Unsteady Heat Transfer to MHD Oscillatory Flow through a Porous Medium under Slip Condition

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ABSTRACT
In this paper, we investigate the effects of slip condition, transverse magnetic field and radiative heat transfer to unsteady flow of a conducting optically thin fluid through a channel filled with porous medium. Exact solution of the governing equations for fully developed flow is obtained in closed form. Detailed computations of the influence of the Grashof number, Hartmann number, slip parameter, porosity parameter, radiation parameter and frequency of the oscillation are discussed.

Keywords: Heat transfers, oscillatory flow, porous medium, slip condition, MHD

1. INTRODUCTION
Convective motion in a porous medium has attracted considerable attention from many researches because of its application in geophysics, oil recovery technique, thermal insulation, engineering and heat storage. The study of electrically conducting fluid has many applications in engineering problems such as magnetohydrodynamic (MHD) generators plasma studies, nuclear reactors, geothermal energy extraction, and boundary layer in the field of aerodynamic (Makinde and Mhone, 2005). In view of the applications of free convective and heat transfer flows through porous medium under the influence of magnetic field many researchers have studied magnetohydrodynamic free convective heat transfer flow in a porous medium. (Raptis et al 1985) studied the unsteady free convective through a porous medium bounded by an infinite vertical plate. (Ram and Mishra, 1976) analyzed unsteady flow through MHD porous media. (Mansutti et al 1993) have studied the steady flow of non- Newtonian fluids past a porous plate with suction or injection. MHD unsteady free convection Walter's memory flow with constant suction and heat sink was studies by (Ramana et al 2007). (Mustafa et al 2008) investigated unsteady MHD memory flow with oscillatory suction, variable free stream and heat sources. (Alagoa et al, 1998) studied MHD optically transparent free convection flow with radiative heat transfer in porous media with time-dependent suction using an asymptotic approximation, showing that thermal radiation exerts a significant effect on the flow dynamics. (El-Hakiem 2000) analyzed thermal radiation effects on transient, two-dimensional hydromagnetic free convection along a vertical surface in a highly porous medium using the Roseland diffusion approximation for the radiative heat flux in the energy equation, for the case where free- stream velocity of the fluid vibrates about mean constant value and the surface absorbs the fluid with constant velocity. In all these investigations "no-slip" boundary condition is considered for the velocity.

The phenomenon of slip-flow regime has attracted the attention of a large number of scholars due to its wide ranging application. The problem of the slip flow regime is very important in this era of modern science, technology and vast ranging industrialization. In practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite tangential velocity, it slips along the surface. The flow regime is called the slip flow regime and its effect cannot be neglected. The fluid slippage phenomenon at the solid boundaries appears in many applications such as micro channels or nano channels. The slip flow regime can also occur in the working fluid containing concentrated suspensions (Soltani and Yilmazer, 1998). Recently, several scholars have suggested that the no-slip boundary conditions may not be suitable for hydrophilic flows over hydrophobic boundaries at both the micro and nano scale (Watanebe et al, 1998), Watanebe et al (1999), Ruckenstein and Rahora (1983). A detailed account of the developments on the subject has been given by Mehmood and Ali (2007). The effect of slip conditions on the MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi (2006). Khaled and Vafai (2004) obtained the closed form solutions for steady periodic and transient velocity field under slip condition. Jain and Gupta analyzed the free convection flow of unsteady magneto polar fluid with variable permeability and constant heat flux in slip flow regime. Mansour et al (2007) studied the free convection flow of micropolar fluid in slip flow regime through porous medium with periodic temperature and concentration.

In the present paper, we investigate the influence of slip conditions, magnetic field and radiative heat transfer on unsteady flow of conducting optically thin fluid through a channel filled with porous medium and surface temperature oscillating.

