Adaptive Coding and Modulation for OFDM Systems using Product Codes and Fuzzy Rule Base System

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ABSTRACT
Adaptive communication has gained attentions of almost all future communication systems, especially Orthogonal Frequency Division Multiplexing (OFDM) Systems. Adaptive nature pays in terms of rate and power while satisfying certain criteria. In fact, this is a constrained optimization problem. In this paper a similar constrained optimization problem is focused. Adaptive modulation has been a key approach in adaptive systems while consideration of practical channel coding is a recent approach. Moreover, convolution codes and parallel concatenated codes (turbo codes) have been very much worked out for such systems while the serially concatenated codes (Product codes) are mostly neglected due their high decoding complexity. In this paper, we have chosen Product codes with a low decoding complexity decoder named Modified Iterative Decoding Algorithm (MIDA) as adaptive coding scheme and Quadrature Amplitude Modulation (QAM) as adaptive modulation. We propose a new scheme to adapt both code rate and modulation size by solving a non-convex optimization problem using a Fuzzy Rule Base System (FRBS) to enhance the achievable data rate in an OFDM system with a fixed target bit error rate and fixed transmit power for each subcarrier. Proposed scheme is compared with various well known adaptive modulation and coding (AMC) schemes.

General Terms
OFDM; Fuzzy Systems; Adaptive Modulation and Coding

Keywords
OFDM; FRBS; Adaptive Modulation and Coding; Modulation Code Pair; Product Codes; MIDA; BER

1. INTRODUCTION
Adaptive communication is one of the prominent candidates for future wireless telecommunication systems. In this technology, transmitter intelligently adapts the transmission parameters like coding scheme, modulation symbol, power etc with respect to the varying wireless channel state information (CSI). In this way, if channel is having poor conditions then a channel code with smaller code rate and a smaller modulation symbol can be used. Similarly, if channel conditions are good a comparatively high code rate or even no coding can be used in contrast to a high modulation symbol etc. Orthogonal Frequency Division Multiplexing (OFDM) has gained major attentions of almost all recent wireless standards like IEEE802.11n (WIFI) [1], IEEE802.16 (WiMAX with fixed devices) and IEEE802.16/e (mobile devices) [2]. In OFDM one big data stream is divided into many small data streams that causes great reduction in inter symbol interference (ISI) then these data streams are modulated over orthogonal subcarriers.

In literature many adaptive power and bit loading techniques have been proposed. In these techniques adaptive modulation was mostly examined while consideration of a practical forward error correction (FEC) technique is quite recent. In 1989, adaptive modulation for OFDM system was initially proposed by Kellet [3]. Later same idea was investigated for Gaussian slowly varying dispersive channel by Chow et al [4]. In 1996, Cyzlwik used concept of adaptive modulation for wideband (WB) radio channel [5]. In 2009, Shastri proposed a Fuzzy Logic based adaptive modulation scheme [6].

A turbo coded adaptive modulation scheme was proposed by Hanzo et al [7], where adaptation was made on behalf of signal to noise ratio (SNR) thresholds. For single antenna OFDM systems, coded bit and power loading problem was addressed by Li et al. using LDPC codes[8] originally motivated by [9]. In all of the above adaptive coding and modulation schemes, either modulation was adaptive with a fixed channel code or channel code was adaptive and modulation was fixed [10].

In above literature survey mostly convolutional codes, turbo codes and LDPC codes has been investigated due to their strong correction capability as well as reasonable decoding techniques. Product codes that are Serially Concatenated Linear Block Codes are mostly neglected due to their enhanced decoding complexity.

In this paper, we have proposed an adaptive coding and modulation scheme in which both code rate and modulation symbol are adapted on per subcarrier bases. In this proposal we have investigated Product Codes as FEC and Quadrature Amplitude Modulation (QAM) as modulation scheme. A modified Iterative Decoding Algorithm (MIDA) [11] that is a hard decision decoder is used for decoding of Product Codes.

For intelligent adaptation we have utilized a Fuzzy Rule Base System (FRBS), which suggests which modulation code pair should be used on which subcarrier, depending upon the individual channel conditions at that subcarrier. Fuzzy systems are considered ideal for the situations that are vague and missing certain information.

Rest of the paper is organized as follows. System model is given in section 2; section 3 contains a brief introduction of Product Codes and their decoding; coded modulation and simulations for various modulation code pairs in given in section 4; section 5 contains rate optimization criteria and cost function to be optimized, in section 6 Fuzzy Rule Base System creation steps and parameters are explained, simulation results of proposed scheme are depicted in section 7 while section 8 concludes the paper.

