Analysis of Peak-to-Average Power Ratio Reduction Techniques for OFDM using a New-phase Sequence

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ABSTRACT

Several properties of OFDM has made it an essential modulation scheme for high speed transmission links. But one major drawback ofOFDM is its large Peak-to- average power ratio. Here we have reviewed some of the PAPR reduction techniques and compared the performance of clipping and filtering,partial transmit sequence and selected mapping methods for QAM modulated OFDM. We have also shown analytically, the relation between passband and baseband PAPR and also the criteria for optimum design of the phase rotation table for the selected mapping technique. Finally we have proposed to use the Chu sequence as the phase sequence in the SLM technique to get reduction in PAPR.

General Terms

Peak-to average power reduction, Orthogonal frequency Division Multiplexing, Quadrature Amplitude Modulation

Keywords

Clipping and filtering; PTS (partial transmit sequence); SLM(selected mapping); Chu-SLM

1. INTRODUCTION

The OFDM signal amplitudes vary widely with high PAPR, as a consequence high power Radio frequency amplifiers will introduce inter-modulation between different subcarriers and introduce additional interference causing increase in bit error rate. Therefore, RF power amplifiers should operate in a very large linear region to avoid the signal peaks from getting into the non-linear region of the power amplifier causing in-band distortion i.e. inter modulation among the subcarriers and out of band radiation. To overcome this, the power amplifiers should be operated with a large power back-offs and this indirectly leads to very inefficient amplification and increase in transmitter power. This rhythms the invention of various PAPR reduction techniques. This article discusses coding, clipping and filtering[1,2], peakwindowing [3], partial transmit sequence [4], selected mapping technique [5].This article describes the basic principle of all these techniques. The selection of any of the PAPR reduction technique may be at the cost of PAPR reduction capability, low coding overhead, synchronization between transmitter and receiver, increase in transmit power, increase in Bit error rate at the receiver, data rate loss, computational complexity, inband and outband distortion. Here we have studied through simulation results the performance of Clipping and filtering, PTS and SLM based PAPR reduction techniques and cited the selection criteria for these techniques basing on various parameters. While calculating the PAPR, the actual time domain OFDM signal that is in analog form must be taken into account since the IFFT outputs will miss some of the signal peaks. So, if we calculate the PAPR by using these sample values then the calculated PAPR will be less than the actual PAPR. This is an Optimistic result which is far from the real results but, these samples values are enough for signal reconstruction. To substantiate this issue, oversampling is performed. After oversampling the increased samples are close to the real analog signal and calculation of PAPR based on the increased sample values will give the true PAPR. PAPR is not a problem with constant amplitude signals as it is a problem with non-constant amplitude signals. Since OFDM and MIMO-OFDM are based on OFDM, they also need PAPR reduction.This article is organized as follows: In section 2, a brief discussion of base band and pass band PAPR is followed by the discussion of various PAPR reduction techniques with their merits and demerits. Then in section 3, the performance of three techniques iscompared through simulation results and those include: clipping and filtering, Partial transmit sequence and selective mapping method. In section 4, at first we have shown the optimal design criteria for the phase rotation table of SLM technique followed by the performance comparison of Chu sequence with all other sequences used so far [6-8] and compared the results of Classical SLM with Chu SLM for QAM modulation schemes. Finally conclusions are drawn in section 5 followed by the references.

2. PAPR AND ITS REDUCTION IN OFDM SYSTEMS

The complex discrete-time baseband equivalent time –domain OFDM signal can be expressed as:

$$x[n] = IFFT\{X[k]\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] \exp\left(\frac{j2\pi nk}{N} \right). \quad (1)$$

Where, n = (0,1,2…….N); N→No of subcarriers; X[k]→ denotes the kth modulated phase shift keying (PSK) or quadrature amplitude modulated (QAM) symbol;X[k] =ak +jbk; where ak and bk are real and imaginary component of X[k].Assuming ak and bk to be independent of each other , if we separate x[n] in equation(1) into its real andimaginary components and evaluate the expectation and variance of them and thereafter apply Central limit theorem for large N,then the probability distribution of x[n]I and x[n]Q will follow the Gaussian distribution. The amplitude of OFDM signal thereafter has a Rayleigh distribution with zero mean and a variance of N times the variance of one complex sinusoid. Let {Zn}be the magnitudes of complex samples. Assuming that the average power of complex passband OFDM signalS (t) is equal to one, the {Zn} are the normalized Rayleigh random variables with its own average power, which has the probability density function [9] as shown below:

$$f_{z_n}(z) = z/\sigma^2 \exp\left(-z^2/2\sigma^2\right) = 2z\exp\left(-z^2\right) \quad (2)$$

where E and D being expectation and variation operator. For (n=0,1,2…N-1) the following equation holds for the condition of: $E\{a_k\} = E\{b_k\} = 0 and D\{a_k\} = D\{b_k\} = \sigma^2$

