Replacement Problem with Grey Parameters

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ABSTRACT
Capital cost, resale value and running cost including maintenance costs, repairing costs, and operation costs of equipment are considered as crisp numbers in ordinary replacement problem. Nevertheless, in a special situation such as operating military equipments during wartime, the working efficiency of the machine and its related costs are no longer crisp but uncertain. This uncertainty can be represented by interval grey numbers. The aim of the paper is to study the replacement problem , where the grey costs are to be considered as interval grey numbers. In the model construction of the problem, we simply use the arithmetic properties of interval grey numbers. We determine a replacement time by considering an average annual cost with grey numbers. Possibility degree of grey numbers is used to make the order preference of average annual costs. We provide a numerical example to demonstrate the potentiality of the proposed method.

General Terms
Replacement Problem

Keywords
Grey number, Grey interval number, Replacement problem.

1. INTRODUCTION
Replacement Problem (RP) is one of the key decision areas in engineering economic policy. In our daily life, replacement of an equipment or item is needed to maintain our desired efficiency level. The main task of the RP is to identify the best policy in determining the time at which the replacement is most economical instead of continuing under the non-profitable situation. Modeling of the replacement problem is presented by many authors. Nakagawa and Osaki [1] introduced the age replacement problem. Bellman [2] studied RP with dynamic programming. Hartman and Rogers [3] extended dynamic programming approaches for equipment replacement problems with continuous and discontinuous technological change. Thi et al [4] discussed an optimal maintenance and replacement decisions under technological change. They used stochastic dynamic programming in dealing with the optimal maintenance and replacement policy of equipment as a function of performance and cost. Abdelwali [5] discussed parametric multi-objective dynamic programming with application to automotive problems. Abdelwali et al. [6] studied optimum replacement policies and applied the concept for Kuwait Passenger Transport Company. They used dynamic programming technique for generating the optimal replacement policies for buses. Ahmed [7] studied an algorithm for a deterministic RP. RP with different criteria were studied [8-14] in the literature. To deal with uncertainty in solving RP, decision-making unit has to model RP with each or some parameters involving imprecise values. At this situation, the concept of fuzzy set theory developed by Zadeh [15] appears to be important to deal with uncertainty. Ward [16] discussed cash flow analysis with a trapezoidal fuzzy numbers to solve fuzzy present worth problem. Buckley [17] developed fuzzy net present value and fuzzy net future value using fuzzy numbers. Chiu and Park [18] also use fuzzy numbers in cash flow analysis. Teng [19] studied fuzzy strategic replacement analysis. Khalil [20] studied the vehicle replacement model using multi-objective fuzzy integer programming. Biswas and Pramanik [21] discussed RP with fuzzy numbers. They determine the replacement time by Yager's ranking method [22]. Choobinch and Behrens [23] have discussed the application of intervals and possibility distribution in economic analysis.

For a conventional replacement problem, the related costs of equipment are represented by crisp numbers. However, if we consider a special case such as during wartime, and other such similar type of situation, the equipment related costs are no longer crisps. In that environment, all costs may have uncertain values. This uncertainty can be described by grey numbers. In this paper, we have proposed RP under uncertainty with grey system theory proposed by Prof Deng [24] in 1982. Grey system theory is the alternative way to study the uncertainty. Here, we consider grey analysis based approach to deal with RP when the distribution between lower and upper bound of information is unknown. We assume that the capital cost $C(\otimes)$, scrap value $S(\otimes)$, maintenance cost or running cost of equipment or item are all interval grey numbers. We also assume that the replacement of equipments or items deteriorates with time and the value of money remains stable. Comparison of minimum average annual cost is considered using grey possibility degree [25]. Using grey possibility degree, we have the minimum average value and it helps us in determining the replacement time of equipment.

This paper is organized as follows. In section 2, preliminaries of grey numbers are discussed. In section 3, operations of interval grey numbers are provided. Replacement model has been formulated in section 4. In section 5, proposed model on replacement problem with grey parameters has been presented. An example has been illustrated to demonstrate the effectiveness of the proposed method in section 6. Finally, section 7 concludes the paper with conclusion and direction of future works.