2. SYSTEM MODEL

The system model considered is OFDM equivalent baseband model with N number of subcarriers. It is assumed that complete channel state information (CSI) is known at both transmitter and receiver. The frequency domain representation of system is given by

$$r_k = h_k.\sqrt{p_k}.x_k + z_k\ ;\ \mathrm{k} = 1,2,......,N \qquad (1)$$

where r_k, h_k, $\sqrt{p_k}$, x_k and z_k denote received signal, channel coefficient, transmit amplitude, transmit symbol and the Gaussian noise of subcarrier $k = 1,2,......,N$, respectively.

The overall transmit power of the system is $P_{total} = \sum_{k=1}^{N} p_k = Np$ since power is same for all subcarriers, and the noise distribution is complex Gaussian with zero mean and unit variance.

It is assumed that signal transmitted on the kth subcarrier is propagated over an independent non-dispersive single-path Rayleigh Fading channel and where each subcarrier faces a different amount of fading independent of each other. Hence, the channel coefficient of kth subcarrier can be expressed as:

$$h_k = \alpha_k e^{j\theta_k}\ ;k = 1,2,......,N \qquad (2)$$

where α_k is Rayleigh distributed random variable of kth subcarrier, and the phase θ_k is uniformly distributed over $[0, 2\pi]$.

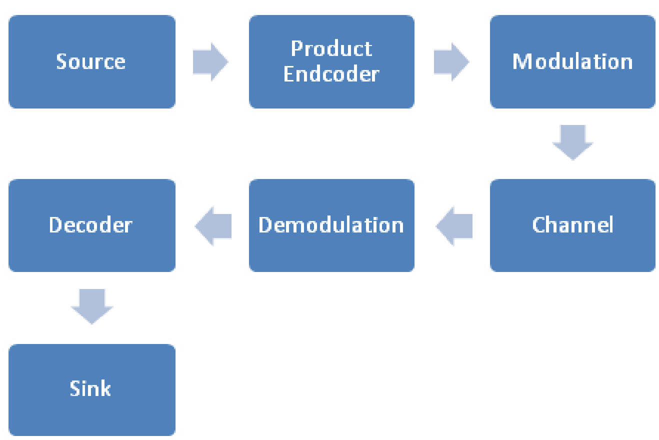

Figure1. System overview

3. PRODUCT CODES AND MODIFIED ITERATIVE DECODER

3.1 Product Codes

Product codes are serially concatenated codes. Product codes were first presented by Elias in 1954 [12]. The concept of Product codes is quite simple as well as powerful, where much shorter constituent block codes are used instead of one long block code. Basically these are matrix codes where rows are encoded by one block code while columns are encoded by another block code. This arrangement enhances their error correction capability since errors are corrected row-wise as well as column-wise. Also these codes are burst error correcting codes since a row-wise burst can easily be corrected column-wise and vice versa. Since burst error in rows will become single error for column code and vice versa.

Consider two linear block codes $\mathbf{A}_1$ and $\mathbf{A}_2$ with parameters $[n_1,k_1,d_1]$ and $[n_2,k_2,d_2]$ respectively, where n_i,k_i and $d_i\,;i = 1,2$ are the length, dimension and minimum Hamming code $\mathbf{A}_i\,(i = 1,2)$ respectively. Code $\mathbf{A}_1$ will be used as row code while $\mathbf{A}_2$ will be used as column code. The rates of individual codes are R_1 and R_2 respectively given by,

$$R_i = \frac{k_i}{n_i}, i = 1,2 \qquad (1)$$

The product code $\mathbf{\Omega}$ can be obtained by codes $\mathbf{A}_i\,, i = 1,2$ in the following manner.

- Place $k_1 \times k_2$ information bits in an array of k_2 rows and k_1 columns
- Encode k_2 rows using code $\mathbf{A}_1$, which will result in an array of $k_2 \times n_1$
- Now encode n_1 columns using code $\mathbf{A}_2$, which will result in $n_2 \times n_1$ product code.

