Viscoelastic Fluid Flow past an Infinite Vertical Plate with Heat Dissipation

I. J. Uwanta
Department of Mathematics,
Usmanu Danfodiyo University,
Sokoto, Nigeria

B. Y. Isah
Division of General Studies,
S.S.C.O.E. Sokoto. Nigeria

M.O Ibrahim
Department of Mathematics,
Usmanu Danfodiyo University,
Sokoto, Nigeria

ABSTRACT
In this paper the effect of Viscoelastic fluid flow past an infinite plate with heat dissipation is investigated. The dimensionless governing equations are solved using perturbation technique. The analytical results for velocity and temperature and there different parameters such as Prandtl number, Eckert number and Viscoelastic parameter number are studied. It is observed that the velocity decreases with increasing material parameters Pr, K_o and Ec. While temperature increases with increasing material parameters Pr and Ec. Only the increase of Viscoelastic parameter K_o decreases the temperature. The skin friction and Nusselt number tends to increase with the increase of material parameter in both tables while the Nusselt number decreases with only increase in Eckert number.

Keywords: heat dissipation, Viscoelastic, infinite plate

1. INTRODUCTION
Of all the fluid Properties, viscosity requires the greatest consideration in the study of fluid flow. Loundin Prandtl in 20th century gives a new dimension to fluid mechanic by introducing viscosity and thus unifying hydraulics and theoretical hydrodynamics.
In many of the studies carried on hydromagnetic flow of a radiating gas inside a vertical channel or the boundary layer problem the effect of this viscous dissipation term in an unsteady state was often neglected. From practical point of view the effect of this heat dissipation function cannot be ignored because of its important in many flow problems. It serves as a source of 'Geodynamic' heating and of the temperature rise that occurs in the lubricant in bearings.

Also in most work, fluid properties (such as viscous and thermal conductivity) are in most cases considered constant when the flow of Radiative fluid is considered. In practical point of view the effect of viscous dissipation upon the temperature distribution is relatively small for many of the low – velocity Process. However, in process involving a dynamic temperature which is comparable to the imposed heat transfer temperature difference, the effect of viscous dissipation may not be ignored. Boundary layer theory has been used to analyze the effect of viscous dissipation for both compressible and incompressible flows. The study of visco-elastic fluids had become of increasing importance in the last few years. Qualitative analyses of these studies have significant bearing on several industrial applications such as polymer sheet extrusion from a dye, drawing of plastic films etc. When the manufacturing process at high temperature need cooling the stretching sheet, the flows may need visco-elastic fluids to produce a good effect to reduce the temperature from the sheet. And also, the fluids have processed many types of effects (i.e. magnetic force, buoyancy and mass diffusion) into the Problem, and have become a hybrid system need to analysis by many different ways. Hsiao (2010) performed a study on heat and mass transfer of a steady laminar boundary-layer flow of a viscous flow past a nonlinearly stretching sheet. Fonsho (2004) identified the effect of viscous dissipation functions with a view to assessing their global contribution to velocity, temperature and magnetic flow distributions of the fluid flow. Sen (1977) studied the behaviours of unsteady free convective flow of an elastics-viscous fluid past an infinite, porous plate with constant suction. It is assumed that the plate temperature oscillates in magnitude about a constant mean but not in direction. Free convective flow of a viscous incompressible flow past an infinite vertical oscillating plate with variable temperature and uniform mass diffusion is studied by Muthucumaraswany (2010). Ahmad (2010) carried out a study on MHD and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in viscous, compressible and electrically conduction fluid over a semi-finite vertical porous plate in a slip-flow region. Ganesan and Luganathen (2002) presented numerical solution of a transient natural convection flow of an incompressible viscous fluid past an impulsively started semi-infinite isothermal plate with mass diffusion. Ghaly and Seddeek (2004) analyzed the effect of variable viscosity; chemical reaction, heat and mass transfer on Laminar flow along a semi-infinite horizontal plate. Muthucumaraswamy and Ganesan (2002) studied numerically the transient incompressible viscous fluid flow regime past a Semi-Infinite Isothermal plate under the conditions of natural convection. Mululani (2000) studied the Laminar natural convection flow over a semi-infinite vertical plate at constant species concentration. They found that in the absence of chemical reaction, a similarity transfer is possible, when chemical reaction occurs, perturbation expansions about an additional similarity variable dependent on reaction rate most be employed. The study of unsteady hydro magnetic free convection flow of viscous incompressible and electrically conducting fluid pass an infinite vertical porous plate in the Presence if constant suction and heat absorbing Sinks has been study by Sahoo *et al.* (2003). Anwar and Ghosh (2009) studied the steady and unsteady magneto hydrodynamic (MHD) viscous, incompressible free and forced convective flow of an electrically – conducting, Newtonian fluid in the Presence of appreciable thermal radiation heat transfer and surface temperature oscillation.

Soundalgekar (1965) has studied analytically the magneto hydrodynamic viscous flow due to uniformly accelerated motion of an infinite flat plate in the presence of a magnetic field fixed relative to the plate sharing that the velocity at any point and at any instant decreases with a rise in magnetic field strength. Modether *et al.* (2001) present the solution of problem of heat and mass transfer of an oscillatory 2-dimensional viscous. Seddeek and Almushigeh (2010) investigate the effect of radiation, chemical reaction and variable viscosity on hydro magnetic heat and mass transfer in the presence of magnetic field.

Sunil *et al.* (2005) have shown that when effect of variable viscosity are taken into consideration, the critical Rayleigh number for the onset of convection is substantially reduced from the classical value, although the associated wave number is nearly the same. Seddeek *et al.* (2010) has studied the effect of chemical reaction and variable viscosity on hydro magnetic mixed convection heat and mass transfer for Hiemenz flow through porous media with radiation. Pantokratoras (2004) has examined the variable viscosity on flow and heat transfer to a continuous moving flat plate. Suneeth *et al.* (2008) analyzed the thermal radiation effects on hydro magnetic free convection flow past on impulsively started vertical plate with variable surface temperature and concentration, taking into account the heat due to viscous dissipation.