2. PRELIMINARIES OF GREY NUMBERS

In grey system, the information is divided into three categories namely, white, grey, and black. White information deals with totally known information. Grey information deals with insufficient information i.e. grey information means one can get some part of information or approximate range of amount of the information instead of getting completely precise information. Finally, black information deals with totally unknown information. Incomplete information is the basic characteristic of the problem considered in grey system theory. In this section, some basic definitions of grey numbers are provided following Sifeng Liu and Yi Lin, 2004 [26].

2.1 Definition: A Grey number is such a number whose exact value is unknown, but a range within which the value lies is known. This grey number is generally written by the symbol "$\otimes$." There are several types of grey numbers.

2.1.1 Grey number with only a lower bound: This kind of grey number $\otimes$ is written as $\otimes \in [\underline{e}, \infty)$ or $\otimes(\underline{e})$, where e stands for the definite, known lower bound of the grey number $\otimes$. The interval $[\underline{e}, \infty)$ is referred to as the field of $\otimes$.

2.1.2 Grey number with only an upper bound: This kind of grey number $\otimes$ is written as $\otimes \in (-\infty, \overline{e}]$ or $\otimes(\overline{e})$, where $\overline{e}$ stands for the definite known upper bound of $\otimes$.

2.1.3 Interval grey number: This kind of grey number $\otimes$ has both a lower $\underline{e}$ and an upper bound $\overline{e}$, written $\otimes \in [\underline{e}, \overline{e}]$.

2.1.4 Discrete grey number: A grey number that only takes a finite number or a countable number of potential values is known as discrete.

2.1.5 Continuous grey number: If a grey number can potentially take any value within an interval, then it is known as continuous.

2.1.6 Black and white numbers: When $\otimes \in (-\infty, +\infty)$, that is, when $\otimes$ has neither any upper nor lower bound, then $\otimes$ is known as a black number.

2.1.7 White number: When $\otimes \in [\underline{e}, \overline{e}]$ and $\underline{e} = \overline{e}$, $\otimes$ is known as a white number.

2.1.8 Essential and non-essential grey numbers: Essential grey number that temporarily cannot be represented by a white number and the non-essential grey number can be represented by a white number obtained through either experience or certain method.

2.1.9 Whitenization value of grey number: There is a class of grey numbers, which vibrate around a base value. This base value can be used as the main whitenization value. So a grey number with a base value e can be denoted as $\otimes(e) = e + \delta_e$ or $\otimes(e) \in (-, e, +)$, where δ_e stands for the vibration variable.

For a general interval grey number $\otimes \in [e, f]$, we take its whitenization value $\tilde{\otimes}$ as $\tilde{\otimes} = \alpha e + (1-\alpha) f$, $\alpha \in [0,1]$ is called equal weight whitenization.

In an equal weight whitenization, the whitenization value, obtained when taking $\alpha = 1/2$, is called an equal weight mean whitenization.

3. OPERATIONS OF INTERVAL GREY NUMBERS

Let us assume the two grey numbers as $\otimes_1 \in [e, f]$, $e < f$ and $\otimes_2 \in [g, h]$, $g < h$.

3.1 Addition: The sum of $\otimes_1 \in [e, f]$ and $\otimes_2 \in [g, h]$, written as $\otimes_1 + \otimes_2$, is defined as follows:
$\otimes_1 + \otimes_2 \in [e + g, f + h]$.

3.2 Inverse: The negative inverse of $\otimes_1 \in [e, f]$ where $e < f$, is written as $-\otimes$. It is defined as $-\otimes = [-f, -e]$.

3.3 Difference: The difference of $\otimes_1 \in [e, f]$ with $\otimes_2 \in [g, h]$, is defined as $\otimes_1 - \otimes_2 \in [e - h, f - g]$.

3.4 Reciprocal: Assume $\otimes_1 \in [e, f]$ $e < f, ef > 0$, the reciprocal of $\otimes$, written and defined as $\otimes_1^{-1} \in [1/f, 1/e]$.

3.5 Multiplication: If $\otimes_1 \in [e, f]$, $e < f$ and $\otimes_2 \in [g, h]$, $g < h$, the product of $\otimes_1$ and $\otimes_2$ is defined as $\otimes_1 * \otimes_2 \in$ [min {eg, eh, fg, fh}, max {eg, eh, fg, fh}].