$$E\left\{{z_n}^2\right\} = 2\sigma^2 = 1 \quad (3)$$

The Max $\{Z_n\}$ =Z $_{\max}$ = Crest Factor=CF; and the cumulative distribution function (CDF) of Z $_{\max}$ is given by

$$\begin{aligned} F_{z_{\max}}(z) &= P\left(Z_{\max} < z\right) \\ &= \left[P\left(Z_0 < z\right)\right].\left[P\left(Z_1 < z\right)\right]......\left[P\left(Z_{N-1} < z\right)\right] \\ &= \left[1-\exp\left(-z^2\right)\right]^N \end{aligned} \quad (4)$$

For baseband condition; CF= (PAPR) $^{1/2}$ and for pass band condition CF= (PMEPR) $^{1/2}$; where PAPR of a complex pass band OFDM signal is given as:

$$PAPR\{S(t)\} = \max\left|S(t)\right|^2 \Big/ E\left\{\left|S(t)\right|^2\right\} =$$

$$\max\left|\mathrm{Re}\left\{S(t)\exp\left(j2\pi f_c t\right)\right\}\right| \Big/ E\left\{\left|\mathrm{Re}\left\{S(t)\exp\left(j2\pi f_c t\right)\right\}\right|^2\right\} \quad (5)$$

In baseband case the peak –to- mean envelop power ratio(PMEPR)is defined as:

$$PMEPR\{S(t)\} = \max\left|S(t)\right|^2 \Big/ E\left\{\left|S(t)\right|^2\right\} \quad (6)$$

Where $s(t)$ is the complex baseband equivalent of the pass band signal . Now to find the probability that the CF exceeds z, we have to consider the complementary CDF (CCDF) as:

$$\begin{aligned} \tilde{F}_{Z_{\max}} &= P\left(Z_{\max} > z\right) = 1 - P\left(Z_{\max} \le z\right) \\ &= 1 - F_{Z_{\max}}(z) = 1 - \left[1-\exp\left(-z^2\right)\right]^N \end{aligned} \quad (7)$$

The PAPR of equation (5) deals with the pass band signal S (t) with a carrier frequency of f_c which is much higher than inverse of one symbol period of $S(t)$,hence the PAPR of the continuous time base band OFDM signal and its corresponding pass band signal will have almost the same PAPR. But,the PAPR of discrete time baseband signal x[n] in equation(1) may not be the same as that for the continuous time baseband signal x(t) and itwill be low, since x[n] may not have all the peaks of x(t). Measurement of the PAPR for x(t) from the PAPR of x[n]can be done by L (≥4) times interpolatingthe x[n].The IFFT output signal x[n] can be expressed in terms of the L times interpolated version as :

$$x'[m] = 1/\sqrt{L}.N \sum_{k=0}^{L.N-1} X'[k].\exp\left[j2\pi mf'k \Big/ L.N\right] \quad (8)$$

where *X'[k]* =X[k], for 0≤ k< N/2 ;and 0,elsewhere andN, f' and X[k]are the no of subcarriers, the subcarrier spacing and the complex symbol carried over a subcarrier k respectively. For x'[m],the PAPR is defined to be :

$$PAPR\{x'[m]\} = \max\left|x'[m]\right|^2 \Big/ E\left\{\left|x'[m]\right|^2\right\} \; for \quad (9)$$

$$m = 0,1,....N.L$$

Various PAPR reduction techniques are discussed below.

2.1 Coding, Clipping and Filtering

In coding technique [1-2] the codewords with minimum PAPR are found from a given set of codewords and the input data blocks are mapped to these selected codewords. The encoding and decoding complexity increases for large no. of carriers and this method could not work for higher order bit rates and reduction of PAPR is at the expense of coding rate. The non-linear clipping process introduces In-band distortion causing degradation in Bit Error Rate performance of the system and also causes out of band noise which reduces the spectral efficiency. Using Filtering with clipping [11] the out of band noise is reduced but the in-band noise is not. To reduce the In-band noise each OFDM block is oversampled by padding the original input with zeros and then taking a longer IFFT. Use of Forward error correction (FEC) codes with clipping and filtering [2] can reduce both the noises and improves the BER performance and spectral efficiency.

2.2 Peak Windowing

This method [3] concerns with removing the less occurred peak values. It provides large reduction in PAPR along with the advantages of easy implementation, independence in no of subcarriers and undisturbed coding rate at the cost of increase in BER and out of band noise. Peak windowing with FEC codes [2] can compensate the increase in BER. Any windowing can be used , provided it has a good spectral properties. Examples are cosine, Kaiser and Hamming. By removing peaks, PAPR cannot be reduced beyond a certain limit since the average value of OFDM signal decreases, thus increasing PAPR. OFDM signal exhibits “Bottoms” similar to peaks. By increasing these bottoms above certain level, the average value of OFDM signal can be shifted up and thus reducing PAPR. After this the samplevalues are amplified and the total method is referred as Peak- windowing with Pre- amplification.