Viscous mechanical dissipation effects are very important in geophysical flows and also in certain industrial operation and are usually characterized by the Eckert Number. Mahajan and Gebhart (1989) reported the influence of viscous heating dissipation in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameters. The influence of viscous dissipation and radiation on an unsteady MHD free convection flow past an infinite heated vertical plate in porous medium with time dependent suction was studies by Cookey *et al.* (2008). Zueco (2007) used network simulation method (NSM) to study the effects of viscous dissipation and radiation on unsteady MHD free convection flow past a vertical porous plate. Panda *et al.* (2003) here analyzed the unsteady free convective flow and mass transfer of a rotating elastico viscous liquid through porous media past vertical porous plate. Soundalgekar *et al.* (1999) here analysed the transient free convection flow of a viscous dissipation fluid past a semi-infinite vertical plate. Chaudhary and Jain (2007) studied the behaviours of unsteady hydro magnetic flow of a Viscoelastic fluid from a Radiative vertical porous plate. Brinkman (1947) estimated the viscous force imparted by a flowing fluid in a dense swarm of particles. Recently Cortell (2007) investigated theoretically the effect of viscous dissipation as well as radiation on the thermal boundary layer flows over non-linearly stretching sheets considering prescribed surface temperature and heat flux. Raptis and Perdikis (2006) studied numerically the steady two dimensional flow of an incompressible viscous and electrically conducting fluid over a non-linear semi-infinite stretching in the Presence of a chemical reaction and under the influence of magnetic field. Rajagopal *et al.* (1983) studied a Falkner-Skan flow field of a second-grade visco-elastic fluid.Shit and Haldar (2010) investigated the effects of thermal radiation and Hall current on magneto hydrodynamic free-convective flow and mass transfer over a stretching sheet with variable viscosity in the Presence of heat generation/absorption. Rapits and Perdikis (1998) and Rapits (1998) studied respectively the flow of a visco-elastic fluid and micropolar fluid past a stretching sheet in the Presence of thermal radiation. Mukhopadhaya *et al.* (2005) investigated the Problem of MHD boundary-layer flow over a heated stretching sheet with variable viscosity. However; Salem (2007) investigated the effect of variable viscosity on MHD Viscoelastic fluid flow and heat transfer over a stretching sheet without considering thermal radiation effect. Shit and Haldar (2009) carried out the study of the effect of thermal radiation on MHD Viscoelastic fluid flow past a stretching surface with variable viscosity. Seddeek *et al.* (2007) analyzed the effects of chemical reaction, radiation and variable viscosity on hydromagnetic mixed convection heat and mass transfer. Nasrin and Alim (2009) studied the effects of temperature dependent viscosity and thermal conductivity on the coupling of conduction and Joule heating with MHD free convection flow along a semi-infinite vertical flat plate has been analyzed. Mamun *et al.* (2007) considered combined effect of conduction and viscous dissipation on MHD free convection flow along vertical flat plate. Alim *et al.* (2008) studied the combined effect of viscous dissipation and Joule heating on the coupling of conduction and free convection along a vertical flat plate. Molla *et al.* (2009) considered the natural convection laminar flow with temperature dependent viscosity and thermal conductivity along a vertical wavy surface. Numerical study on a vertical plate with variable viscosity and thermal conductivity has been reported by Palani and Kim (2010). Rajagopal (1983) investigated the heat transfer in the forced convection flow of a visco-elastic fluid of Walters's model. Chowdhury *et al.* ((2000) studied the MHD free convection flow of visco-elastic fluid past a vertical porous plate.

This present work extends the work of Ramdas Sen (1977) to include viscous dissipation on the natural convection flow of an incompressible fluid flow past an infinite vertical plate**.** The equations were solved by perturbation technique and the solutions were obtained for velocity and temperature. We discuss the effect of the material parameters involved on the flow and compute the skin friction at the wall and rate of heat transfer.

2. PROBLEM FORMULATION

We consider the origin to be taken at any point in the plate as Sen. (1977). The X'-axis is chosen vertically upwards and the Y'-axis perpendicular to it. U' is the velocity in the X'-direction and V', the normal to the plate; t' is the time variable: ξ_0 the limiting viscosity at small rate of shear; and k_0 the elastic constant. f_x is acceleration due to gravity: β the coefficient of expansion; C_P, the specific heat at constant Pressure , λ' the thermal conductivity ; T' the temperature far away from the plate while the normal component is $v' = -v_0$, where v_0 is the constant velocity . The governing equations are given in equation (1 to 4) with boundary condition and assume solution respectively. It is assumed that heat dissipation is taken into consideration while viscous assumed neglected. Velocity follows the exponential increase or decrease of perturbation law. It is also assumed that the plate temperature and suction velocity varies exponentially with time. The governing equations are represented by the continuity equation, momentum and energy equations.

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\rho'\frac{\partial u'}{\partial t'}+\rho'v'\frac{\partial u'}{\partial y'}=\rho' f_x\beta\left(T'-T'_\alpha\right)+\xi_0\frac{\partial^2 u'}{\partial y'^2}$$
$$-k_0\left[\frac{\partial^3 u'}{\partial y'^2\partial t}+v'\frac{\partial^3 u'}{\partial y'^3}-3\frac{\partial^2 v'}{\partial y'^2}-2\frac{\partial v'}{\partial y'}\frac{\partial^2 u'}{\partial y'^2}\right] \tag{2}$$

$$\rho'\frac{\partial v'}{\partial t'}+\rho'v'\frac{\partial v'}{\partial y'}=-\frac{\partial \rho'}{\partial y'}+2\xi_0\frac{\partial^2 v'}{\partial y'^2}$$
$$-2k_0\left(\frac{\partial^3 y'}{\partial y'^2\partial t}+v'\frac{\partial^3 v'}{\partial y'^3}-3\frac{\partial v'}{\partial y'}\frac{\partial^2 v'}{\partial y'^2}\right) \tag{3}$$

$$\rho' C'_p\left(\frac{\partial T}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right)=\lambda'\frac{\partial^2 T'}{\partial y'^2}+\left(\frac{\partial u'}{\partial y'}\right)^2$$
$$-k^*\left(\frac{\partial u'}{\partial y'}\frac{\partial^2 u'}{\partial t'\partial y'}-v'\frac{\partial u'}{\partial y'}\frac{\partial^2 u'}{\partial y'^2}\right) \tag{4}$$