3.6 Scalar multiplication: If $\otimes_1 \in [e, f]$, $e < f$ and k is a positive real number, the scalar multiplication of k and $\otimes$ is defined as $k * \otimes \in [ke, kf]$.

3.7 Division: If $\otimes_1 \in [e, f]$, $e < f$, and $\otimes_2 \in [g, h]$, $g < h$, and $gh > 0$. The division of $\otimes_1$ and $\otimes_2$ is defined as:

$\otimes_1 / \otimes_2 = \otimes_1 * \otimes_2^{-1}$ i.e.

$$\frac{\otimes_1}{\otimes_2} \in \left[\min\left\{\frac{e}{g}, \frac{e}{h}, \frac{f}{g}, \frac{f}{h}\right\}, \max\left\{\frac{e}{g}, \frac{e}{h}, \frac{f}{g}, \frac{f}{h}\right\}\right].$$

3.8 Length of grey number: For a grey number $\otimes_1 \in [e, f]$, $e < f$ defined on the information field [e, f], the length of grey number is defined by $L(\otimes_1) = |f - e|$.

3.9 Comparison of grey numbers:

To compare the grey numbers, grey possibility degree can be used.

3.9.1 Definition: The possibility degree of two grey numbers $\otimes_1 \in [e, f]$ $e < f$, $\otimes_2 \in [g, h]$, $g < h$. can be expressed as

$$P\{\otimes_1 \le \otimes_2\} = \frac{Max(0, L^* - Max(0, f - g))}{L^*} \quad (1)$$

where $L^* = L(\otimes_1) + L(\otimes_2)$. The following position relationship between two grey numbers $\otimes_1 \in [e, f]$, $e < f$ and $\otimes_2 \in [g, h]$, $g<h$ can be determined as:

3.9.1.1 If e = g and f = h, then $\otimes_1 = \otimes_2$ and $P\{\otimes_1 \le \otimes_2\} = .5$ (2)

3.9.1.2 If $g > f$, then $\otimes_1 < \otimes_2$ and $P\{\otimes_1 \le \otimes_2\} = 1.$ (3)

3.9.1.3 If $h < e$, then $\otimes_1 > \otimes_2$ and $P\{\otimes_1 \le \otimes_2\} = 0.$ (4)

3.9.1.4 Finally, if there is an intercrossing between two grey numbers and $P\{\otimes_1 \le \otimes_2\} > 0.5$ then $\otimes_1 < \otimes_2$.
If $P\{\otimes_1 \le \otimes_2\} < 0.5$. then $\otimes_1 > \otimes_2$. (5).

4. FORMULATION OF REPLACEMENT PROBLEM

The study of replacement of equipment is a field of application rather than a method of analysis. This study concerns with the comparison of alternative replacement policies. In this case, the deterioration process is predictable and is represented by an increased maintenance cost and decreased in scrap cost and increased production cost per unit. We calculate the cost per unit of time, without considering the changes of value of currency. Here, we determine the total cost up to the period. Then, we divide the total cost by time unit to find out the average cost for making a decision of replacement time.

Let C: Capital cost of equipment.
S: Scrap value of equipment.
n: Number of years that equipment would be used.
m_t : Maintenance cost for time t.
$\Phi(n)$: Average total cost.

Here we consider the time t is discrete variable.

Let us assume that the equipment would be used for n-years, and then the total cost incurred on the equipment during t- years is:

Total Cost = Capital Cost – Scrap Value + Maintenance cost

$T(c) = C - S + \sum_{t=1}^{n} z_t$ and its average cost $\Phi(n)$ is given by

$$\Phi(n) = \frac{C-S}{n} + \frac{1}{n}\sum_{t=1}^{n} z_t$$

Here $\Phi(n)$ will be minimum value for the year n,

if $\Phi(n-1) \ge \Phi(n) \le \Phi(n+1)$.

$$\text{Now } \Phi(n+1) = \frac{C-S}{n+1} + \frac{1}{n+1}\sum_{t=1}^{n+1} z_t$$

$$= \frac{1}{n+1}\left[C - S + \sum_{t=1}^{n} z_t\right] + \frac{1}{n+1} z_{(n+1)}$$

$$= \frac{1}{n+1}\,[n\ \Phi(n) + z_{(n+1)}]$$

Therefore, $\Phi(n+1) - \Phi(n) = \frac{1}{n+1}\,[\, z_{(n+1)} - \Phi(n)\,]$

Therefore, $\Phi(n+1) \ge \Phi(n)$, implies $z_{(n+1)} \ge \Phi(n)$.

