The Mutual Coupling Effect on the MUSIC Algorithm for Direction of Arrival Estimation

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ABSTRACT
The smart antenna systems combine antenna arrays with digital signal processing (DSP) algorithms. In a smart antenna system a specialized signal processor computes the direction of arrival (DOA) of a user and also adds the strength of the signals from each antenna element together to form a beam towards the direction as computed by DOA. If additional users join in the system, the adaptive antenna processor can tune out unwanted interferers by placing nulls towards the signals not of interest, and concentrate on the desired user by the main beam toward the signal of interest. Smart antenna systems integrate with radio intelligence with antenna array technology to increase the channel capacity, coverage range and improve link quality. In adaptive array smart antenna, to locate the desired signal, various DOA estimation algorithms are used. This paper investigates the effect of mutual coupling on the Multiple Signal Classification (MUSIC) algorithm for DOA estimation and compares its performance with Bartlett algorithm. The half wavelength dipole antenna elements are used in the linear array antenna to carry out a performance study of the MUSIC and Bartlett algorithms by investigating the effect of the mutual coupling between the array elements. However simulation results in this paper show that MUSIC algorithm is highly accurate and stable and provides high angular resolution compared to Bartlett and hence applying the MUSIC algorithm is preferred in mobile communication to estimate the DOA of the arriving signals.

Keywords
Smart Antenna, DOA, MUSIC, Bartlett, Mutual Coupling.

1. INTRODUCTION
One of the most promising techniques for enhancing the communication systems performance and improving the link quality for transmission and reception is the adaptive array smart antenna. The smart antenna technology is based on antenna arrays where the radiation pattern is controlled by adjusting the weight (amplitude and relative phase) on the different elements via DSP. If several transmitters are operating simultaneously, each source creates many multipath components at the receiver and hence the received array must be able to estimate the angles of arrival in order to determine which emitters are presented and what are their angular locations [1]. This information in turn can be used by the smart antenna to eliminate or combine signals for greater fidelity or suppress interferers to improve the capacity and mitigate the fading of cellular mobile communication. In general, the smart antenna systems estimate the DOA of all users and their multipath components and form a main beam toward desired user and reject the interferer users. The various known DOA estimation algorithms are Bartlett, Capon, Min-norm, MUSIC and ESPRIT [2-3]. Most of the DOA estimation algorithms are applied in the antenna array assumed to be isotropic point sources which are impractical. The MUSIC algorithm is a high resolution and an accurate method which is widely used in the design of smart antennas and hence, in this paper, its performance is evaluated in the absence and presence of mutual coupling. The effect of mutual coupling between real antenna array elements, [4], is studied on the MUSIC algorithm and comparison between its performance and Bartlett algorithm performance, [5-6], is investigated to see if these algorithms can be applied in actual antenna array elements or not. The finite size antenna elements in the array are considered. Here, the antenna elements receive the incident fields resulting in mutual coupling between the antenna elements. Parallel thin wire dipole antennas are considered, for example, to account for mutual coupling among the antenna elements [5]. A computer simulation tool using “MCAD” or MATLAB Codes are developed to calculate the voltages induced at the antenna elements, and utilizing these measured voltages to the two algorithms to estimate the direction of the signals and to carry out a performance study of the two algorithms. Also additive white Gaussian noise is considered in the simulation results.

2. DOA ESTIMATION USING BARTLETT AND MUSIC ALGORITHMS
The purpose of DOA estimation is to use the data received by the antenna array to estimate the direction of arrival of the signals. The results of DOA estimation are then used to design the adaptive beamformer in such way as to maximize the power radiated towards the desired user and to suppress the interferences. These are accomplished by placing beam maxima and beam minima, ideally nulls, towards desired and interfering signals, respectively. In short the successful design of adaptive array smart antenna depends highly on the performance of DOA estimation algorithm. Number of DOA estimation algorithms have been developed and categorized into two methods, conventional and subspace [7]. Conventional methods also called classical methods which first compute a spatial spectrum and then estimate DOAs by local maxima of the spectrum (such as Bartlett). But MUSIC algorithm is one of the subspace methods.

