

Design and Implementation of a Noise Tolerant Polynomial Nonlinear ARX Model using the Averaging Wavelet Method

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ABSTRACT

In this paper, a new nonlinear wavelet identification structure is proposed for high noise resistive soft sensors. This method uses proposed Polynomial Nonlinear Auto Regressive Exogenous Model, which can be solved with linear Gaussian Least Square Method, alongside the Averaging Wavelet Method (AWM) filter. AWM uses the approximation spaces for analyzing the signals and reduce the noise by a mean filtering over sub-resolutions. Conventional wavelet modeling methods use the detail spaces of the decomposed signal for signal modeling. The application results show that this method can be more accurate in high level noisy environments than the conventional wavelet modeling methods can tolerate.

General Terms

Wavelets, Nonlinear Auto Regressive Exogenous Modeling

Keywords

Averaging Wavelet Method, NARX, Noise tolerant modeling

1. INTRODUCTION

The real world systems are rarely linear and well defined for controlling. Most systems show nonlinear behaviors. The low power μ -sensors produce low power results in Pico-voltages or Pico-Amperes, which by transmitting over wires or wireless would be highly corrupted by noise. Employing these sensors to acquire data from nonlinear systems deteriorates the overall Signal to Noise Ratio (SNR). Analog hardware or digital hardware smoothing filters are conventionally employed to solve this problem. These kinds of filters reject the high frequencies or pass a frequency band that is needed in the process.

However, nonlinear systems may contain vital frequency components in the rejected region of the employed smoothing filters. These frequency components must be preserved to generate appropriate signals which will be used for system identification purposes. The solution is to develop an algorithm which truly extracts the nonlinear system structures and parameters using the acquired unfiltered sensor signals. The developed algorithm achieves this goal by simultaneously denoising and identifying the system.

Studies on wavelet algorithms for denoising have recently been started [1-3]. These algorithms reduce the noise in both time and frequency scales through analyzing different resolutions of

frequencies in space bounds, which makes this technique different from conventional cut-off filters.

In section (2), Polynomial Nonlinear Auto Regressive Exogenous (PNARX) modeling will be introduced. Section (3) discusses the Averaging Wavelet Method. In section (4) the Conventional Wavelet Method (CWM) for identification is studied. The newly proposed PNARX Averaging Wavelet Method (PNARX-AWM) has been discussed in section (5). In section (6), PNARX-AWM is compared to the conventional wavelet identification method.

2. THE PNARX MODELING

For a SISO nonlinear system the general nonlinear mathematical

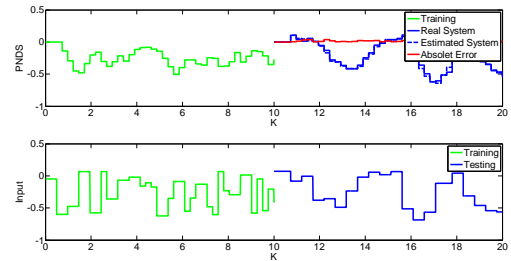


Figure 1. The PNARX identification method.

model can be given as:

$$y(t) = f \left(\frac{d^n y}{dt^n}, \frac{d^{n-1} y}{dt^{n-1}}, \dots, \frac{dy}{dt}, \frac{d^n u}{dt^n}, \frac{d^{n-1} u}{dt^{n-1}}, \dots, \frac{du}{dt}, u; P, W \right) \quad (1)$$

According to Shannon sampling theory we can write this model as the following nonlinear difference equation [4]:

$$y[k] = f(y[k-1], \dots, y[k-p], u[k-1], u[k-2], \dots, u[k-r]; P, W) \quad (2)$$

y is the output, u is the input, k is the sample index, P is the parameters vector and W is the uncertainty of the model.

If we consider the effect of each shifted input and output individually, then the equation (2) can be separated as:

$$y[k] = f_{1y}(y[k-1]) + f_{2y}(y[k-2]) + \dots + f_{py}(y[k-p]) + f_{1u}(u[k-1]) + f_{2u}(u[k-2]) + \dots + f_{ru}(u[k-r]) + W[k] \quad (3)$$

We can find the Taylor series expansion of each function in equation (3) at its operating point as:

$$f_{iy}(y[k-i]) = \sum_{n=0}^{n_p-1} h_n (y[k-i] - y_{0i})^n \quad (4)$$

$$f_{iu}(u[k-i]) = \sum_{n=0}^{n_r-1} g_n (u[k-i] - u_{0i})^n$$

Equation (4) can be further simplified to:

$$\begin{aligned} y[k] &= a_{11} + a_{12}y[k-1] + a_{13}y[k-1]^2 + \dots \\ &+ a_{1n_p}y[k-1]^{n_p-1} + a_{22}y[k-2] + a_{23}y[k-2]^2 + \dots \\ &+ a_{pn_p}y[k-p]^{n_p-1} + \dots \\ &+ b_{12}u[k-1] + b_{13}u[k-1]^2 + \dots + b_{mr}u[k-r]^{n_r-1} + W[k] \end{aligned} \quad (5)$$