Similarly, $\Phi(n) \le \Phi(n-1)$, implies $z_n \le \Phi(n-1)$.

Therefore, the replacement policy is as follows:

Replacement the equipment at the end of n-th years, if the maintenance cost in the (n+1)-th years is more than the average total cost in the nth year and the nth years maintenance cost is less than the previous year's average total cost.

5. PROPOSED MODEL OF RP WITH GREY PARAMETERS

To get realistic view, we consider a replacement problem with grey parameters. The capital cost, scrap value or salvage value, operating cost or maintenance cost of equipment or item are considered as a grey numbers. Our purpose is to determine the optimal replacement time of an equipment or item, whose running cost or maintenance cost increases with time and the value of money remains static during the period. We assume time variable as a discrete case and

$C(\otimes)$: Grey capital cost of equipment.
$S(\otimes)$: Grey scrap value or salvage value of equipment.
n: The number of years that equipment would be in use.
$M_n(\otimes)$: Grey maintenance cost for the corresponding year n.
$\Phi_n(\otimes)$: Average total annual cost.

If the equipment is used for n years, then the total grey cost incurred during this period n is given by

Total grey cost $=$ grey capital cost $-$ grey scarp value $+$ grey maintenance cost.

$$T(\otimes) = C(\otimes) - S(\otimes) + \sum_{i=1}^{n} M_i(\otimes)$$

Therefore, $\frac{T(\otimes)}{n} = \frac{C(\otimes) - S(\otimes)}{n} + \frac{1}{n}\sum_{i=1}^{n} M_i(\otimes)$, If we consider

$\Phi_n(\otimes) = \frac{1}{n} T(\otimes)$. Then,

$$\Phi_n(\otimes) = \frac{1}{n}[C(\otimes) - S(\otimes)] + \frac{1}{n}\sum_{i=1}^{n} M_i(\otimes) \text{ and}$$

$$\Phi_{n+1}(\otimes) = \frac{1}{n+1}[C(\otimes) - S(\otimes)] + \frac{1}{n+1}\sum_{i=1}^{n} M_i(\otimes)$$

$$\Phi_{n+1}(\otimes) = \frac{1}{n+1}\left[C(\otimes) - S(\otimes) + \sum_{i=1}^{n} M_i(\otimes)\right] + \frac{1}{n+1} M_{n+1}(\otimes)$$

$$= \frac{1}{n+1}[n\Phi_n(\otimes) + M_{n+1}(\otimes)]$$

$$\text{So, } \Phi_{n+1}(\otimes) - \Phi_n(\otimes) = \frac{1}{n+1}[M_{n+1}(\otimes) - \Phi_n(\otimes)]$$

Here, $\Phi_{n+1}(\otimes)$ and $\Phi_n(\otimes)$ denote total average annual grey cost of equipment for (n+1)-th and n-th year respectively. As these grey costs are considered as interval grey numbers therefore, we have to rank these interval grey numbers (for n= 1, 2,…,n) for searching smallest value, which is discussed on section 3.9. This smallest representative value helps us in making a decision for optimal replacement of equipment as per replacement policy.

6. NUMERICAL EXAMPLES

6.1 Example. The purchasing grey cost of a certain type of truck is \$ [6000, 6200]. A manager of Transport Company finds from his past records that the running costs and the salvage value of this type of trucks are presented by grey interval numbers (See Table 1). Determine at which time, the manager would take a decision for replacement.

