *N* is the time interval in seconds over which the error index is computed.

Step-5: Adjustment of reduced model coefficients using GA

The coefficients A_0, B_0 and B_1 are adjusted based the cumulative error index (J) using Genetic Algorithm (GA). The resultant reduced order model will closely matches with the corresponding higher order model. If the cumulative error produced by reduced order model obtained as in step-4 is larger than the same as in step-2/step-3, then reduced model obtained in step-2/step-3 will be treated as a resultant reduced model.

4. GENETIC ALGORITHM

A genetic algorithm (or GA for short) is a programming technique that mimics biological evolution as a problem-solving strategy. Given a specific problem to solve, the input to the GA is a set of potential solutions to that problem, encoded in some fashion, and a metric called a fitness function that allows each candidate to be quantitatively evaluated. This is a method for moving from one population of "chromosomes" (e.g., strings of ones and zeros, or "bits") to a new population by using a kind of "natural selection" together with the genetics inspired operators of crossover, mutation, and inversion. Each chromosome consists of "genes" (e.g., bits), each gene being an instance of a particular "allele" (e.g., 0 or 1). The selection operator chooses those chromosomes in the population that will be allowed to reproduce, and on average the fitter chromosomes produce more offspring than the less fit ones. Crossover exchanges subparts of two chromosomes, roughly mimicking biological recombination between two single chromosome ("haploid") organisms; mutation randomly changes the allele values of some locations in the chromosome.

Genetic algorithms are now widely applied in science and engineering as adaptive algorithms for solving practical problems. Certain classes of problem are particularly suited to being tackled using a GA based approach.

5. ILLUSTRATION

Consider a sixth order two input two output system described by the transfer function matrix [24]

$$[G(s)] = \begin{bmatrix} \frac{2(s+5)}{(s+1)(s+10)} & \frac{(s+4)}{(s+2)(s+5)} \\ \frac{(s+10)}{(s+1)(s+20)} & \frac{(s+6)}{(s+2)(s+3)} \end{bmatrix} \qquad (33)$$

$$= \frac{1}{D(s)}\begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix} \qquad (34)$$

Where,

D(s) is the common denominator of given MIMO system and is given by,

$$D(s) = (s+1)(s+2)(s+3)(s+5)(s+10)(s+20)$$

$$D(s) = \mathrm{s}^6 + 41\mathrm{s}^5 + 571\mathrm{s}^4 + 3491\mathrm{s}^3 + 10060\mathrm{s}^2 + 13100\mathrm{s} + 6000 \qquad (35)$$

and

$a_{11}(s) = 2s^5+70s^4+762s^3+3610s^2+7700s+6000$ (36)

$a_{12}(s) = s^5+38s^4+459s^3+2182s^2+4160s+2400$ (37)

$a_{21}(s) = s^5+30s^4+331s^3+1650s^2+3700s+3000$ (38)

$a_{22}(s) = s^5+42s^4+601s^3+3660s^2+9100s+6000$ (39)

By applying the proposed scenario, the corresponding reduced model is obtained as,

$$[G_r(s)] = \begin{bmatrix} R_{11}(s) & R_{12}(s) \\ R_{21}(s) & R_{22}(s) \end{bmatrix}$$

$$[G_r(s)] = \begin{bmatrix} \frac{2s+10}{s^2+11s+10} & \frac{s+4}{s^2+7s+10} \\ \frac{2s+10}{s^2+21s+20} & \frac{s+6}{s^2+5s+6} \end{bmatrix} \qquad (40)$$

The step response of corresponding reduced order systems is shown in the Figure.1 (a)-(d). The unit step time response of $[G_r(s)]$ is analyzed with a computer program and the time domain characteristic parameters were noted down in the respective tables from Table-I-IV. Where, initial approximation model and model parameters tuned using genetic algorithm were tabulated. The results listed in Table-I-IV shows the validity of the proposed scenario in the approximation process. The reduced order parameter adjustment using genetic algorithm over number of iterations for the $G_{21}(s)$ is shown in Figure.2. The Table-V gives the comparison of proposed scenario with some of the existing methods based in Integral Squared Error value.

6. CONCLUSION

The proposed model reduction method uses the pade approximation technique in its procedure and gave the stable reduced order models for linear time invariant continuous MIMO dynamic systems. The algorithm has also been extended to the design of compensators, sub-optimal controllers for continuous and discrete systems, IIR filter. The algorithm is simple, rugged and computer oriented. The matching of step response is assured reasonably well in this method with the help of Genetic Algorithm. The algorithm preserves more stability and avoids any error in between the initial or final values of the responses of original and reduced order models.