2.3 Partial Transmit Sequence

In this method [4, 12, 13, 14] the input data vector is partitioned into P no. of sub blocks. and then IFFT for each sub block is taken. Then each IFFT output goes through scrambling (rotating the phase independently) followed by the multiplication of a corresponding complex phase factor

b^p *=exp(j∅p), where p= 1, 2,.....P and* at this point theOFDM signal is given by*:*

$$\underline{x} = IFFT\left\{\sum_{p=1}^{P} b^p x^p\right\} = \sum_{p=1}^{P} b^p.IFFT\left\{X^p\right\} == \sum_{p=1}^{P} b^p x^p$$

(10)Where { $\underline{X}^p$} is a partial transmit sequence and {X^p} are the consecutively located data sub blocks. The phase vector is chosen such that the PAPR is minimized. Initially all the phase factors (bp) are set to 1 for all p=1:P;it is set to be PAPRmin.Then in the second step the PAPR of equation(5) is determined for bp=-1. If PAPR exceeds PAPRmin bp is set back to 1; else PAPRmin=PAPR and this process continues till p<P and the process stops with the result of optimal phase factors . PAPR performance of this technique is affected by the no of sub blocks, the sub block partitioning method and the allowed phase factors.

2.4 Selective Mapping Technique

In this method [5, 14, 15, 16] the parallel input data vector is multiplied with V different phase sequences (each of length N) to create V modified data blocks with different phases before the IFFT operation. Then after the IFFT operation, among the modified data blocks the block having minimum PAPR is selected for transmission. Information about the selected phase sequence should be transmitted to the receiver as side information and this is the reason for complexity. SLM can be used for any no of subcarriers and for any signal constellation. It provides significant gain with moderate complexity. Channel coding is needed to protect the side information[6].

3.COMPARISION OF PAPRREDUCTION TECHNIQUES

Here we have compared the performance of clipping and filtering, PTS and Classical SLM based OFDM. Figure.1(a-b) shows the results for N=255 and 510.The simulation is done taking the clipping ratio to be 0.8 for both N values. Clipping ratio is the ratio of clip level (M) and the r.m.s power of OFDM signal. From these figures we can see that as N increases the CCDF plots for all techniques occurs at a larger distance from thevertical axis, however the SLMtechnique is giving betterperformance in both the cases and the Clipping and filtering is giving the worst performance.

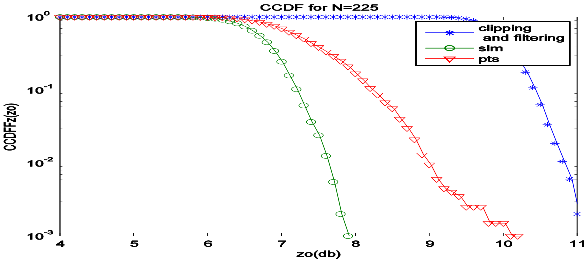

Fig.1a CCDF Comparison for N=255 of16-QAM OFDM

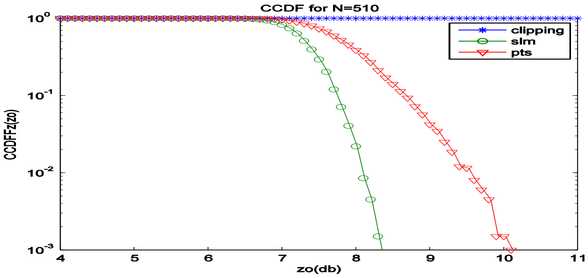

Fig.1b CCDF Comparison for N=510 of 16 QAM OFDM

Table.1 Comparison of various phase sequences

Serial no	Name of the Sequence	Baseband PAPR in dB	Passband PAPR in dB	Computational Time complexity(in Secs)
1	Chu	0	4.27	0.026929
2	Hadamard	2.84	4.28	0.066156
3	Hilbert	10.53	10.53	0.052609
4	Circulant	11.24	11.24	0.171689
5	Riemann	5.11	5.95	0.235032
6	Newman	9.07	9.07	0.027521

3. OPTIMAL SLM AND PERFORMANCE OF CHU-SEQUENCE

After applying SLM, the OFDM signal is expressed [5] below for $0 \le t < NT$ and v= 0,1,…V-1 as:

$$x^v(t) = 1/\sqrt{N}\sum_{0}^{N-1} x_n b_{v,n}.\exp(j2\pi n\nabla f t) \qquad (11)$$