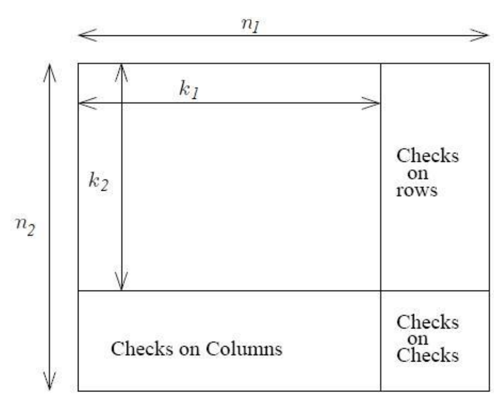

Figure 1. Structure of the Product code

The resultant product code $\mathbf{\Omega}$ has the parameters $[n_1n_2, k_1k_2, d_1d_2]$ and the rate will be R_1R_2. In this way long block codes can be constructed using much shorter constituent block codes.

This concept can also be viewed as that product code Ω is intersection of two codes $\mathbf{A}_1'$ and $\mathbf{A}_2'$. Where $\mathbf{A}_1'$ is a code represented by all $n_2 \times n_1$ matrices whose each row is a member of code $\mathbf{A}_1$, similarly $\mathbf{A}_2'$ is a code represented by all $n_2 \times n_1$ matrices who's each column is a member of code $\mathbf{A}_2$. This can be written as;

$$\mathbf{\Omega} = \mathbf{A}_1' \cap \mathbf{A}_2' \qquad (3)$$

3.2 Modified Iterative Decoding Algorithm

Iterative decoding algorithm (IDA) for product codes was originally presented by [10] in his Doctoral thesis that is based upon List Decoding also designated as Maximum Likelihood (ML) decoding of product codes[9]. ML decoding is an optimum decoding with an exponential complexity. The iterative decoder was proposed to reduce the complexity of ML decoding, but yet it exhibits a huge complexity.

Modified Iterative Decoding Algorithm (MIDA) is modification of IDA. For reference purpose it is written here in brief, interested readers may visit original paper by the same author [11].

3.1.1 Algorithm

Let $\mathbf{R}_{n_2 x n_1}$ be the received code matrix

1. while (i <= Max no of iterations) do
   a) If ((each row of $\mathbf{R}$ is the member in the row code) and (each column of $\mathbf{R}$ is the member in column code)) then go to step b, otherwise to step c
   b) Return $\mathbf{R}$ as the decoded solution and go to step 2
   c) Mark those rows/columns in $\mathbf{R}$ that are members in the row/column code (using Syndrome check), respectively
   d) Make lists for unmarked rows ($n_2^{'}$) and columns ($n_1^{'}$), with decoding radii ($t_{A_1}+1$) and ($t_{A_2}+1$) respectively
   e) Take decisions for suggested solution in row/column decoders respectively
   f) $\mathbf{R}=\mathbf{S}^i$
   g) $i=i+1$ go to 2
2. Exit

4. CODED MODULATION

4.1 Coding Scheme

Coding schemes used for this framework are set of product codes. Since product codes are matrix codes, where rows contain one code and column contains another code. The set of row codes and column codes used in this paper are listed in table1. All of these codes are BCH codes.

TABLE I. Coding Parameter

Sr	Row Code	Column Code	Product Code	Code rate
C1	[63,63,1]	[63,63,1]	[3969,3969,1]	1
C2	[63,57,3]	[63,63,1]	[3969,3591,3]	0.9
C3	[63,51,5]	[63,63,1]	[3969,3213,5]	0.8
C4	[63,36,11]	[63,63,1]	[3969,2268,11]	0.57

So set of code is consisted of four different product codes. That is

$$C=\{C_1,C_2,C_3,C_4\} \qquad (4)$$

The reasons for selection of these codes are as under.

- All codes are of same length would be helpful in hardware implementation
- Same length of row codes make it possible for decoding since if we use different length codes then upon receiving received matrix may not be formulated

4.2 Modulation Scheme

The modulation scheme used for this experiment is Quadrature Amplitude Modulation (QAM) which is recommended by many OFDM standards. Following set of modulation schemes is used. That is

$$M=\{2,4,8,16,32,64,128\} \qquad (5)$$

So with these coding and modulation sets we have 28 possible modulation code pairs (MCP) by a Cartesian product of the sets C and M. This can be given by the expression.

$$P=CxM \;=\{(c_i,m_j);\forall c_i\in C,\forall m_j\in M\} \quad (6)$$

After deciding modulation and coding schemes for this framework, all of the possible combinations of modulation code pairs are plotted in subsequent figures.
In figure 2, all modulation schemes namely from 2QAM to 128QAM are plotted using Product Code C1 as listed in table 1. Similarly, in figure 3, 4 and 5 different QAM modulations are plotted using Product codes C2, C3 and C4 respectively.
In subsequent sections, these graphs are being used as the fundamentals of Fuzzy Rule Base System (FRBS) and the conscious knowledge of Fuzzy Inference Engine (FIE).