Table 1 Year wise grey running costs and salvage values

Year (n)	Running cost or maintenance cost $M_n(\otimes)$	Resale value or Salvage value $S_n(\otimes)$
1	[950, 1000]	[3000, 3100]
2	[1200, 1230]	[1480, 1500]
3	[1350, 1390]	[720, 750]
4	[1770, 1780]	[380, 400]
5	[2310, 2330]	[190, 210]
6	[2790, 2800]	[180, 192]
7	[3100, 3420]	[170, 181]
8	[3550, 4000]	[160, 173]

Solution: To deal with this problem, we simply use the operations of interval grey numbers. Using operations of grey numbers, we have average annual costs (see Table 4).

From Table 4, we see that the average annual grey costs for the year of 4th, 5th, 6th and 7th are intercrossing interval grey numbers. We now determine the possibility degree of interval grey numbers (average annual cost) for comparison. Using (3), the result $\Phi_2(\otimes) > \Phi_3(\otimes) > \Phi_4(\otimes)$ can be easily determined, as they do not intercrossed each other. Similarly, we have $\Phi_8(\otimes) > \Phi_7(\otimes)$. As $\Phi_4(\otimes)$, $\Phi_5(\otimes)$, $\Phi_6(\otimes)$ and $\Phi_7(\otimes)$ are intercrossed each other, to rank these, we have to use the definition 6.1.4.

Considering the 4th and 5th year average annual cost, we have $L^* = L(\Phi_4(\otimes)) + L(\Phi_5(\otimes))$.

Therefore, we have $L^* = |2795 - 2710| + |2748 - 2674| = 159$.

Therefore,

$$P\{\Phi_4(\otimes) \le \Phi_5(\otimes)\} = \frac{Max(0, 159 - Max(0, 2795 - 2674))}{159}$$

$= 0.23899$. (6)

Therefore, we have $\Phi_4(\otimes) > \Phi_5(\otimes)$.

Similarly, on computing, we have $P\{\Phi_5(\otimes) \le \Phi_6(\otimes)\} = \frac{Max(0, 136 - Max(0, 2748 - 2696.33))}{136} =$ 0.62007, $P\{\Phi_6(\otimes) \le \Phi_7(\otimes)\} = 0.98283$. These three result yield a conclusion as $\Phi_7(\otimes_1)$ is greater than $\Phi_6(\otimes)$ and $\Phi_6(\otimes_1)$ is greater than $\Phi_5(\otimes)$. Therefore the resulting order is $\Phi_7(\otimes) > \Phi_6(\otimes) > \underline{\Phi_5(\otimes)} < \Phi_4(\otimes_1) < \Phi_3(\otimes) < \Phi_2(\otimes) < \Phi_1(\otimes)$. So the manager can take a decision for replacing a new truck after the end of 5th year.

6.2 Example.

The data collected in running a machine, the grey cost of which is \$ [60000, 60100], are given in Table 2. Determine the optimum period for replacement of the machine.

Table 2 Year wise grey spares costs, grey labor costs and salvage values

Year (n)	Cost of spares	Labor costs	Resale value or Salvage value $S_n(\otimes)$
1	[3990, 4000]	[13995, 14005]	[41900, 42100]
2	[4265, 4272]	[16000, 16015]	[31050, 31060]
3	[4875, 4880]	[17995, 18015]	[20390, 20410]
4	[5705, 5710]	[21000, 21010]	[14380, 14420]
5	[6796, 6805]	[24990, 25020]	[9640, 9660]

Solution: The operating or maintenance grey cost of machine in successive years is shown in Table 3.

Table 3 Year wise grey operating costs and salvage values

Year (n)	Operating costs	Resale value or Salvage value $S_n(\otimes)$
1	[17985, 18005]	[41900, 42100]
2	[20265, 20287]	[31050, 31060]
3	[22870, 22895]	[20390, 20410]
4	[26705, 26720]	[14380, 14420]
5	[31786, 31825]	[9640, 9660]

The cost of spares and labor together determine the operating cost or running cost or maintenance cost. Then the average total annual grey cost is computed in Table 5.