Where λ' is thermal conductivity, k_0 is the elastic parameter, U' is the velocity, t' is the time variable, ξ_0 the limiting viscosity, f_x is acceleration due to gravity, β the coefficient of expansion, C_P the specific heat at constant pressure and T' the temperature far away from the plate.

$$\left.\begin{aligned}&u'=0, T'=T'_{\omega(t)}\ at\ y'=0\\&u'\to 0, T'\to\infty\ as\ y'\to\infty\end{aligned}\right\} \tag{5}$$

To obtain the constant suction velocity v_0 we integrate (1)

$$v'=-v_0 \tag{6}$$

The dimensionless variables and parameters of the problems are

$$\left.\begin{aligned}&y=\frac{y'v_0}{v}, t=\frac{v_0^2 t'}{4v}, u=\frac{u'}{Gv_0}, k=\frac{k^* v_0^2}{v^2},\\&T=\frac{T'-T'_\alpha}{T'_\omega-T'_\alpha}\ G=\frac{v f_x\beta\left(T'_\omega-T'_\alpha\right)}{v_0^3},\\&\rho=\frac{\xi_0 C'_p}{\lambda'}, f_x=\frac{Gv_0^3}{v\beta\left(T'_\omega-T'_\alpha\right)}\end{aligned}\right\} \tag{7}$$

Substituting (6) and (7) into equation (2) to (5). We obtain the following equations

$$\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}-\frac{1}{4}\frac{\partial u}{\partial t}-k_0\left(\frac{1}{4}\frac{\partial^3 u}{\partial y^2\partial t}-\frac{\partial^3 u}{\partial y^3}\right)=-T \tag{8}$$

$$\frac{\partial T}{4\partial t}-\frac{\partial T}{\partial y}=\frac{\partial^2 T}{Pr\partial y^2}+E_c\left(\frac{\partial u}{\partial y}\right)^2-Rk_0\left(\frac{\partial u}{4\partial y}\frac{\partial^2 u}{\partial t\partial y}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right) \tag{9}$$

Subject to the boundary conditions

$$\left.\begin{aligned}&u=0,\ T=T_{\omega(t)} && at\ y=0\\&u\to 0, T\to\infty && as\ y\to\infty\end{aligned}\right\} \tag{10}$$

Where Pr is Praandtl number, Ec is Eckert number, k_0 Viscoelastic parameter, T represents the temperature and R is a constant which is taken as one (1) for convenience throughout the work.

To solve the above system of partial differential equations (8) and (9) subject to boundary conditions (10), let the temperature and the velocity on the neighborhood of the plate Pop (1968) be taken as

$$T\left(y,t\right)=\left(1-Q_1\left(y\right)\right)+\varepsilon e^{i\omega t}\left(1-Q_2\left(y\right)\right) \tag{11}$$

$$u\left(y,t\right)=S_1\left(y\right)+\varepsilon e^{i\omega t}S_2\left(y\right)\Big\} \tag{12}$$

Substitution of (11) and (12) into (9) and (10) we obtain the following ordinary differential equations by collecting and comparing the harmonic terms and neglecting the coefficients higher order of $\varepsilon^{(\cdot)}e^{(\cdot)}$,

$$Q_1''+PrQ_1'=PrE_cS_1'^2-Prk_0RS_1''S_2'' \tag{13}$$

$$Q_2''+PrQ_2'-\frac{i\omega}{4}PrQ_2=-\frac{i\omega Pr}{4}+Pr\left(2E_cS_1'S_2'-\frac{R}{4}S_1'S_2'-k_0RS_1'S_2''-RS_1''S_2''\right) \tag{14}$$

$$k_0S_1'''+S_1''+S_1'=Q_1-1 \tag{15}$$

$$k_0S_2'''+\left(1-\frac{k_0}{4}\right)S_2''+S_2'-S_2=Q_2-1 \tag{16}$$

and the corresponding boundary conditions are

$$\begin{aligned}&Q_1=0=Q,\ S_1=0=S_2 && at\quad y=0\\&Q_1\to 1, Q_2\to 1,\ S_1\to 0, S_2\to 0 && as\quad y\to\infty\end{aligned} \tag{17}$$

To solve equations, we shall use the perturbation technique employed by Soundalgekar (1971) for non-magnetic case. The aim is that when Ec = 0, Equation (3) reduces to Newtonian fluid flow. When the solutions of (13) to (17) are expanded in powers of the small parameter Ec, the zero-order terms will be those of Newtonian case. We assume, therefore, the solution of (13) to (17) to be expanded in power of Ec, where Ec is small. Hence

$$S_1 = S_{11} + E_c S_{11},\ S_2 = S_{20} + E_c S_{22} \tag{18}$$

$$Q_1 = Q_{10} + E_c Q_{11},\ Q_2 = Q_{20} + E_c Q_{22} \tag{19}$$

Collecting and comparing the harmonic terms and neglecting the coefficients of the higher order of $Ec^{(\cdot)}$ in (13) to (17), we obtain the following

$$Q_{10} + PrQ'_{10} = -Prk_0 RS'_{10}S''_{10} \tag{20}$$

$$Q''_{11} + pQ'_{11} = PrS'^2_{10} - Prk_0 RS'_{10}S''_{11} - Prk_0 RS''_{10}S'_{11} \tag{21}$$