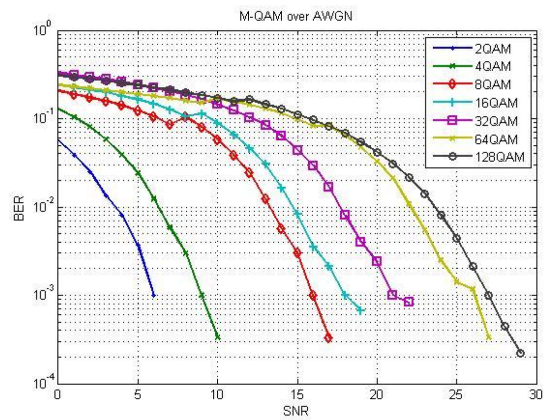

Figure 2: Performance of different QAM schemes using Product Code C1

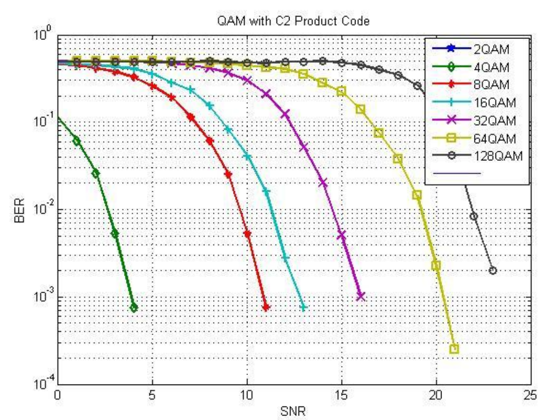

Figure 3: Performance of different QAM schemes using Product Code C2

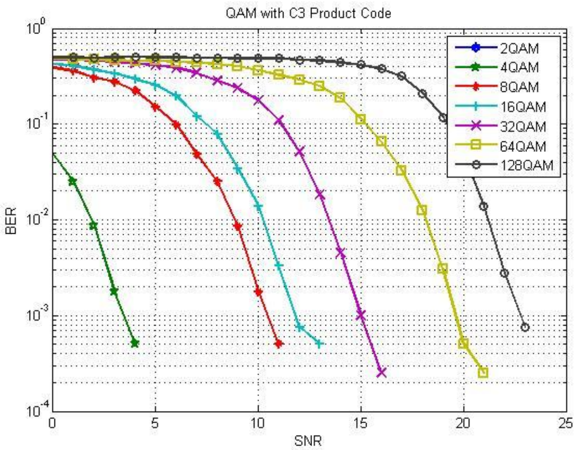

Figure 4: Performance of different QAM schemes using Product Code C3

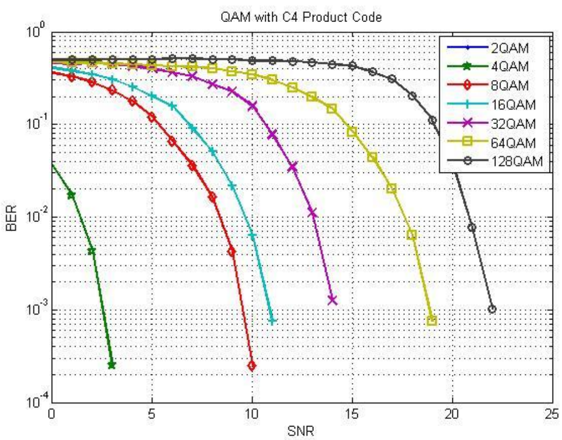

Figure 5: Performance of different QAM schemes using Product Code C3

5. RATE OPTIMIZATION

In order to maximize the rate for OFDM symbol following constrained optimization problem will be considered.

$$\begin{aligned} &\max \quad R_{Total} = R\log_2(M) \\ &\text{s.t,} \\ &BER_{Total} \le BER_T \qquad (7) \\ &and \\ &P_{Total} = \sum_{i=1}^{N} p_i = Np < P_T \end{aligned}$$

Where R is rate of product code used and M is modulation symbol used in OFDM system. P_T is the available transmit power and p is power transmitted per subcarrier. Power distribution per subcarrier is assumed to be flat. BER_T is target BER that depends upon a specific quality of service (QoS) request or application requirement like $BER_T = 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}$ while N is total number of subcarriers in OFDM system. So the above cost function will be optimized by the proposed Fuzzy Rule Base System.