2. MATHEMATICAL FORMULATION
Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer. It is assumed that the fluid has a small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system(x, y), where x lies along the center of the channel, y is the distance measured in the normal section. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given as:

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{K'}u' - \frac{\sigma_e B_0^2 u'}{\rho} + g\beta\left(T' - T'_0\right) \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q'}{\partial y'} \quad (2)$$

With boundary conditions

$$\left.\begin{array}{ll} u' - \gamma^*\dfrac{\partial u'}{\partial y'} = 0, \quad T' = T'_\omega & on\ y' = 0 \\ u' = 0, T' = T'_0 + (T'_w - T'_0)\cos\omega' t', & on\ y' = 1 \end{array}\right\} \quad (3)$$

Where u' is the axial velocity, t' is the time, ω' is the frequency of the oscillation, T' the fluid temperature, P the pressure, g gravitational force, C_p the specific heat at constant pressure, k the thermal conductivity, q the radiative heat flux, β the coefficient of volume expansion, K' the porous medium permeability coefficient, Bo the electromagnetic induction,σe the conductivity of the fluid, ρ the density of the fluid, v is the kinematics viscosity coefficient. It is assumed that walls temperature T'o, T'_ω are high enough to induce radiative heat transfer, and γ^* is the dimensionless slip parameter. Following Makinde and Mhone (2005), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q'}{\partial y'} = 4\alpha^2\left(T'_0 - T'\right) \quad (4)$$

Where α is the mean radiation absorption coefficient. The following dimensionless variables and parameters are introduced:

$$\left.\begin{array}{l} \mathrm{Re} = \dfrac{Ua}{\nu}, x = \dfrac{x'}{a}, y = \dfrac{y'}{a}, u = \dfrac{u'}{U}, \theta = \dfrac{T' - T'_0}{T'_\omega - T'_0}, \\ H^2 = \dfrac{a^2\sigma_e B_0^2}{\rho\nu}, t = \dfrac{t'U}{a}, p = \dfrac{aP'}{\rho\nu U}, D_a = \dfrac{K'}{a^2}, \\ Gr = \dfrac{g\beta\left(T'_\omega - T'_0\right)a^2}{\nu U}, Pe = \dfrac{Ua\rho C_p}{k}, \\ N^2 = \dfrac{4\alpha^2 a^2}{k}, \gamma = \dfrac{\gamma^*}{a}. \end{array}\right\} \quad (5)$$

Where U is the flow mean velocity, the dimensionless governing equations together with appropriate boundary conditions can be written as

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial \rho}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(s^2 + H^2\right)u + Gr\theta \quad (6)$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta \quad (7)$$

$$\left.\begin{array}{ll} u - \gamma\dfrac{\partial u}{\partial y} = 0, \quad \theta = 0 & \text{on } y = 0 \\ u = 0, \quad \theta = \cos\omega t & \text{on } y = 1 \end{array}\right\} \quad (8)$$

Where Gr is the thermal Grashof number, H is the Hartmann number, N is the radiation parameter, Pe is the Peclet number, Re is the Reynolds number, Da is the Darcy number, Pr is the Prandtl number, γ is the slip parameter and s is the porous medium shape factor.

3. METHOD OF SOLUTION

In order to solve (6) to (8) for purely oscillatory flow, let the pressure gradient, fluid velocity and temperature be

$$-\frac{\partial \rho}{\partial x} = \lambda(e^{i\omega t} + e^{-i\omega t}) \quad (9)$$

$$u(y,t) = u_0(y)e^{i\omega t} + u_1(y)e^{-i\omega t}$$

$$\theta(y,t) = \theta_0(y)e^{i\omega t} + \theta_1(y)e^{-i\omega t} \quad (10)$$

Where $\lambda < 0$ for favorable pressure, ω is the frequency of the oscillation. Substituting the above expressions in (9) and (10) into (6), (7) and (8), we obtained (1)

$$\frac{d^2u_0}{dy^2} - m_1^2 u_0 = -\lambda - Gr\theta_0 \quad (11)$$

$$\frac{d^2u_1}{dy^2} - m_2^2 u_1 = -\lambda - Gr\theta_1 \quad (12)$$

$$\frac{d^2\theta_0}{dy^2} + N_1^2\theta_0 = 0 \quad (13)$$

$$\frac{d^2\theta_1}{dy^2} + N_2^2\theta_1 = 0 \quad (14)$$

$$\left.\begin{array}{l} u_0 - \gamma \dfrac{du_0}{dy} = 0, u_1 - \gamma \dfrac{du_1}{dy} = 0, \theta_0 = \theta_1 = 0, \; on \;\; y = 0 \\ u_0 = u_1 = 0, \quad \theta_0 = \theta_1 = 1/2, \qquad on \;\; y = 1 \end{array}\right\} \quad (15)$$