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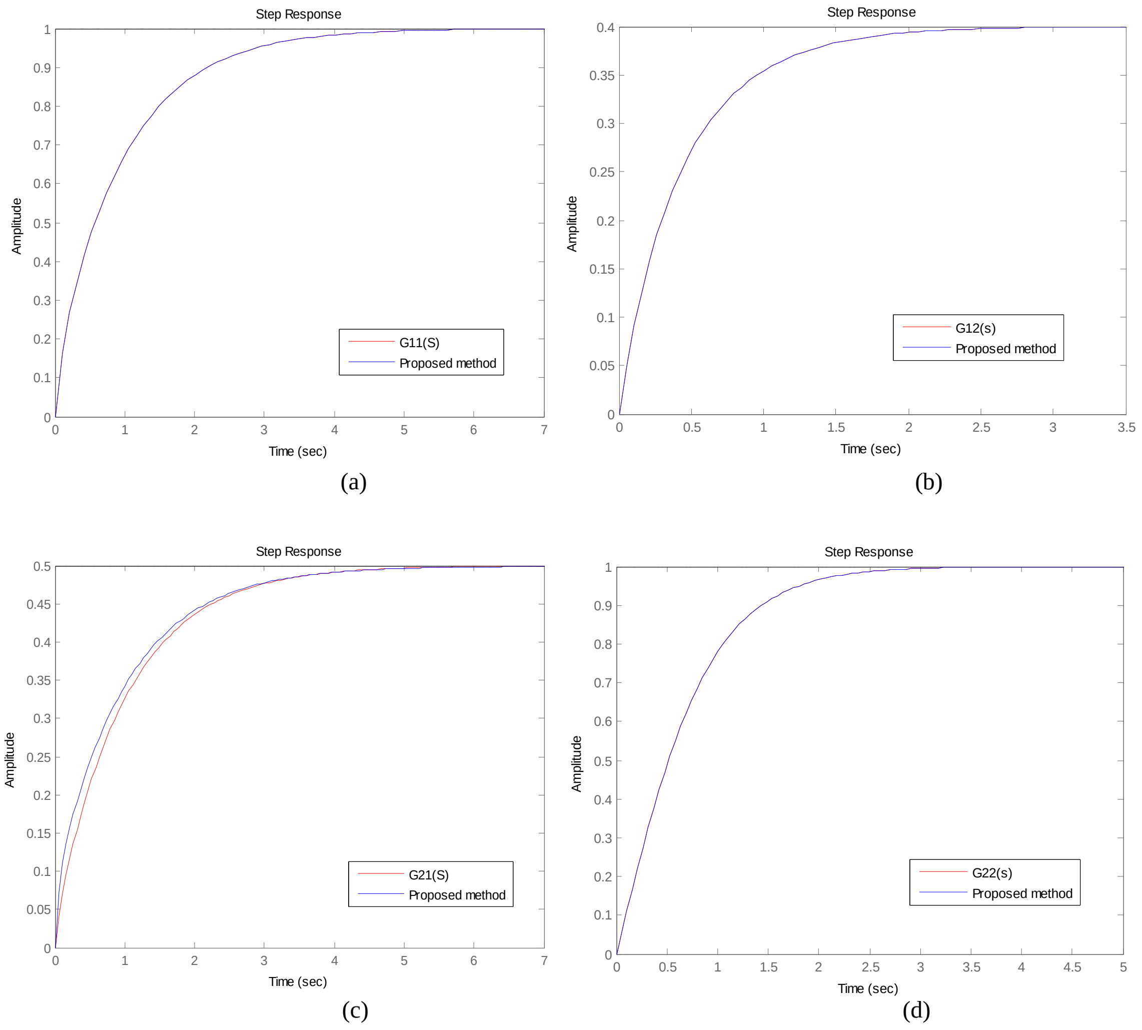

Fig 1: (a) Unit step time response for $G_{11}(s)$ (b) Unit step time response for $G_{12}(s)$ (c) Unit step time response for $G_{21}(s)$ (d) Unit step time response for $G_{22}(s)$