From the results obtained above, those code-modulation pairs that fulfill different BER demands depending upon different quality of services are obtained. These are obtained by drawing straight lines on the graphs shown in fig2-5, on certain BER points. Then the points of intersection of these lines and the curves (representing a code and a modulation) are noted that gives the appropriate SNR value. So the information obtained can be expressed as "for a given average received SNR and specific QoS which modulation code pair can be used". All of these pairs are listed in a table, and table is used for creation of Fuzzy Rule Base in next section.

Without loss of generality we can say that this table represents a function (mapping) in which the throughput can be expressed in terms of BER and SNR.

$$R = MCP = f(SNR, BER) \qquad (8)$$

The mapping shown in eq-4 is a non-convex function that cannot be optimized using convex optimal techniques unless it is made convex according to [14]. However, this function is optimized by the proposed Fuzzy Rule Base System described in next section. The steps involved in creation of FRBS are described in the flowchart given in figure 6. The brief description of each phase of the flowchart is given below.

A. Graphs

Graphs for different combinations of Codes and Modulation schemes are obtained that are depicted in figure 2 to 5.

B. Data Acquisition

Data is obtained from the graphs in terms of input/output (IO) pairs. This is taken by putting the horizontal lines for various bit error rates and then points of intersection of these lines with the curves are noted.

C. Rule Formulation

Rules for each pair are obtained by the appropriate fuzzy set used. That is by putting complete pair in input/output set and a rule generated for each pair.

D. Elimination of Conflicting Rule

The rules having same IF part but different THEN parts are known as conflicting rules. This appears when more than one modulation code pair (MCP) are available for given specification. For instance, there is a rule whose THEN part contains two different MCP namely, [32, C3] and [32, C4]. Now [32, C3] is best since its throughput is 5x0.8=4 b/s/Hz while others have 5x0.57=2.58 respectively.

Similarly, sometime there could be two different pairs with same throughput like [32,C3] and [16,C1] both have same throughput that is 4 b/s/Hz, then [16,C1] will be chosen since it exhibits less modulation/demodulation and coding/decoding cost.

E. Completion of Lookup Table

Since in lookup table scheme we may not have complete number of IO pairs, then those parts are filled by heuristic or expert knowledge. For example, a modulation code pair is suggested by rule for a certain SNR and QoS. Since if a modulation code pair performs for lower SNR, then it can easily sustain in higher SNR situations. Similarly, if a MCP performs for a good QoS then it can sustain for poor QoS demands.

F. Fuzzy Rule Base Creation

Using the Lookup table in above phase Fuzzy Rule Base is created using Fuzzy Logic Toolbox in MATLAB. Further details are given in next section.

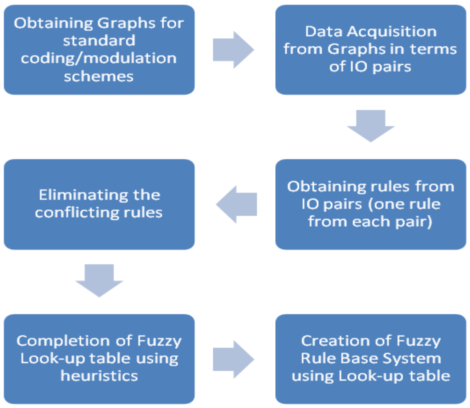

Figure 6. Steps involved in creation of Fuzzy Rule Base System

6. FUZZY RULE BASE SYSTEM

We propose a fuzzy rule base system (FRBS), which is capable of deciding the best modulation code pair (MCP) for the next transmission, based upon the heuristics. Fuzzy logic is best suited for the situations that are vague, ambiguous, noisy or missing certain information. Also fuzzy systems are very easy to implement in hardware. There are many ways we o build a Fuzzy Rule Base System, we have used table lookup scheme for this purpose. The lookup table is given in figure 7 that is used for creation process.

This table shows the facts extracted for simulated performance of different codes and modulation pairs in previous section. Received signal to noise ratio is expressed in level 1 to level 9 and Quality of Service are given like poor, med, good and high that is $10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$ respectively.