2.1 Bartlett Algorithm
The first attempt to automatically localize signal sources using antenna arrays was through beamforming techniques. The idea is to "steer" the array in one direction at a time and measure the output power. The steering locations which result in maximum power yield the DOA estimates. The array (composed of N

elements) response is steered by forming a linear combination of the sensor noiseless outputs

$$y(t)=\sum_{n=1}^{N} w_n^* x_n(t)=\sum_{n=1}^{N} w_n^* a_n(\theta)s(t)=W^H X(t) \quad (1)$$

where * denote the conjugate, ^{H} denotes the conjugate transpose, W is the weighting vector, $X(t)$ is the received signal vector, $s(t)$ is the base band time varying signal and $a(\theta)$ is, the steering vector at the DOA θ. Given samples $y(1), y(2),\ldots, y(M)$, where M is the number of samples, the output power is measured by

$$P(W)=\frac{1}{M}\sum_{m=1}^{M}|y(m)|^2=\frac{1}{M}\sum_{m=1}^{M} W^H X(m)X(m)^H W = W^H R_{xx} W \quad (2)$$

where R_{xx} is the array correlation matrix and is given by

$$R_{xx}=\frac{1}{M}\sum_{m=1}^{M} X(m)X(m)^H \quad (3)$$

The conventional (or Bartlett) beamformer is a natural extension of classical Fourier-based spectral analysis to sensor array data. For an array of arbitrary geometry, this algorithm maximizes the power of the beamforming output for a given input signal. Suppose we wish to maximize the output power from a certain direction θ. Given a signal emanating from direction θ, a measurement of the array output, $X(t)$, is corrupted by additive noise, $V(t)$, and written as

$$X(t)=a(\theta)s(t)+V(t) \quad (4)$$

The problem of maximizing the output power is then formulated as,

$$\max_W \mathrm{E}\{W^H X(t)X^H(t)W\}= \max_W W^H \mathrm{E}\{X(t)X^H(t)\}W = \max_W \left\{\mathrm{E}|s(t)|^2 \left|W^H a(\theta)\right|^2+\sigma^2|W|^2\right\} \quad (5)$$

where $\mathrm{E}[.]$ denotes statistical expectation, σ^2 is the noise covariance matrix and the assumption of spatially white noise is used. To obtain a non-trivial solution, the norm of W is constrained to $|W|=1$ when carrying out the above maximization. The resulting solution is then

$$W_{BF}=\frac{a(\theta)}{\sqrt{a^H(\theta)a(\theta)}} \quad (6)$$

The above weight vector can be interpreted as a spatial filter, which has been matched to the impinging signal. Intuitively, the array weighting equalizes the delays (and possibly attenuations) experienced by the signal on various sensors to maximally combine their respective contributions.

Inserting the weighting vector of (6) into (2), the classical spatial spectrum is obtained as

$$P_{BF}(\theta)=\frac{a^H(\theta)R_{xx}a(\theta)}{a^H(\theta)a(\theta)} \quad (7)$$

For a uniform linear array (ULA) of isotropic sensors, the steering vector $a(\theta)$ takes the form

$$a_{ULA}(\theta)=\left[1 \quad e^{j\psi} \quad \cdots \quad e^{j(N-1)\psi}\right]^T \quad (8)$$

where ^{T} denotes the transpose and

$$\psi=-kd\cos\theta=\frac{-2\pi d}{\lambda}\cos\theta \quad (9)$$

is termed the electrical angle and d denotes the inter-element distance.

2.2 MUSIC Algorithm

MUSIC is an acronym which stands for Multiple Signal Classification. This approach is a popular high resolution eigenstructure method. MUSIC promises to provide unbiased estimates of the number of signals, the angles of arrival, and the strengths of the waveforms. MUSIC makes the assumption that the noise in each channel is uncorrelated making the noise correlation matrix diagonal. The incident signals may be somewhat correlated creating a non-diagonal signal correlation matrix. However, under high signal correlation the traditional MUSIC algorithm breaks down and other methods must be implemented to correct this weakness. One must know in advance the number of incoming signals or one must search the eigenvalues to determine the number of incoming signals.
If the number of signals is N, the number of signal eigenvalues and eigenvectors is D, and the number of noise eigenvalues and eigenvectors is $N-D$. Because MUSIC exploits the noise eigenvector subspace, it is sometimes referred to as a *subspace method*.
As before the array correlation matrix is calculated assuming uncorrelated noise with equal variances. Next find the eigenvalues and eigenvectors for R_{xx}. Will be found produce D eigenvectors associated with the signals and $N-D$ eigenvectors associated with the noise. Will be chosen the eigenvectors associated with the smallest eigenvalues. For uncorrelated signals, the smallest eigenvalues are equal to the variance of the noise. Then it can construct the $N\times$ ($N-$ D) dimensional subspace spanned by the noise eigenvectors such that