Where $m1 = \sqrt{s^2 + H^2 + \mathrm{Re}\, i\omega}$, $m2 = \sqrt{s^2 + H^2 - \mathrm{Re}\, i\omega}$,

$N1 = \sqrt{N^2 - Pei\omega}$, $N2 = \sqrt{N^2 + Pei\omega}$

Equations (11) to (15) are solved and the solution for fluid temperature and velocity are given as follows:

$$\theta(y,t) = \frac{1}{2}\left[\frac{\sin N1y}{\sin N1} e^{i\omega t} + \frac{\sin N2y}{\sin N2} e^{-i\omega t}\right] \quad (16)$$

$$u(y,t) = \left[A \cosh m1y + B \sinh m1y + \lambda_1 + \eta_1 Gr \sin N1y\right] e^{i\omega t} + \left[C \cosh m2y + D \sinh m2y + \lambda_2 + \eta_2 Gr \sin N2y\right] e^{-i\omega t} \quad (17)$$

The rate of heat transfer across the channel's wall is given as

$$\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = \frac{1}{2}\left[\frac{N1}{\sin N1} e^{i\omega t} + \frac{N2}{\sin N2} e^{-i\omega t}\right] \quad (18)$$

The shear stress at the lower wall of the channel is given by

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = \left[Bm1 + \eta_1 GrN1\right] e^{i\omega t} + \left[Dm2 + \eta_2 GrN2\right] e^{-i\omega t} \quad (19)$$

Where

$\lambda_1 = \dfrac{\lambda}{m_1^2}$, $\lambda_2 = \dfrac{\lambda}{m_2^2}$, $\eta_1 = \dfrac{1}{(N_1^2 + m_1^2)\sin N_1}$,

$\eta_2 = \dfrac{1}{(N_2^2 + m_2^2)\sin N_2}$, $d1 = \sinh m_1 + \gamma m_1 \cosh m_1$,

$d2 = \sinh m_2 + \gamma m_2 \cosh m_2$,

$A = \dfrac{-B \sinh m_1}{\cosh m_1} + \dfrac{\lambda}{m_1^2 \cosh m_1} + \dfrac{Gr}{\cosh m_1 (N_1^2 + m_1^2)}$,

$C = \dfrac{-D \sinh m_2}{\cosh m_2} + \dfrac{\lambda}{m_2^2 \cosh m_2} + \dfrac{Gr}{\cosh m_2 (N_2^2 + m_2^2)}$,

$B = \dfrac{Gr}{(N_1^2 + m_1^2)d1} + \dfrac{\lambda}{m_1^2 d1} + \dfrac{\lambda \cosh m_1}{m_1^2 d1} \ldots - \dfrac{\gamma N_1 Gr \cosh m_1}{\sinh m_1 (N_1^2 + m_1^2) d2}$,

$D = \dfrac{Gr}{(N_2^2 + m_2^2)d2} + \dfrac{\lambda}{m_2^2 d2} + \dfrac{\lambda \cosh m_2}{m_2^2 d2} \ldots - \dfrac{\gamma N_2 Gr \cosh m_2}{\sinh m_2 (N_2^2 + m_2^2) d2}$

4. GRAPHICAL RESULTS AND DISCUSSION

To study the effects of wall slip, magnetic field, radiation parameter, thermal buoyancy force, porosity of medium, Peclet number, Reynolds number and oscillations on flow-field numerical values are computed from analytical solution. We made use of the following parameter values except otherwise indicated, Re = 1, S = 1, H = 1, Gr = 1, Pe = 1, N = 1, $\lambda = -1$, $\gamma = 2$ and $\omega t = \pi/2$.