$$Q''_{20} + PrQ'_{20} - \frac{i\omega Pr}{4}Q_{20} = -\frac{i\omega Pr}{4} - Prk_0 R\left(\frac{S'_{10}S'_{20}}{4} + S'_{10}S''_{20} + S''_{20}S''_{10}\right) \tag{22}$$

$$Q''_{22} + \Pr Q'_{22} - \frac{iPr}{4}Q_{22} = 2PrS'_{10}S'_{20} - Prk_0 R(\frac{S'_{10}S'_{22}}{4} + S'_{11}S'_{22} + S'_{10}S''_{22} + S''_{11}S''_{20} + S''_{10}S''_{22} + S''_{11}S''_{20}) \tag{23}$$

$$k_0 S'''_{10} + S''_{10} + S'_{10} = Q_{10} - 1 \tag{24}$$

$$k_0 S'''_{11} + S''_{11} + S'_{11} = Q_{11} \tag{25}$$

$$k_0 S'''_{20} + \left(1 - \frac{k_0}{4}\right)S''_{20} + S'_{20} - S_{20} = Q_{20} - 1 \tag{26}$$

$$k_0 S'''_{22} + \left(1 - \frac{k_0}{4}\right)S''_{22} + S'_{22} - S_{22} = Q_{22} \tag{27}$$

The boundary conditions are

$$\left.\begin{aligned} &S_{10} = S_{11} = S_{20} = S_{22} = 0, \\ &Q_{10} = Q_{11} = Q_{20} = Q_{22} = 0 \qquad at\ y = 0 \\ &S_{10} \to 0, S_{11} \to 0, S_{20} \to 0, S_{22} \to 0, \\ &Q_{10} \to 1, Q_{11} \to 0, Q_{20} \to 1, Q_{22} \to 0\ as\ y \to \infty \end{aligned}\right\} \tag{28}$$

Next we further reduce the ordinary differential equation using the following assume solution Soundalgekar, (1971) by expanding on power of k_0, where k_0 is small as follows.

$$\left.\begin{aligned} &Q_{10} = L_0 + k_0 L_1, S_{10} = h_0 + k_0 h_1, Q_{11} = L_2 + k_0 L_3, \\ &S_{11} = h_2 + k_0 h_3, Q_{20} = b_0 + k_0 b_1, \\ &S_{02} = C_0 + k_0 C_1, Q_{22} = b_2 + k_0 b_3, \\ &S_{22} = C_2 + k_0 C_3 \end{aligned}\right\} \tag{29}$$

Using (28) and (29) in equations (20), (21), (22), (23), (24), (25), (26), (27),with boundary condition (28) and (29), by collecting and comparing the harmonic terms and neglecting the coefficient the higher order of $k^{(\cdot)}$, we obtain the following;

$$L''_0 + PrL'_0 = 0 \tag{30}$$

$$L''_1 + PrL'_1 = -PrRh'_0 h''_0 \tag{31}$$

$$h''_0 + h'_0 = L_0 - 1 \tag{32}$$

$$h''_1 + h'_1 = L_1 - h'''_0 \tag{33}$$

$$L''_2 + PrL'_2 = Prh'^2_0 \tag{34}$$

$$L''_3 + PrL'_3 = 2Prh'_0 h'_1 - PrRh'_0 h'_2 - PrRh''_0 h'_2 \tag{36}$$

$$h''_2 + h'_2 = L_2 \tag{37}$$

$$b''_0 + Prb'_0 - \frac{i\omega Pr}{4}b_0 = -\frac{i\omega Pr}{4} \tag{38}$$

(24)

$$b''_1 + Prb'_1 - \frac{i\omega Pr}{4}b_1 = -\frac{PrR}{4}h'_0 C'_0 - PrRh'_0 C''_0 - PrRC''_0 h''_0 \tag{39}$$

(26)

$$C''_0 + C'_0 - C_0 = b_0 - 1 \tag{40}$$

(27)

$$C'''_0 + C''_1 - \frac{C''_0}{4} + C'_1 - C_1 = b_1 \tag{41}$$

$$b''_2 + Prb'_2 - \frac{i\omega}{4}Prb_2 = 2Prh'_0 C'_0 \tag{42}$$

$$b''_3 + Prb'_3 - \frac{i\omega Pr}{4}b_3 = 2Prh'_0 C'_1 + 2Prh'_1 C'_0 - \frac{PrR}{4}h'_0 C'_2 - PrRh'_0 C''_2 - PrRh'_2 C''_0 - PrRh''_0 C_2 - PrRh''_2 C''_0 \tag{43}$$

$$C_2'' + C_2' - C_2 = b_2 \quad (44)$$

$$(47)\; C_2''' + C_3'' - \frac{C_2''}{4} + C_3' - C_3 = b_3 \quad (45)$$

Subject to the boundary conditions

$$\left.\begin{array}{l} h_0 = h_1 = h_2 = h_3 = C_0 = C_1 = C_2 = C_3 = 0 \; at\, y = 0 \\ h_0, h_1, h_2, h_3, C_0, C_1, C_2 \;\; and, C_4 \to 0 \;\; as\, y \to \infty \end{array}\right\} \quad (46)$$

and

$$\left.\begin{array}{l} L_0 = L_1 = L_2 = L_3 = b_0 = b_1 = b_2 = b_3 = 0 \; at\, y = 0 \\ \left.\begin{array}{l} L_0 \to 1, L_1 \to 0, L_2 \to 0, L_3 \to 0, b_0 \to 1, \\ b_1 \to 2, b_2 \to 0, b_3 \to 0 \end{array}\right\} \quad as\, y \to \infty \end{array}\right\} \quad (47)$$