This structure is nonlinear, however the equation is linear with respect to parameters which can be found by Gaussian Least Square method as:

$$m = \max\{p, r\} \quad (6)$$

$$\begin{aligned} Y^T &= [y(1), y(2), \dots, y(m), y(m+1), \dots, y(L)]; \\ m+N &= L; \\ p(n_p-1) + r(n_r-1) + 1 &\leq N \end{aligned} \quad (7)$$

$$\begin{aligned} y_k &= \left\{ \left[y^i(m-j+k) \right]_{i=1}^{n_p-1}; j=0, \dots, p-1 \right\} \\ u_k &= \left\{ \left[u^i(m-q+k) \right]_{i=1}^{n_r-1}; q=0, \dots, r-1 \right\} \end{aligned} \quad (8)$$

$$\varphi = \begin{bmatrix} 1 & y_0 & u_0 \\ \cdot & \cdot & \cdot \\ 1 & y_{N-1} & y_{N-1} \end{bmatrix} \quad (9)$$

$$\theta^T = [a_{11}, a_{12}, \dots, a_{1n_p}, \dots, a_{pn_p}, b_{12}, \dots, b_{mr}] \quad (10)$$

$$\hat{\theta} = (\varphi^T \varphi)^{-1} \varphi^T Y \quad (11)$$

Figure (1) demonstrates the input and output signals of a typical PNARX model. The upper figure shows the Polynomial Nonlinear Dynamic System (PNDS) of a 19 parameters system (in green), output of the identified system using the PNARX model (in dashed blue line) and the error between identified and the real system (in red). The figure shown at the bottom is the APRBS input to the system. The green portion of this figure is used for the training time and the blue portion is employed for testing the system.

3. AVERAGING WAVELET METHOD (AWM)[5]

Here we use, the Haar wavelet transform explained in [6]. A function $f(x)$ defined on $[0,1]$ has an expansion in terms of Haar functions as follows:

Given any integer $J \geq 0$ in L^2 on $[0,1]$ we have:

$$f(x) = \sum_{j=J}^{\infty} \sum_{k=0}^{2^j-1} \langle f, h_{j,k} \rangle h_{j,k}(x) + \sum_{k=0}^{2^J-1} \langle f, p_{J,k} \rangle p_{J,k}(x) \quad (12)$$

In order to motivate a discrete version of this expansion, the Discrete Haar Transform (DHT), we assume that we have only a finite, discrete approximation to $f(x)$. In this case, the most natural approximation is by the dyadic step function $P_N f(x)$, where $N \in \mathbb{N}$ and $N > J$. That is, given $f(x)$,

$$f(x) \approx P_N f(x) = \sum_{k=0}^{2^N-1} \langle f, P_{N,k} \rangle P_{N,k}(x) \quad (13)$$

Thus, the Haar coefficients of $f(x)$ can be approximated by the Haar coefficients of $P_N f(x)$. That is:

$$\langle f, h_{j,k} \rangle \approx \langle P_N f, h_{j,k} \rangle \quad \text{and} \quad \langle f, p_{j,k} \rangle \approx \langle P_N f, p_{j,k} \rangle \quad (14)$$

Therefore, we can make the following definition:

Given $J, N \in \mathbb{N}$ with $J < N$ and a finite sequence $c_0 = \{c_0(k)\}_{k=0}^{2^N-1}$, the DHT of c_0 is defined by:

$$\begin{aligned} \{d_j(k): 1 \leq j \leq J; 0 \leq k \leq 2^{N-j} - 1\} \\ \cup \{c_j(k): 0 \leq k \leq 2^{N-j} - 1\} \end{aligned} \quad (15)$$

, where:

$$c_j(k) = \frac{1}{\sqrt{2}} c_{j-1}(2k) + \frac{1}{\sqrt{2}} c_{j-1}(2k+1) \quad (16)$$

$$d_j(k) = \frac{1}{\sqrt{2}} c_{j-1}(2k) - \frac{1}{\sqrt{2}} c_{j-1}(2k+1) \quad (17)$$