SNR-->>				
L1	P2	P2	P1	P1
L2	P4	P3	P2	P2
L3	P6	P5	P5	P2
L4	P7	P7	P6	P5
L5	P8	P10	P10	P6
L6	P15	P12	P12	P7
L7	P17	P14	P15	P9
L8	P17	P16	P17	P11
L9	P18	P18	P18	P13
	Poor	Med	Good	High

QoS-->>

Figure 7. Lookup Table for FRBS Creation

This fuzzy rule base system (FRBS) is used to optimize the cost function given in equation 5. It will be decided that which modulation code pair is suitable for transmission based upon the average channel state information (CSI) at the subcarriers and the Quality of Service demand.

We have used the table look-up scheme for design of this fuzzy rule base system using the following steps. The input-output pairs needed for design of FRBS are provided in figure 8. They are of the form;

$$(x_1^p, x_2^p; y^p); p = 1, 2, 3.......M \qquad (9)$$

Where x_1^p represents received SNR, x_2^p represents required BER (QoS) and y^p represents the output MCP suggested by FRBS, so the rule format will be given as below;

{IF (x_1 is Good and x_2 is L7) THEN y is P15} (10)

Following is the brief description of different components of fuzzy rule based system used. Design of the FRBS is carried out in MATLAB 7.0 standard Fuzzy System Toolbox.

6.1 Fuzzy Sets

Sufficient numbers of fuzzy sets are used to cover the input output spaces. There are two input variables average received SNR and minus log bit error rate (MLBER) that represents a QoS. The reason taking MLBER is because BER of a required QoS is given by $10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$ etc while the range of fuzzy variable should be equally spaced and quantifiable. So to get this, following operation is done first.

$$\begin{aligned} MLBER &= -\log(BER) \\ BER &= 10^{-q} \\ MLBER &= -\log(10^{-q}) = q \end{aligned} \qquad (11)$$

There is one output variable for modulation code pair MCP. All of these input and output variables are depicted in figures 8, 9 and 10 respectively. There are nine, four and eighteen fuzzy sets used for the variables SNR, MLBER and MCP, respectively.

6.2 Fuzzifier

Standard triangular fuzzifier is used with AND as MIN and OR as MAX.

6.3 Rule Base

Rule base contains rules against all the IO pairs. As there are nine sets (L1 to L9) for first input variable named SNR and about four sets (low, medium, good and high) for input variable MLBER. Hence there are 36 rules in rule base. Rule base is complete in a sense that rules are defined for all possible combinations of input spaces.

6.4 Inference Engine

Standard Mamdani Inference Engine (MIE) is used that will infer which input pair will be mapped on to which output point. It is shown in figures 11 and 12.

6.5 De-Fuzzifier

Standard Center Average Defuzzifier (CAD) is used for defuzzification.