The velocity and temperature profiles are as follows

$$\begin{aligned} U(y,t) &= G_1 e^{-y} + G_2 y e^{-y} + G_3 e^{-Pry} + G_4 e^{-(Pr+1)y} \\ &+ G_5 e^{-(Pr+1)y} + G_6 e^{-2Pry} + G_7 e^{-3y} + G_8 e^{-2y} \\ &+ G_9 y e^{-2y} + G_{10} e^{-(Pr+2)y} + G_{11} e^{-(2Pr+1)y} \\ &+ G_{10} e^{-3Pry} + G_{13} e^{-2Pry} + [G_{14} e^{-ny} + G_{15} y e^{-ny} \\ &+ G_{16} e^{-Przy} + G_{17} e^{-(Prz+1)y} + G_{18} e^{-(Pr+n)y} \\ &+ G_{19} e^{-(n+1)y} + G_{20} y e^{-(n+1)y} \\ &+ [G_{14} e^{-ny} + G_{15} y e^{-ny} + G_{16} e^{-Przy} + G_{17} e^{-(Prz+1)y} \\ &+ G_{18} e^{-(Pr+n)y} + G_{19} e^{-(n+1)y} + G_{20} y e^{-(n+1)y} \\ & G_{28} e^{-(Prz+2)y} + G_{29} y e^{-(Prz+1)y} + G_{35} e^{-(n+2)y}]\varepsilon e^{i\omega t} \end{aligned}$$
(48)

$$\begin{aligned} T(y,t) &= 1 - H_1 e^{-Pry} - H_2 e^{-(Pr+1)y} - H_3 y e^{-(Pr+1)y} \\ &- H_4 e^{-2Pry} - H_5 e^{-2y} - H_6 y e^{-2y} - H_7 e^{-(2Pr+1)y} \\ &- H_8 e^{-(Pr+2)y} - H_9 e^{-3Pry} - H_{10} e^{-3y} - 1 + \\ &\Big[1 - H_{11} e^{-\Pr y} - H_{12} e^{-(Prz+Pr)y} - H_{13} e^{-(Prz+1)y} - H_{14} e^{-(Pr+n)y} \\ &- H_{15} e^{-(n+1)y} - H_{16} e^{-(Pr+n)y} - H_{17} e^{-(Prz+1)y} - H_{18} y e^{-(n+1)y} \\ &- H_{19} e^{-(Prz+2Pr)y} - H_{20} e^{-(Prz+Pr+1)y} - H_{21} e^{-(2Pr+1)y} \\ &- H_{22} e^{-(Pr+n+1)y} - H_{23} e^{-(\Pr z+2)y} + H_{24} e^{-(n+2)y} - 1\Big] e^{i\omega t} \end{aligned} \quad (49)$$

Where

$$z = \frac{Pr}{2}\left(1 + \sqrt{1 + i\frac{\omega}{Pr}}\right) \qquad and\; n = \frac{1}{2}\left(1 + \sqrt{5}\right)$$

The skin friction expression is;

$$\begin{aligned} U(y,t) &= -G_1 + G_2 - PrG_3 - (Pr+1)G_4 + G_5 \\ &-2PrG_6 - 3G_7 - 2G_8 + G_9 - (Pr+2)G_{10} \\ &-(2Pr+1)G_{11} - 3GrG_{12} - 2PrG_{13} + \\ &+[-nG_{14} + G_{15} - PrzG_{16} e^{-Przy} - (Prz+1)G_{17} \\ &-(Pr+n)G_{18} - (n+1)G_{19} + G_{20} - \\ &(Prz+Pr)G_{21} + G_{23} - (Prz+2Pr)G_{24} \\ &-(Prz+Pr+1)G_{25} - (2Pr+n)G_{26} \\ &-(Pr+n+1)G_{27}(Prz+2)G_{28} \\ &+G_{29} - (n+2)G_{35}]\varepsilon e^{i\omega t} \end{aligned} \quad (50)$$

The constant $G_1 \cdots, H_1 \cdots H_{24}$ are constants too long to present

While the Nusselt number is expressed as;

$$\begin{aligned} T(y,t)\Big|_{y=0} &= pH_1 + (p+1)H_2 - H_3 + 2pH_4 + 2H_5 \\ &-H_6 + (2p+1)H_7 + (p+2)H_8 + 3pH_9 + 3H_{10} \\ &+ \begin{bmatrix} pzH_{11} + (pz+p)H_{12} + (pz+1)H_{13} \\ +(p+n)H_{14} + (n+1)H_{15} - H_{16} - H_{17} \\ -H_{18} + (pz+2p)H_{19} + (pz+p+1)H_{20} \\ +(2p+n)H_{21} + (p+n+1)H_{22} \\ +(pz+2)H_{23} + (n+2)H_{24} \end{bmatrix} \varepsilon e^{i\omega t} \end{aligned} \quad (51)$$

3. RESULTS AND DISCUSSION

For numerical validation of our analytical results, we have taken the real part of the results obtained in equation (48) to (51).

In Figures 1 to 3 the variation of velocity field along y-axis are shown, the effects of Prandtl number (Pr = 0.6, 0.63, 0.66, 0.7), Viscoelastic parameters (k_0 = -0.012, -0.008, -0.004, 0) and Eckert numbers (Ec = 0, 0.012, 0.014, 0.016) were indicated. The velocity decreases with the increase of Prandtl Pr in Figure 1. The varying values of viscous parameter $k_{0,}$ in increase order decreases the velocity as indicated in Figure 2. Increasing Eckert number decreases the velocity as in Figure 3. Figures 4 to 6 shows the temperature profiles. Temperature increases with increase on Prandtl number (Pr = 0.6, 0.71, 0.85) in figure 4. The increasing effect of Eckert number (Ec = 0, 0.12, 0.14, 0.1) also increases the temperature in Figure 5. In Figure 6 the increase of Viscoelastic parameter (k_0 = 0, 0.4, 0.8, 1.2) has an opposing influence on the temperature.

Table 1 depicts the effects of the Prandtl number Pr, Eckert number Ec, and Viscoelastic parameter k_0 on skin friction coefficients parameter τ . It is observed from table 1 that as Ec, Pr and k_0 increases the skin friction increases.