The inverse DHT is given by the formula:

$$c_{j-1}(2k) = \frac{1}{\sqrt{2}} c_j(k) + \frac{1}{\sqrt{2}} d_j(k) \quad (18)$$

$$c_{j-1}(2k+1) = \frac{1}{\sqrt{2}} c_j(k) - \frac{1}{\sqrt{2}} d_j(k) \quad (19)$$

Based on the Haar transform the AWM is constructed. The traditional wavelet denoising method is based on computing the two composed and detail vectors of signal, then thresholding details in each d_j and consequently reconstructing the signal c_{j+1} . However, the detail vectors are important and the noise is unwanted. We use the DHT's equation (16) and the detail vectors are ignored. From equation (16) we find the composition vectors for $L \leq N/2$. Then we write the S_i sequences using c_i as:

$$S_i(k) = \frac{c_i\left(\left\lfloor \frac{k}{2^i} \right\rfloor\right)}{\sqrt{2^i}}; 1 \leq i \leq L; 0 \leq k \leq 2^N - 1 \quad (20)$$

The denoised sequences will then be constructed by averaging of all S_i as:

$$S(k) = \frac{\sum_{i=1}^L S_i(k)}{L} \quad (21)$$

Then we can repeat this procedure R times. Each time $S(k)$ is used as input to the algorithm. The codes and results can be downloaded from [4].

4. OFFLINE PNARX USING THE CONVENTIONAL WAVELET METHOD (PNARX-CWM)

For illustration, consider the nonlinear system shown in figure (2). The output of the system is the summation of two

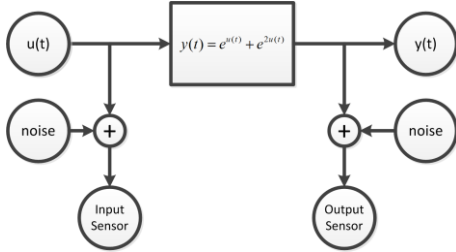


Figure 2. Nonlinear system in the presence of noise.

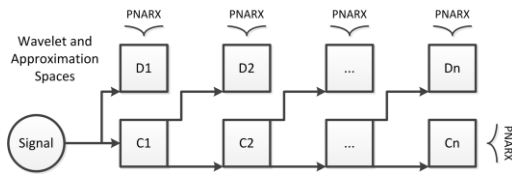


Figure 3. Each wavelet space and the last approximation space have its own PNARX identification.

exponential functions of input signal (with coefficient constants of 1 and 2) as:

$$y(t) = e^{u(t)} + e^{2u(t)} \quad (22)$$

The structure of the PNARX-CWM can be seen in figure (3). Each wavelet space and the last approximation space have their own PNARX model.

Figure (4) shows the Amplitude-modulated Pseudo Random Binary Signal (APRBS) described in [7]. In this work, this signal is used as the excitation signal. The length, amplitude and sampling time of the excitation signal must be chosen to capture the nonlinearity and frequency range of the system to be identified.

As shown in figure (4), after identification of the system, the identified parameters have been testified by a variable frequency non-stationary input and the simulation results are shown in figure (6):

$$u(t) = \begin{cases} 0.5 \cos(2t) + 1 & 6\pi \leq t < 8\pi \\ 0.5 \cos(4t) + 1 & 8\pi \leq t < 10\pi \\ 0.5 \cos(8t) + 1 & 10\pi \leq t \leq 12\pi \end{cases} \quad (23)$$

By adding noise to the signal from 1% to 100%, as shown in table (1), the MSE obtained using PNARX-CWM will be around 43 to 68. As shown in this table the identification technique deteriorates the MSE if the level of added noise is less than 50%. Improvement in the magnitudes of MSE is achieved for the level of noise greater than 50%. At 100% noise level, the MSE of the identified signal will almost be half of the MSE of the original signal. According to table (2), the SNR of this method will vary from 4 to 6.

5. OFFLINE PNARX USING THE AVERAGING WAVELET METHOD (PNARX-AWM)

Unlike PNARX-CWM, this method identifies each approximation space in lower resolutions and reconstructs the

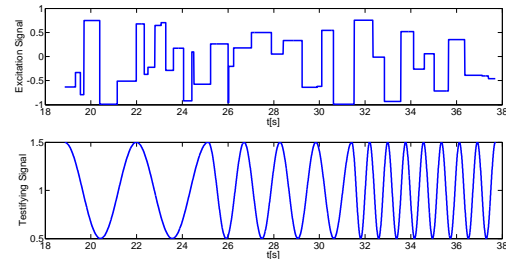


Figure 4. The APRBS excitation signal (the upper signal). The variable frequency signal (the lower signal) used for testing the identified model.

original signal by the AWM algorithm explained in section (3). The structure of the identification algorithm is shown in figure (5) and the simulation results are shown in figure (7).