$$E_n=\left[\bar{e}_1 \quad \bar{e}_2 \quad \cdots \quad \bar{e}_{N-D}\right] \quad (10)$$

The noise subspace eigenvectors are orthogonal to the array steering vectors at the angles of arrival $\theta_1,\theta_2,\cdots,\theta_N$. Because of this orthogonality condition, one can show that the Euclidean distance $d_2=a(\theta)^H E_n E_n^H a(\theta)=0$ for each and every arrival angle $\theta_1,\theta_2,\cdots,\theta_N$. Placing this distance expression in the denominator creates sharp peaks at the angles of arrival. The MUSIC pseudospectrum is now given as

$$P_{MUSIC}(\theta)=\frac{1}{\left|a(\theta)^H E_n E_n^H a(\theta)\right|} \quad (11)$$

3. ACOUNTING FOR MUTUAL COUPLING AMONG AN ARRAY OF DIPOLES

Parallel thin wire dipole antennas are considered, for example, to account for mutual coupling among the antenna elements. The dipoles are assumed to be *z*-directed of length *l* and radius *a* and are placed along the *x*-axis, separated by a distance d_x. The port of each antenna element is located at the centre and is loaded with an impedance of Z_L ohms as shown in Figure 1, [3].

If an incoming electric field is linearly polarized, the *z*-component of the field is formulated as

$$E_z = E_0 e^{-j\bar{k}\cdot\bar{r}} \tag{12}$$

where

$$\bar{k} = -k[\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta] \tag{13}$$

is the wave vector associated with the direction of arrival of the incident field (θ,ϕ). Here θ is the elevation angle defined from the *z*-axis and ϕ is the azimuth angle defined in the *x*-*y* plane with the starting angle from the *x*-axis.

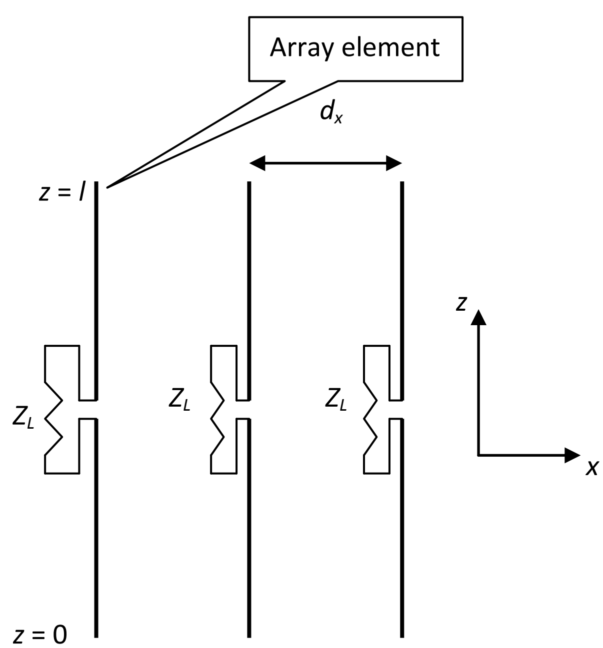

Fig. 1: Model of the receiving antenna as linear array.