The temperature profiles have been studied and presented in Figures 1 to 3. The temperature profiles for different values of the radiation parameter (N = 0.1, 0.5, 1.0, 1.5), frequency of the oscillation (ω =0.2, 0.4, 0.6, 0.8) and Peclet number (Pe = 1, 2, 3, 4) are shown in Figures 1, 2, and 3 respectively. It is observed that the temperature increases with increasing radiation parameter, frequency of the oscillation and Peclet number. The velocity profiles have been studied and presented in Figures 4 to 7. The velocity profiles for different values of the slip parameter (γ =0.0, 0.5, 1.0, 2.0) are presented in Figure 4. It is observed that the velocity increase with increasing slip parameter. The velocity profiles for different values of the Hartmann number (H = 0.0, 0.5, 1.0, 2.0), porosity parameter (S = 0.0, 0.5, 1.0, 2.0), and the Grashof number (Gr = 0.0, 0.5, 1.0, 2.0) are shown in Figures 5, 6 and 7 respectively. It is observed that the velocity decrease with increasing Hartmann number, porosity parameter and Grashof number. The variation of the Nusselt number and Skin friction on the porous plate with material parameters is presented in table 2, and table 1 respectively.

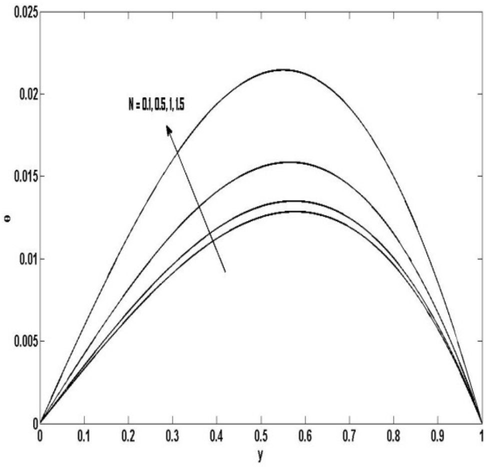

Figure 1: Temperature profiles for different values of the radiation paramete

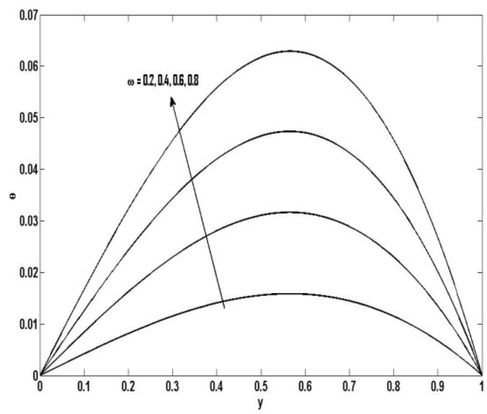

Figure 2: Temperature profiles for different values of the frequency of the oscillation