The effects of the Prandtl number Pr, Eckert number Ec, and Viscoelastic parameter k_0, on the Nusselt number Nu are given in Table 2. It is seen from this table that as Ec and k_0 increases, the Nusselt number increase whereas the Nusselt number decrease as Pr increases.

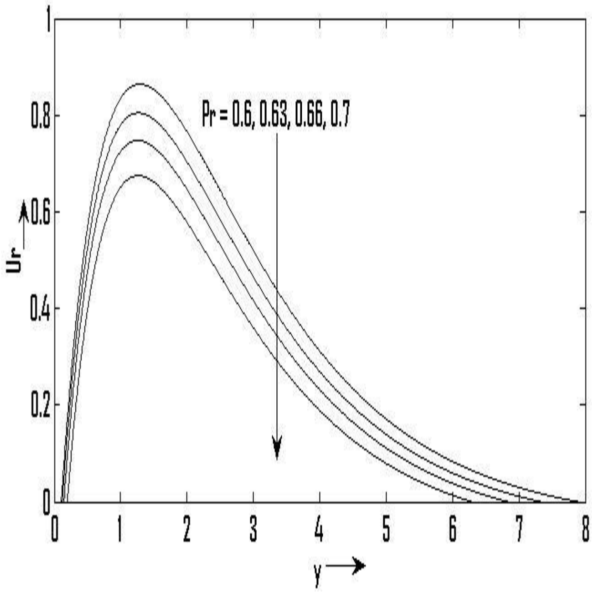

Figure 1; Velocity profile for different values of Prandtl number

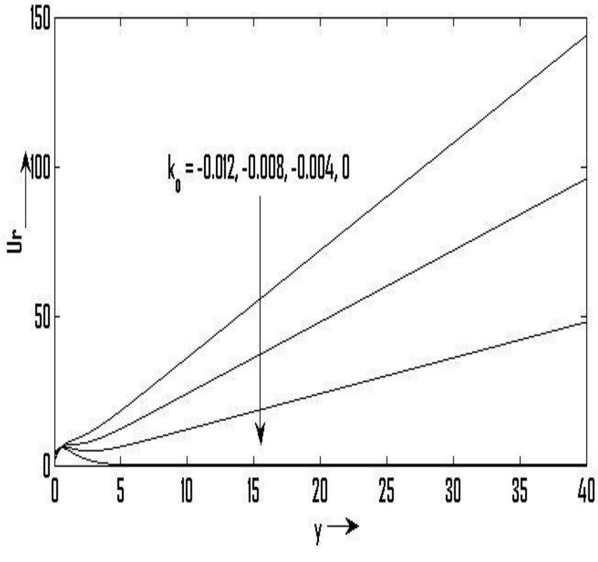

Figure 2; Velocity profile for different values of Viscoelastic parameter.

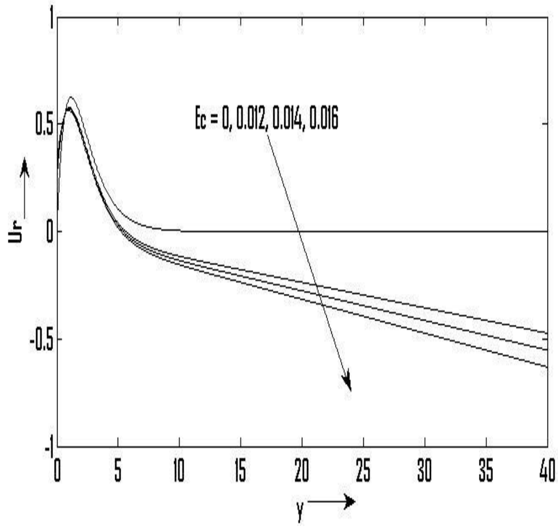

Figure 3; Velocity profile for different values of Eckert number.