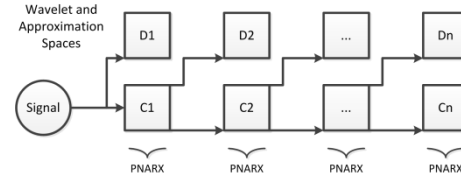


Figure 5. Each approximation space has its own PNARX identification.

For identification, we use an APRBS signal in offline mode and then testify the identified system by non-stationary input given by equation (26). As shown in table (1) the identification technique deteriorates the MSE if the level of added noise is less than 20%. Reduction in the magnitudes of MSE is achieved for the level of noise greater than 20%. At 100% noise level, the MSE of the identified signal will almost be one tenth of the MSE of the original signal. Therefore, the level of achieved MSE for this technique is always less than the MSE obtained in PNARX-CWM. According to table (2), the SNR of this method will vary from 10 to 13, which are always better than the SNR quoted for PNARX-CWM.

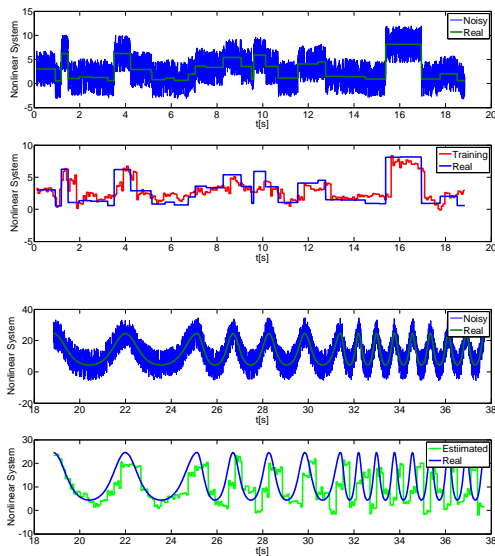


Figure 6. Identification of the nonlinear system using PNARX-CWM (the training is performed using 50% input-output noisy data).

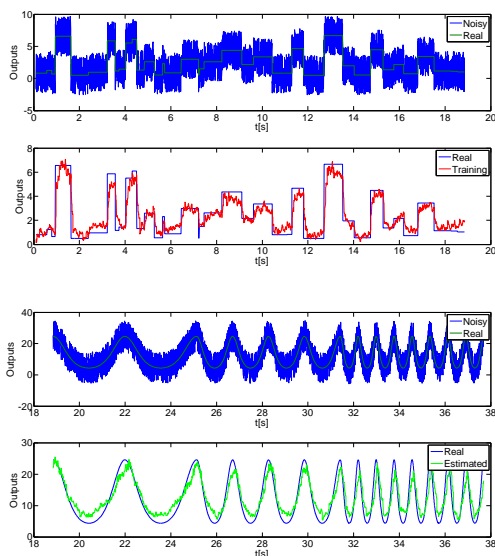


Figure 7. Identification of the nonlinear system using PNARX-AWM (the training is performed using 50% input-output noisy data).

6. CONCLUSIONS

The results in table (1) and table (2) show that the developed PNARX-AWM method outperforms the conventional wavelet identification method for signals with high level of noises. The proposed method results a substantial reduction of MSE. In addition, a significant improvement in SNR is achieved. These

Table 1. MSE comparison of PNARX-CWM and PNARX-AWM.

Noise Percent	Noisy System MSE	MSE of PNARX-CWM	MSE of PNARX-AWM
1%	0.013	68.71	8.161
5%	0.339	51.79	7.665
10%	1.388	50.37	9.189
20%	5.440	58.65	10.27
50%	33.80	53.65	10.86
80%	86.75	43.83	12.65
100%	138.3	63.17	14.17

Table 2. SNR comparison of PNARX-CWM and PNARX-AWM.

Noise Percent	Noisy System SNR	SNR of PNARX-CWM	SNR of PNARX-AWM
1%	41.62	4.61	13.11
5%	27.71	5.161	13.44
10%	21.61	4.599	12.49
20%	15.73	4.427	12.55
50%	8.352	6.299	11.84
80%	5.205	5.693	11.91
100%	3.770	4.338	10.73

data are derived using the PNARX and Wavelet structure with the following parameters: $N=12$, $v=6$, $p=5$, $r=1$, $np=3$, $nr=2$. Codes and results of simulation can be downloaded from: http://ekoshv.persiangig.com/MATLABCODES/PNARX_ACWM/PNARXACWM.rar

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