From Table 5, we observe that the average total annual grey cost of a machine for 2nd and 3rd year are inter crossing interval grey numbers. Therefore, for comparison we have to determine possibility degree of interval grey numbers. Using 6.1.4, we have $\Phi_3(\otimes) > \Phi_4(\otimes)$; $\Phi_5(\otimes) > \Phi_3(\otimes)$; $\Phi_5(\otimes) > \Phi_4(\otimes)$; $\Phi_1(\otimes) > \Phi_5(\otimes)$; $\Phi_1(\otimes) > \Phi_2(\otimes)$; and $\Phi_2(\otimes) > \Phi_5(\otimes)$. (7)

To compare between 2nd and 3rd year average total annual grey cost (here interval grey numbers), we use 6.1.4. Then,

$$L^* = L(\Phi_2(\otimes)) + L(\Phi_3(\otimes)).$$

$$L^* = |33671 - 33595| + |336323 - 33570| = 76 + 62.3 = 139.3$$

Therefore,

$$P\{\Phi_2(\otimes) \le \Phi_3(\otimes)\} = \frac{Max(0, 139.3 - Max(0, 33671 - 33570))}{139.3} = 0.2749.$$

Therefore, the relation between the grey average costs is given by

$$\Phi_2(\otimes) > \Phi_3(\otimes). \quad (8)$$

Combining (7) and (8), we have the resulting order as $\Phi_1(\otimes) > \Phi_2(\otimes) > \Phi_3(\otimes) > \underline{\Phi_4(\otimes)} < \Phi_5(\otimes)$. The calculation in the Table 5 reflects that the average grey cost is lowest during the fourth year. Hence, it is concluded that the machine should be replaced after every four year.

Table-4 Calculation to determine the economic life of equipment

Year(n)	Cumulative running cost $M_n(\otimes)$	Depreciation cost $[C(\otimes)-S(\otimes)]$	Total cost $T(\otimes)$ $[C(\otimes)-S(\otimes)]+\sum M_n(\otimes)$	Average cost $\Phi_n(\otimes)$
1	[950, 1000]	[2900, 3200]	[3850, 4200]	[3850, 4200]
2	[2150, 2230]	[4500, 4720]	[6650, 6950]	[3325, 3475]
3	[3500, 3630]	[5250, 5480]	[8750, 9100]	[2916.66, 3033.33]
4	[5270, 5400]	[5570, 5780]	[10840, 11180]	[2710, 2795]
5	[7580, 7730]	[5790, 6010]	[13370, 13740]	**[2674, 2748]**
6	[10370, 10530]	[5808, 6020]	[16178, 16550]	[2696.33, 2758.33]
7	[13470, 13950]	[5819, 6030]	[19289, 19980]	[2755.57, 2854.28]
8	[17760, 17950]	[5827, 6040]	[23587, 23990]	[2898.375, 2998.75]

Table-5 Calculation to determine the optimal replacement time of equipment

Year(n)	Cumulative operating cost $M_n(\otimes)$	Depreciation cost $[C(\otimes)-S(\otimes)]$	Total cost $T(\otimes)$ $[C(\otimes)-S(\otimes)]+\sum M_n(\otimes)$	Average cost $\Phi_n(\otimes)$
1	[17985, 18005]	[17900, 18200]	[35885, 36205]	[35885, 36205]
2	[38250, 38292]	[28940, 29050]	[67190, 67342]	[33595, 33671]
3	[61120, 61187]	[39590, 39710]	[100710, 100897]	[33570, 33632.3]
4	[87825, 87907]	[45590, 45710]	[133415, 133617]	**[33353.75, 33404.25]**
5	[119611, 119732]	[50340, 50450]	[169951, 170182]	[33990.2, 34036.4]

7. CONCLUSION

In this paper, we proposed grey replacement problem, which is realistic in nature. The possibility degree of grey numbers is used for the purpose of comparison of average total costs. The proposed method is effective to deal with replacement problem with grey parameters.

Replacement analysis is one of the important issues in different management system such as hospitals, chemical industries, etc. We hope the proposed method presented here may be used for future study of the grey replacement problem, where the value of money is unstable with time, sudden failure of equipments, consideration a technological improvement of equipments. Our future study will focus on a more extensive campaign of experiments in order to improve the grey model performance.

8. ACKNOWLEDGMENTS

The authors are very grateful to the anonymous referees for their constructive comments and suggestions, which have led to improve the quality and presentation of the paper to its current standard.

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