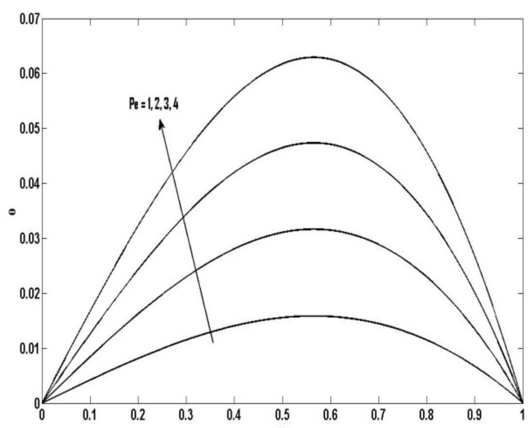

Figure 3: Temperature profiles for different values of the Peclet number

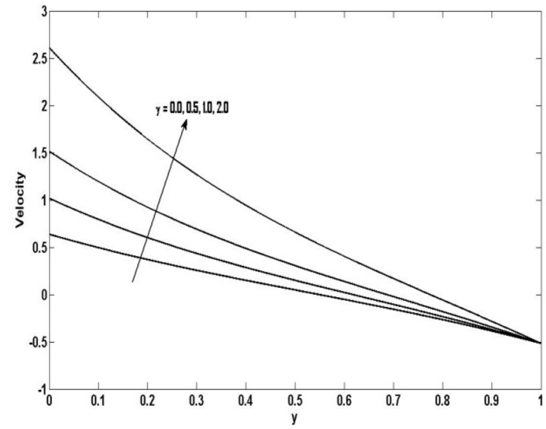

Figure 4: Velocity profiles for different values of the slip parameter

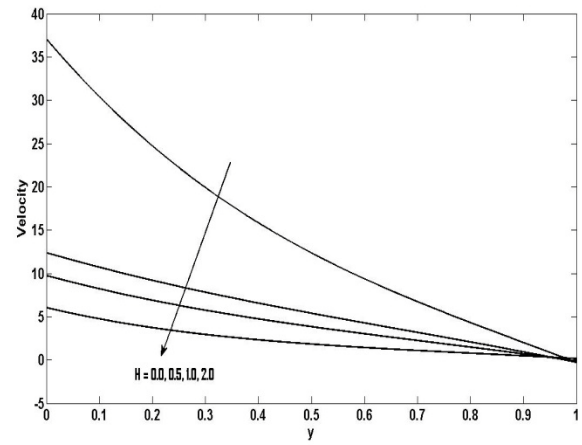

Figure 5: Velocity profiles for different values of the Hartmann number

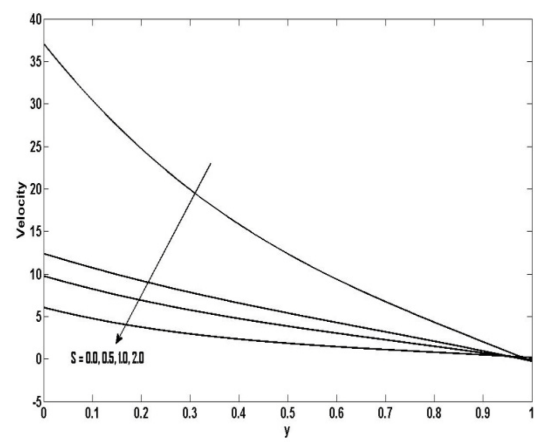

Figure 6: Velocity profiles for different values of the porosity parameter

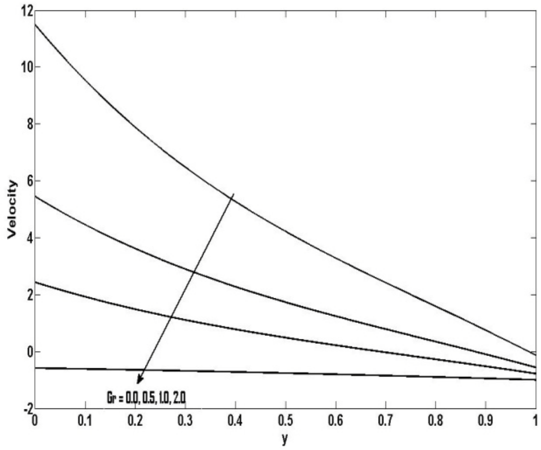

Figure 7: Velocity profiles for different values of the Grashof number

Table 1: Values of the Nusselt number with different values of material parameters

N	Pe	ω	Nu
1	1	2	0.7862
5	1	2	0.1998
1	5	2	2.1272
1	1	5	1.4316
1	10	2	3.0842
1	1	10	2.1272
10	1	2	0.1000

Table 2: Values of Shear Stress with different values of material parameters

Re	H	S	γ	Gr	ω	τ
1	1	1	2	1	2	10.7776
5	1	1	2	1	2	10.7776
1	1	1	5	1	2	24.0003
1	5	1	2	1	2	10.7776
1	1	1	2	5	2	52.7150
1	1	5	2	1	2	10.7776
1	1	1	2	1	5	7.9258

5. CONCLUSIONS

This paper investigates the transient heat transfer to MHD oscillatory flow through porous medium under slip condition and oscillating temperature. The velocity and temperature profiles are obtained analytically. The effect of different parameters namely, the radiation parameter, Grashof number, Hartmann number, Porosity parameter, Slip parameter, frequency of the oscillation and Peclet number are studied. The conclusions of the study are as follows.

(i) It is observed that the temperature increase with increasing radiation parameter, Peclet number or frequency of the oscillation.

(ii) It is observed that the velocity increase with increasing slip parameter

(iii) It is observed that the velocity decrease with decreasing magnetic parameter, porosity parameter or Grashof number.

(iv) An increase in the Peclet number or frequency of the oscillation causes an increase in Nussetl number, while increasing radiation parameter weaken the Nusselt number.

(v) An increase in Grashof number or slip parameter lead to the increasing in shear stress, while increasing frequency of the oscillation decreases the shear stress.

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