Then the computed induced voltages are given by

$$V_i = \frac{E_0 e^{jkx_m \sin\theta\cos\phi}}{k\sin(k\Delta z)\sin^2\theta} 2e^{jkz_{q,m}\cos\theta}[\cos(k\Delta z\cos\theta) - \cos(k\Delta z)] \tag{14}$$

where x_m is the *x*-coordinate of the axis of the *m*th antenna, $i = [(m-1)P + q]$ and *P* is the number of segments. The term V_i tends to zero as θ tends to the end fire case, which corresponds to $\theta = 0$ and $\theta = \pi$. The entries of the impedance matrix $[Z]$ are given by

$$Z_{i,t} = \frac{j30}{\sin^2(k\Delta z)} \int_{z_{q-1,m}}^{z_{q,m}} \sin[k(z - z_{q-1,m})] \left\{ \frac{e^{-jkR_1}}{R_1} - 2\cos(k\Delta z)\frac{e^{-jkR_2}}{R_2} + \frac{e^{-jkR_3}}{R_3} \right\} dz + \frac{j30}{\sin^2(k\Delta z)} \int_{z_{q,m}}^{z_{q+1,m}} \sin[k(z_{q+1,m} - z)] \left\{ \frac{e^{-jkR_1}}{R_1} - 2\cos(k\Delta z)\frac{e^{-jkR_2}}{R_2} + \frac{e^{-jkR_3}}{R_3} \right\} dz \tag{15}$$

where

$$R_1 = \sqrt{(x_m - x_n)^2 + (z - z_{p-1,n})^2} \tag{16}$$

$$R_2 = \sqrt{(x_m - x_n)^2 + (z - z_{p,n})^2} \tag{17}$$

$$R_3 = \sqrt{(x_m - x_n)^2 + (z - z_{p+1,n})^2} \tag{18}$$

and $t = [(n-1)P + p]$. If $m = n$, the term $(x_m - x_n)$ is set equal to the radius *a*. The method of moment (MOM) admittance matrix is the inverse of the impedance matrix. To take the load impedance into account, the load impedance is added to the entries of the diagonal elements of $[Z]$ that correspond to the port of excitation. Define a new impedance matrix $[Z']$ such that

$$Z'(i,t) = Z(i,t) + Z_L \quad \text{if } i \text{ corresponds to a port} \tag{19}$$

$$Z'(i,t) = Z(i,t) \quad \text{otherwise} \tag{20}$$

The coefficients of the MOM current expansion for the loaded antennas are given by

$$[I] = [S][V] \tag{21}$$

where $[S] = [Z']^{-1}$ is the new admittance matrix of size $NP \times NP$. Due to the choice of an odd number of basis functions, only a single basis function is nonzero at the port. The measured voltages at the ports are therefore given by

$$[V_{meas}] = [Z_L][I_{port}] = [Z_L][S_{port}][V] = [C][V] \tag{22}$$

where

$$[Z_L] = diag[Z_L \quad Z_L \quad \cdots \quad Z_L]_{N\times N} \tag{23}$$

is the diagonal load matrix, $[I_{port}]$ is the vector of currents at the *N* ports, and $[S_{port}]$ is the $N \times NP$ rectangular matrix corresponding to the N_w row of $[S]$ that corresponds to the N_w port.

The $N \times NP$ matrix $[C] = [Z_L][S_{port}]$ has dimensionless entries.

4. SIMULATION RESULTS

In this section we present the simulation results to demonstrate the effect of mutual coupling between the array elements on the accuracy of the MUSIC algorithm, which is also compared to Bartlett algorithm. A linear array of $N = 12$ parallel thin wire dipole antennas are considered as actual antenna elements instead of the isotropic point sources. The inter-element spacing is $d = \lambda/2$. Each element of the array is divided into 7 segments and identically loaded by 50Ω at the centre. The dipoles are *z*-directed of length $l = \lambda/2$ and radius $a = \lambda/200$.

This linear array is used to estimate two uncorrelated signals (K = 2) arrived at angles 60^o and 120^o, based on a batch of $M = 10$ noisy data samples. Assume additive white Gaussian noise zero mean with SNR=10 dB.

Figure 2 and 3 illustrate the performance of the MUSIC and Bartlett algorithms, respectively, in the absence and presence of mutual coupling. It can be noted from the two Figures that there is a difference in the normalized pseudospectrum between the two cases for both algorithms. The two signal directions can be resolved by applying any algorithm of the two algorithms but the noise floor is increased due to the effect of mutual coupling between real antenna elements which degrades the performance significantly. Also, as been expected, the peak 3-dB beamwidth of MUSIC algorithm (about two degrees) for mutual coupling case is less than Bartlett algorithm (about ten degrees), so, the resolution of the MUSIC algorithm is better than the Bartlett algorithm.