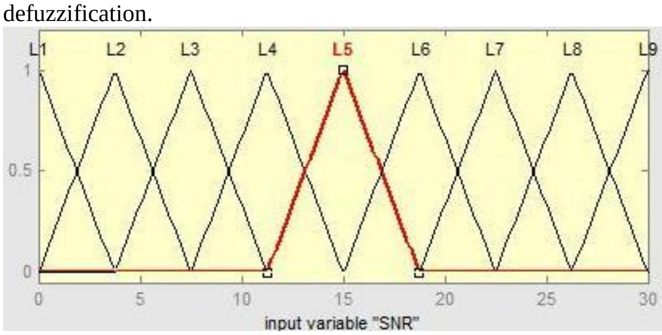

Figure 8. Fuzzy Sets for input variable average received SNR

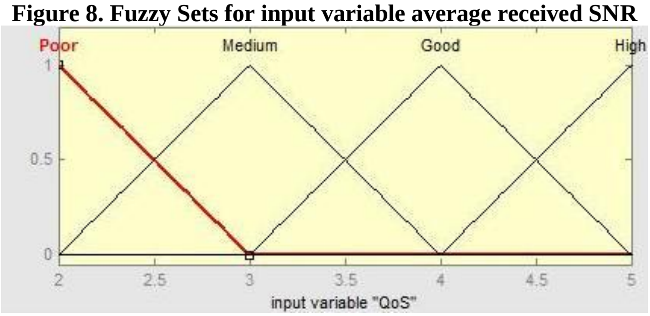

Figure 9. Fuzzy Sets for input variable QoS

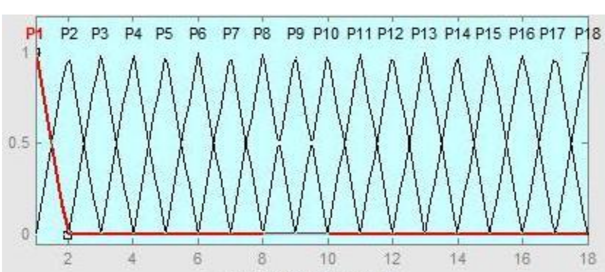

Figure 10. Fuzzy Sets for output variable Modulation Code Pair

Figure 13 shows the rule surface which reveals that by increasing SNR the throughput is maximized. Also on the other hand for poor QoS throughput is more than that of high QoS. A combined effect of both input variables namely SNR and QoS can be seen in that figure. For the highest value of SNR and lowest value of QoS, throughput of the system approaches to 5bits/s/Hz.

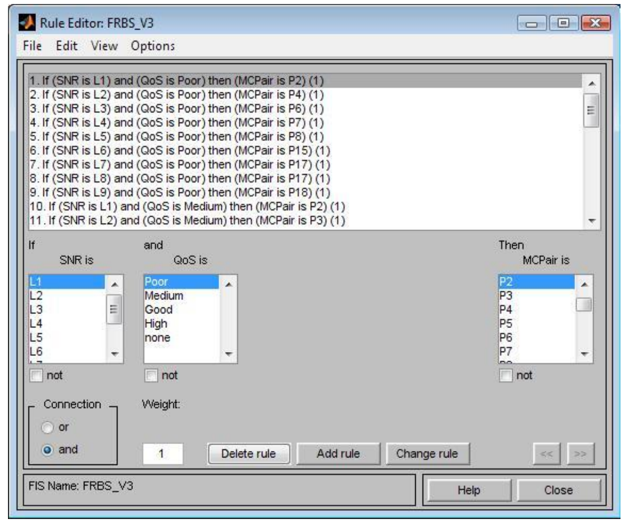

Figure 11. Fuzzy Rule Editor

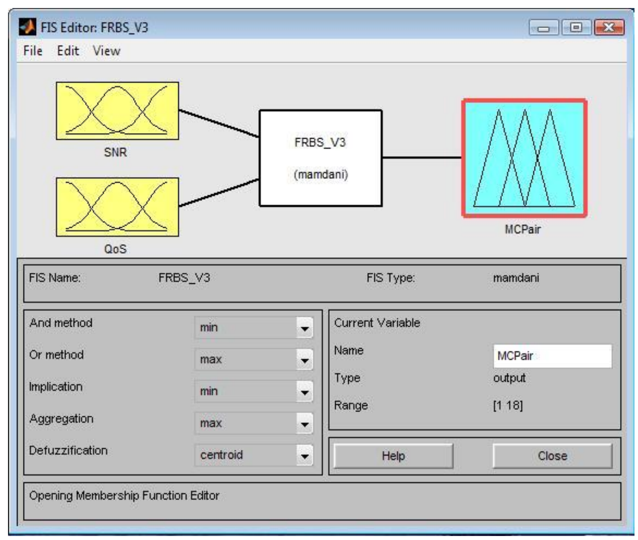

Figure 12. Fuzzy Rule Base System at a glance

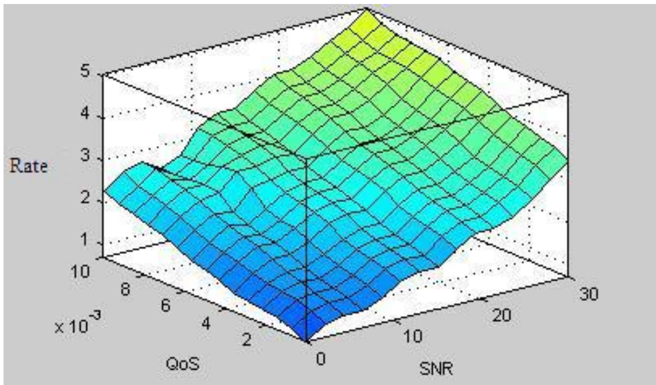

Figure 13. Rule surface

7. RESULTS

In this section proposed scheme is compared with the well-known adaptive technique and with the HYPERLAN/2 standard. Simulation parameters are enlisted in table2.

In figure 14, proposed scheme is compared for various quality of service (QoS) like average BER=10e-1, 10e-2, 10e-3 and 10e-4. In this way QoS was fixed initially then depending upon the received signal to noise ratio (SNR), most appropriate modulation code pair (MCP) was chosen using Fuzzy Rule Base System (FRBS), for entire OFDM system, then the product of modulation rate and code rate so called modulation-code-product is considered as throughput is plotted.