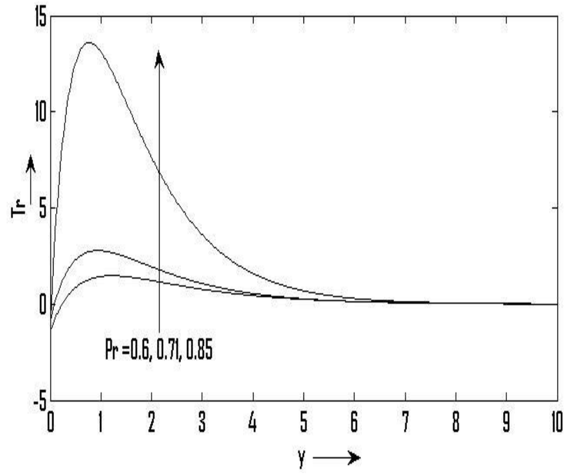

Figure 4; temperature profile for different values of Prandtl number

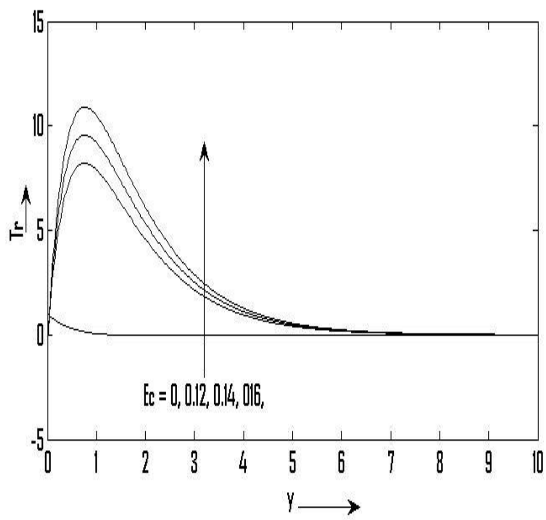

Figure 5; temperature profile for different values of Eckert number

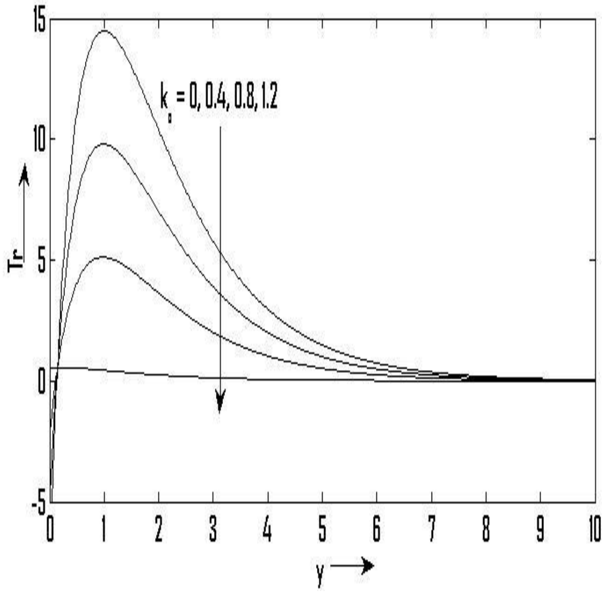

Figure 6; temperature profile for different values of Viscoelastic parameter

Table 1: Values of the Skin friction

S/NO.	Pr	Ec	R	k_0	τ
1	0.71	0.04	1	0.02	4.1138
	0.71	0.08	1	0.02	9.7780
	0.71	0.12	1	0.02	15.4422
2	0.71	0.04	1	0.04	8.9115
	0.71	0.04	1	0.08	18.5069
	0.71	0.04	1	0.12	37.6979
3	0.60	0.04	1	0.02	0.9992
	0.71	0.04	1	0.02	4.1138
	0.85	0.04	1	0.02	37.3304

Table 2: Values of the Nusselt Number

S/NO.	Pr	Ec	R	k_0	Nu
1	0.71	0.04	1	0.02	0.6896
	0.71	0.08	1	0.02	0.4522
	0.71	0.12	1	0.02	0.2147
2	0.71	0.04	1	0.04	0.1663
	0.71	0.04	1	0.08	0.8801
	0.71	0.04	1	0.12	1.9266
3	0.60	0.04	1	0.02	0.6454
	0.71	0.04	1	0.02	0.6896
	0.85	0.04	1	0.02	0.8492

4.CONCLUSION

This paper studied Viscoelastic fluid flow past an infinite vertical plate with heat dissipation. The dimensionless governing equations are solved using perturbation technique. The effect of different parameters such as the thermal Prandtl number, Viscoelastic parameter and Eckert number are studied. The conclusions of the study are as follows.

- ❖ The increase of material parameters result to the decrease of the stream wise velocity.
- ❖ The temperature field increases with the increase of the material parameters.
- ❖ Only the increase of Viscoelastic parameter k_0 decreases the temperature.
- ❖ The skin friction and Nusselt number tends to increase with the increase of material parameter in both Table 1 and 2 while the Nusselt number decreases with only increase in Eckert number on table 2.

5. REFERENCES

[1] Ahmad S. (2010), Influence of Chemical reaction on Transient MHD Free Convection flow over vertical plate in slip-flow regime". Emirate Journal for engineering Research, IS (i), Pp.25 – 34.

[2] Anwar, O. B. and Ghosh. S.K.P. (2009). "Analytical Study of Magnetohydrodynamic Radiation Convection with surface temperature oscillation and secondary flow effects". International Journal of Applied Mathematics and Machinery, Vol. 6(6), Pp. 1 – 22.

[3] Alim, M. A., Alam, Md. M., Mamun, A. A. and Hossain, Md. B. (2008). "Combined Effect of Viscous Dissipation & Joule Heating on the Coupling of Conduction & Free Convection along a Vertical Flat Plate". Int.Commun. in Heat & Mass Transfer, Vol. 35, No. 3, pp. 338-346.

[4] Brinkman, H. C. (1947). "A calculation of Viscous Force Extended by Flowing Fluid in a Sense Swarm of particles". Applied Sciences Research A (1), Pp. 27 – 34.

[5] Chaudhary, R. C. and Jain, P. (2007). "Hall Effect on MHD Mixed Convection Flow of a viscous elastic fluid past an infinite vertical porous plate with mass transfer and Radiation". UUR J. Phys. 2007 Vol. 52(10), Pp.110-127

[6] Cortell, (2007). "Viscous flow and heat transfer over a non-linear stretching sheet". Applied Mathematics Compute, Vol.184 (2), Pp. 864 – 873.

[7] Chowdhury, M. K. and Islam, M. N. (2000). "MHD free convection flow of visco-elastic fluid past an infinite vertical porous plate". Heat and Mass Transfer, Vol.36, Pp.439-447.

[8] Fonsho, A. J. (2004). "The effect of Dissipation function and variable fluid Properties on unsteady MHD flow of a radiating gas inside a vertical thermal". Being a paper Presented at Annual conference of MAN at Usmanu Danfodiyo University Sokoto, 3rd September, 2004.

[9] Ghaly, A. Y. and Seddeek, M. A. (2004). "Chebysheu Finite difference Method for the effect of chemical reaction, heat and mass transfer on laminar flow along a semi-horizontal plate with temperature dependent viscosity". Chas solution and Practical, Vol.19 (1), Pp. 61 – 70.

[10] Ganesan, and Luganathen, P. (2002). "Heat and Mass Flow effect on a Moving vertical plate with chemically reactive species diffusion". Diffusion Journal for Engineering Physics and Theomorphic, Vol. 75(4), Pp. 899 – 909.

[11] Muthucumaraswamy, R. and Ganesan, P. (2002). "Natural Convection on a Moving Isothermal Vertical Plate with Chemical Reaction". Journal for Engineering Physics and Theomorphic, Vol. 75(1), Pp 113 –119.

[12] Madather, M., Rashid, A. M and Chamkha (2001). "An analytical study of MHD heat and mass a vertical permeable plate in a Porous Medium". Turkish Journal of engineering Sciences, Vol. 33(2), Pp. 245 – 257.