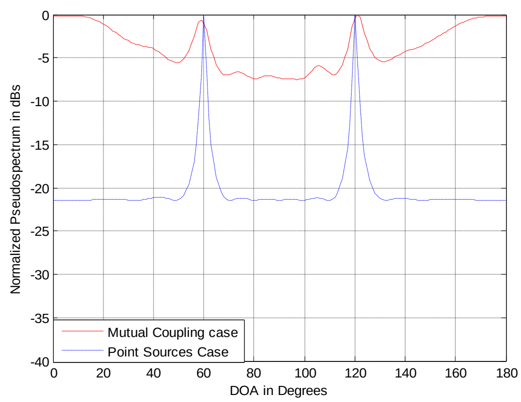

Fig. 2: Effect of the mutual coupling between the antenna elements on the MUSIC Pseudospectrum.

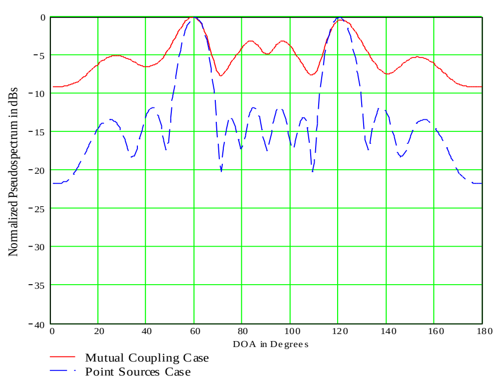

Fig. 3: Effect of the mutual coupling between the antenna elements on the Bartlett Pseudospectrum.

To enhance the performance of the MUSIC algorithm it must increase the number of the data samples (snapshots) and this is not satisfied on the Bartlett algorithm and this is illustrated in Figures 4 and 5. Here we increase the number of snapshots to $M = 100$ with SNR =10 dB.

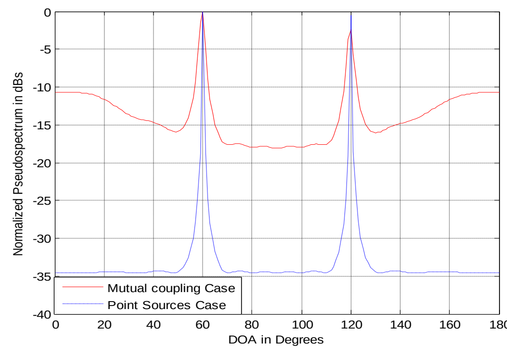

Fig. 4: Effect of the number of snapshots on MUSIC Pseudospectrum.

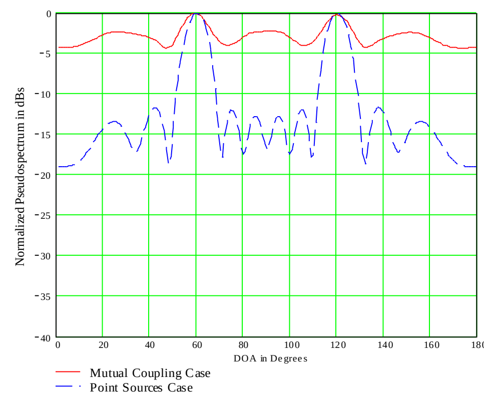

Fig. 5: Effect of the number of snapshots on MUSIC Pseudospectrum.

5. CONCLUSION

In adaptive array smart antenna, to locate the desired signal, various DOA estimation algorithms are used. Most of the DOA estimation algorithms are applied in the antenna array assumed to be isotropic point sources which are impractical. The simulation results illustrate that the MUSIC and Bartlett algorithms for DOA estimation can be applied on real (dipoles) array elements. Many numerical examples were introduced to measure the performance of the both algorithms and their ability to resolve incoming signals accurately and efficiently. The performance of the MUSIC algorithm depends on the number of the data snapshots. The MUSIC algorithm has highly accurate and stable and provides high angular resolution.

6. REFERENCES

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