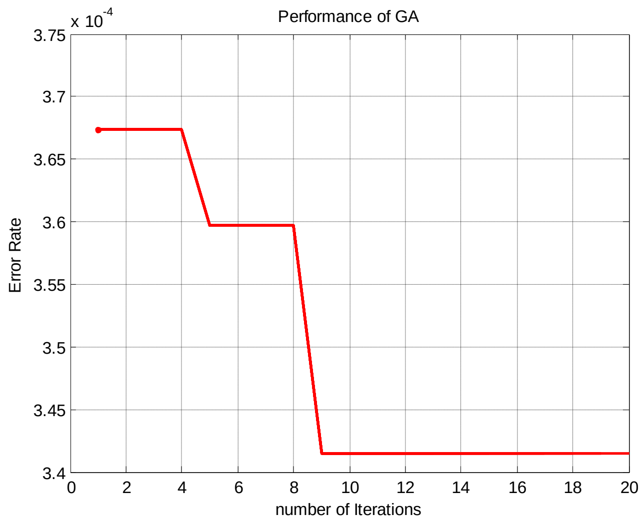

Fig 2: Approximation using GA for $G_{21}(s)$

Table 1. Stages of Proposed Method for $G_{11}(S)$

System characteristics	Higher order system	Initial second order approximant	Second order model from GA based approach
Damping Ratio (ξ)	-	1.739	1.739
Undamped natural frequency of oscillation (ω_n)	-	3.162	3.162
Transient gain (T_g)	2	2	2
Steady state gain (S_g)	1	1	1
Rise time (T_r)	2.12	2.12	2.12
Peak amplitude (P_a)	0.9999	1	1
Settling time (T_s)	3.8	3.8	3.8
Transfer function $R_{11}(s)$	-	$\frac{2s+10}{s^2+11s+10}$	$\frac{2s+10}{s^2+11s+10}$
Cumulative error index (J)	-	2.3099×10^{-29}	2.3099×10^{-29}

Table 2. Stages of Proposed Method for $G_{12}(S)$

System characteristics	Higher order system	Initial second order approximant	Second order model from GA based approach
Damping Ratio (ξ)	-	1.107	1.107
Undamped natural frequency of	-	3.162	3.162
Transient gain (T_g)	1	1	1
Steady state gain (S_g)	0.4	0.4	0.4
Rise time (T_r)	1.02	1.02	1.02
Peak amplitude (P_a)	0.4	0.4	0.4
Settling time (T_s)	1.87	1.87	1.87
Transfer function $R_{12}(s)$	-	$\frac{s+4}{s^2+7s+10}$	$\frac{s+4}{s^2+7s+10}$
Cumulative error index (J)	-	1.3867×10^{-31}	1.3867×10^{-31}

Table 3. Stages of Proposed Method for $G_{21}(S)$

System characteristics	Higher order system	Initial second order approximant	Second order model from GA based approach
Damping Ratio (ξ)	-	2.348	2.365
Undamped natural frequency of	-	4.472	4.461
Transient gain (T_g)	1	1	1
Steady state gain (S_g)	0.5	0.5	0.5
Rise time (T_r)	2.18	2.09	2.12
Peak amplitude (P_a)	0.499	0.5	0.499
Settling time (T_s)	3.86	3.74	3.78
Transfer function $R_{21}(s)$	-	$\frac{2s+10}{s^2+21s+20}$	$\frac{2s+9.9356}{s^2+21.0974s+19.9034}$
Cumulative error index (J)	-	4.3357×10^{-4}	2.9998×10^{-5}

Table 4. Stages of Proposed Method for $G_{22}(S)$

System characteristics	Higher order system	Initial second order approximant	Second order model from GA based approach
Damping Ratio (ξ)	-	1.02	1.02
Undamped natural frequency of	-	2.45	2.45
Transient gain (T_g)	1	1	1
Steady state gain (S_g)	1	1	1
Rise time (T_r)	1.34	1.34	1.34
Peak amplitude (P_a)	1	1	1
Settling time (T_s)	2.28	2.28	2.28
Transfer function $R_{22}(s)$	-	$\frac{s+6}{s^2+5s+6}$	$\frac{s+6}{s^2+5s+6}$
Cumulative error index (J)	-	$9.5773x10^{-30}$	$9.5773x10^{-30}$

Table 5. Comparison of Integral Square Error

Model reduction method	Cumulative error index *(J)* for 10 s			
	$G_{11}(s)$	$G_{12}(s)$	$G_{21}(s)$	$G_{22}(s)$
Proposed Method	$2.3099x10^{-29}$	$1.3867x10^{-31}$	$2.9998x10^{-5}$	$9.5773x10^{-30}$
Swadhin Ku. Mishra et.al.[30]	$4.0656x10^{-4}$	$7.772x10^{-5}$	$3.2448x10^{-5}$	0.0068
C.B.Vishwakarma[24]	0.001515	7.845×10^{-5}	0.000299	0.004681
S. N. Sivanandam et al. [25]	0.0067	0.0020	0.0028	0.0235
Parmar et al. [26]	0.014498	0.008744	0.002538	0.015741
Prasad and Pal [27]	0.136484	0.002446	0.040291	0.067902
Prasad et al. [28]	0.030689	0.000256	0.261963	0.021683
Safonov and Chiang [29]	0.590617	0.037129	0.007328	1.066123
Prasad et al.[31]	0.1676	0.0955	0.0307	0.1970
Prasad [32]	0.2301	0.0887	0.0468	0.2114