[13] Mululani, I. (2000), Convective Diffusive Transport Chemical Reaction in Natural Convection Flows, Theory and Comp. Fluid Dynamics, Vol. 13(5),Pp. 291 – 304.

[14] Mahajan R. L, Gebhart, B. B. (1989). "Viscous Dissipation Effects in Buoyancy induced Flows". Int. Journal Heat Mass Transfer, VOL. 32(7), Pp. 1380 – 1382.

[15] Mukhopadhaya, S. Layek, G. C. and Samad, A. (2005). "Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity". Int. J. Heat Mass Transf. Vol. 48, Pp. 4460-4466.

[16] Molla, MD, Hossain, M.A, Reddy, and Gorla, R. S. (2009). "Natural Convection Laminar Flow with Temperature Dependent Viscosity and Thermal Conductivity along a Vertical Wavy Surface". Int. J. of FluidMech. Research, Vol. 36, pp. 272- 288.

[17] Mamun, A. A., Azim, N. H. and Maleque (2007). "A Combined Effect of Conduction and Viscous Dissipation on MHD Free Convection Flow along a Vertical Flat Plate". Journal of Naval Archit. and Marine Eng., Vol. 4, No. 2, pp. 87-98, 2007.

[18] Palani, G., Kim, Kwang-Yong (2010). "Numerical Study on a Vertical Plate with Variable Viscosity and Thermal Conductivity". Arch Appl. Mech., Vol. 80, No. 7, Pp. 711-725.

[19] Pantokratoras, A. (2004). "Further results on the variable viscosity on flow and heat transfer to a continuous moving flat". Int. J. Engng.Sc., Vol.42, Pp. 1891-1896.

[20] Panda, J. P., Dash, G. C., and Das, S. S. (2003). "Unsteady Free Convective Flow and Mass Transfer of a Rotating Elastics-Viscous Liquid through Porous Media past Vertical Porous plate". AMSEJ. Mod Meas. Cont. B. Vol.72 (3).Pp. 47 – 59.

[21] Pop, I. (1968). "Effect of periodic suction on the unsteady free convection flow past a vertical porous flat plate". Rev. Roum. Sci. Tech., Vol.13, Pp.41-46.

[22] Raptis, A., and Perdikis, C. (2006) "Viscous Flow over a non- linear stretching sheet in the Presence of a chemical reaction and magnetic field". Int. J. Non-Linear Machinery. Vol. 63, Pp.527– 529.

[23] Raptis, A. and Perdikis, C. (1998). "Viscoelastic flow by the presence of radiation". ZAMM. Vol78 Pp. 277-279.

[24] Rajagopal, K. R. (1983). "On Stokes problem for a non-Newtonian fluid". Acta Mechanica, Vol.48, Pp.233-239.

[25] Rajagopal, K.R., Gupta. A.S. and Na, T.Y. (1983) "A note on the Falkner-Skanflows of a non-Newtonian fluid". Int. J. Non-linear Mech., Vol. 35 Pp.313- 320.

[26] Sen, R. (1977). "On Visco-Elastic free convection Boundary Layer flows past an infinite plate with constant suction". Indian journal of pure and applied Mathematics, Vol. 9(3), Pp. 229- 241.

[27] Seddeek, M. A., and Almushigeh AA (2010); Effect of Radiation and Variable Viscosity on MHD Free Convective Flow and Mass Transfer over a Stretching sheet with chemical". Reaction, Application and Applied Mathematics; Vol.5 (1), Pp.181 – 197.

[28]Suneetha, (2008). "Thermal Radiation Effects on MHD Free Convection Flow Past an Impulsively Started vertical plate with variable surface temperature and concentration". Journal of Naval Architecture and Marine Engineering Vol.2,Pp.57 – 70.

[29] Shit, G. C. and Haldar, R. (2010). "Combined effects of thermal radiation and Hall current on MHD free-convective flow and mass transfer over a stretching sheet with variable viscosity". Physics Flu-dyn, ar XIv: 1008.0165v1.

[30] Soundalgekar, V. M., Jaisawal, BS, A. G., Uplekar and Takhar, H. S. (1999). "Transient Free Convection flow of a Viscous dissipative fluid past a semi-infinite vertical

Plate". Journal for Applied Machinery Engineering, Vol. 4, Pp. 203 – 218.

[31] Shit, G.C., and Haldar R. (2009). "Effect of thermal radiation on MHD Viscoelastic fluid flow over a stretching sheet with variable viscosity". Communicated for publication in Int. J. Fluid Mech.

[32] Salem, A. M. (2007). "Variable viscosity and thermal conductivity effects on MHD flow and heat transfer in Viscoelastic fluid over a stretching sheet". Physics Letters A Vol.369 Pp. 315-322.

[33] Seddeek, M. A., Darwish, A. A, Abdelmeguid, M .S (2007). "Effect of chemical reaction and variable viscosity on hydromagnetic mixed convention heat and mass transfer for Hiemenz flow through porous media with radiation". Communication in Nonlinear Science and Numerical Simulation Vol.12, Pp. 195-231.

[34] Sahoo, P. K., Datta, N. and Bisnal, S. P. (2003) "Magneto hydrodynamic Unsteady Free Convection flow Past an Infinite Vertical Plate with Constant Suction and Heat Sink". Indian Journal of Pure and Applied Mathematics, Vol.34 (1); Pp.145 – 155.

[35] Soundaigekar, V.M. (1965). "Hydro magnetic flow near an accelerated plate in the Presence of a magnetic field". Applied Scientific Research, Vol.12 (1), Pp. 151 – 156.

[36] Sunil, Anupama, and Sharma, R.C. (2005). "The effect of magnetic field dependent viscosity on thermosolutal convection in ferromagnetic in fluid". Applied Maths. and Pomp,Vol.163,Pp. 119-1214.

[37] Zueco, J. (2007). "Network simulation method applied to radiation and viscous dissipation effect on porous plate". Applied mathematical Modeling, Vol. 31, and Pp.2019-2033.