For poor required QoS like 10e-1, average achievable throughput at high SNR is 4.5bit/s/Hz. Similarly, as QoS become rich e.g, average BER=10e-4, average achievable throughput is 2bits/s/Hz.

In figure 15, proposed scheme is compared with the Adaptive Coding scheme proposed by Al-Askary in this PhD thesis [10], where HYPERLAN/2 standard was compared, the adaptation criteria was based upon SNR thresholds. As simulation results reveal, proposed scheme profoundly performs better than that of proposed by Al-Askary as well as HYPERLAN/2 standard. Another plus point is the decoding complexity of product codes quite affordable using Modified Iteration Decoding Algorithm [11] which is a hard decision decoding algorithm.

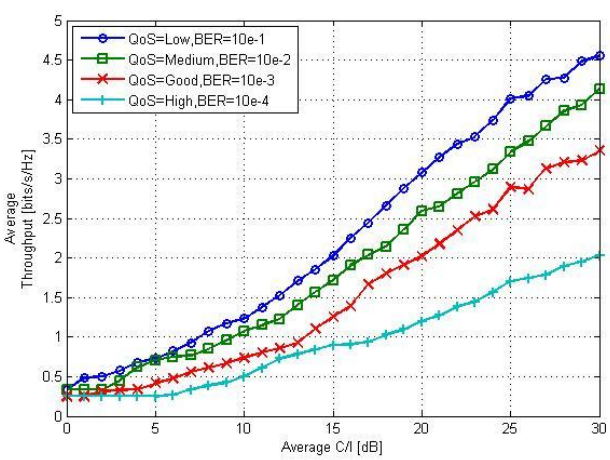

Figure 14. Comparison of proposed scheme for various QoS in a HYPERLAN/2 environment

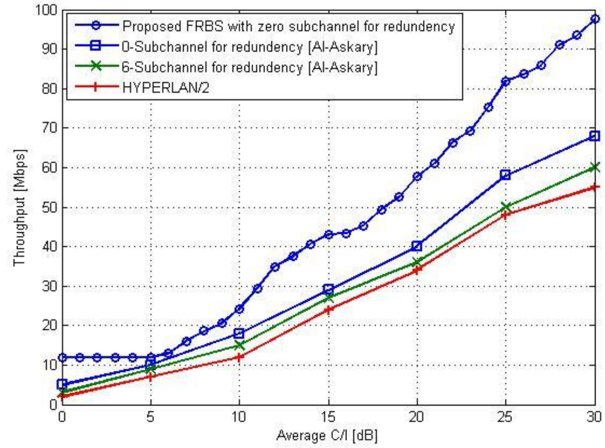

Figure 14. Comparison of proposed scheme with different schemes

TABLE II. Simulation Parameter

Sr	Parameter name	Value/s
1	Coding Schemes	Product code C1,C2,C4,C5
2	Code rates	1, 0.9, 0.8, 0.57
3	Modulation Schemes	2, 4, 8, 16, 32, 64, 128 QAM
4	Bits/symbols in modulation	1, 2, 3, 4, 5, 6, 7
5	Total MCPs	4x7=28
5	OFDM Standard used	HYPERLAN/2
6	Number of subchannel	64
7	Minimum throughput MCP	0.57x1=0.57bits/s/Hz
8	Maximum throughput MCP	1x7=7bits/s/Hz
9	Adaptation	Both modulation and code
10	Adaptation Criteria	Fuzzy Rule Base System

8. CONCLUSIONS

An adaptive coding and modulation scheme is proposed using Fuzzy Rule Base System, where product codes are utilized with Quadrature Amplitude Modulation. Fuzzy Systems are best suited for the situations that are vague and certain information is missing, also once the Fuzzy Rule Base System is created can easily be implemented in hardware and its robustness make is suitable for real time systems. The proposed scheme was tested for OFDM HYPERLAN/2 standard and compared to a similar work namely Adaptive Coding for OFDM System [10] and significance of proposed scheme is shown by using simulation results.

Significance of proposed scheme is due to the following factors,

- Better Code Rates (zero code rate penalty for column codes)
- Better Modulation Schemes (large constellations like 128QAM)
- Fuzzy Rule Base System to choose suitable most combination of code and modulation scheme based upon a specific Quality of Service and average received channel power to interference noise ratio.

9. ACKNOWLEDGMENTS

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