Where, $b_{v,n} = |bv,n|\exp(j\,\phi\, v_{,n})$ and the V uncorrelated N length phase shift vectors are defined as:

$$\{B_v\} = b_{v,n}\Big|_{n=0}^{N-1} \tag{12}$$

The frequency domain OFDM (unmodified) signal is given by the vectors $\{X_k\}$; for k=0,1,2….N-1 and k is the sub carrier index and N is the no of. Sub carriers. During the V no of multiple representation of the same OFDM signal the average power is unchanged means that;

$$E\left\{\left|X^v\right|^2\right\} = E\left\{\left|X_n\right|^2\right\} = E\left\{\left|X_k\right|^2\right\} = \sigma^2 \tag{13}$$

The equation (13) assumes that Xk has zero mean and variance σ2 , hence all $\{x_n^v\}$ for v=0,1,….V -1, contain the same information about X k. If for each v = 0,1,2….V-1 and for each n= 0,1,…N-1; $\{x_n\ v\}$is independently and identically distributed complex Gaussian distribution andif it has independent real and imaginary part, the optimum design of the phase rotation table will be obtained at the condition of asymptotic mutual independence between {xn}v and {xn}l for all v≠l and with E[exp(jϕ)]=0 for ϕ to be uniformly distributed in [0,2π). Hence, this is the optimal SLM condition.

The Chu sequence is given by :

$$Y_i(k) = \begin{cases} e^{j\pi k(i*i)/N}, & \text{For N even} \\ e^{j\pi k i(i+1)/N}, & \text{For N odd, gcd (k,N) = 1} \end{cases} \tag{14}$$

The proposed Chu sequence [10] is giving 0 dB PAPR without oversampling and 4.27 dB with a oversampling factor of 4 . Table-1 compares these above values with those for the Hadamard , Hilbert, Circulant, Riemann and Newman sequences selected from the rows of corresponding matrices. Chu-sequence is giving same PAPR with normalization also. So, no need of normalization as in [6] for Riemann matrix.It is giving less PAPR compared to all sequences[6-8]. The computational time complexity is also less compared to all the sequences in the literature. Since the Chu matrix has a particular structure, there is no need of sending the side information with coding for accurate detection of signal. The receiver itself can generate Chu sequence for decoding. Using Chu sequence as phase with SLM, we have shown the performance of 16-QAM modulated OFDM signal with Chu-SLM and compared it with the 16-QAM modulated OFDM signal with Classical SLM for different values of N. Fig 2(a-b) shows these results for different values of N. The Chu-SLM has shown very much better performance than the Classical SLM.

4. CONCLUSION

The SLM technique is a promising technique to be used with higher no. of subcarriers. The true objective of SLM OFDM scheme is to reduce the probability of crest factor exceeding some threshold level rather than to reduce the crest factor of alternative symbol sequence.A phase sequence set that makes as many crest factors of alternative symbol sequences and look statistical independent can perform well.With Chu SLM we have got better performance than the classical SLM. The complexity is very less with Chu- SLM, since no need to send side Information to the receiver. Since the Pass band PAPR of Chu SLM is very less, it can be a promising PAPR reduction technique for high data rate Passband applications. Also the Computational time complexity is very less compared to all other sequences. So, it is practicable to use in a complex communication network .

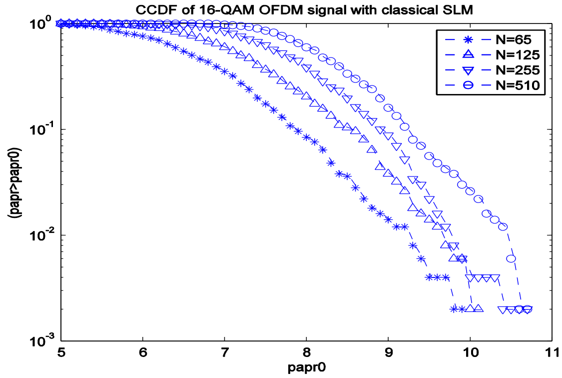

Fig. 2a CCDF of 16-QAM OFDM with Classical-SLM

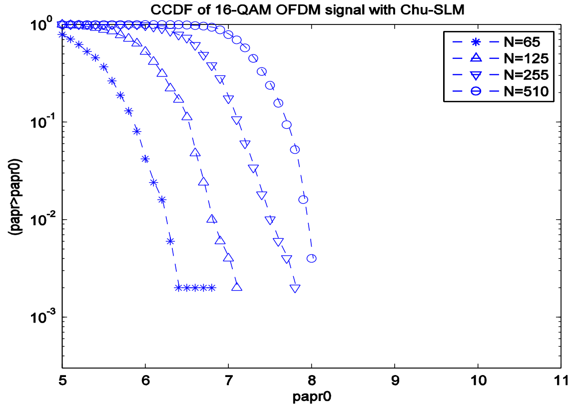

Fig. 2 b CCDF of 16-QAM OFDM with Chu-SLM

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