Model Order Reduction of Interval Systems Using Mihailov Criterion and Cauer Second Form

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ABSTRACT

This paper presents a new mixed method for reducing the large scale interval systems using the Mihailov Criterion and Cauer second form. The reduced order model of denominator is determined by using Mihailov Criterion and numerator coefficients are obtained by using Cauer second form. We show that the mixed method is simple and guarantees the stability of the reduced model if the original system is stable. A numerical examples are illustrated and verified its stability.

Keywords

Mihailov Criterion, Cauer second form, Reduced order, Stability, Mixed method.

1. INTRODUCTION

Model order reduction of interval systems has been considered by several researchers. The technique of Routh approximation [1] based on direct truncation guarantees the stability of reduced order model. γ-δ Routh approximation [2] evaluated for order reduction. Hwang [3] has proposed comments on the computation aspects of interval Routh approximation. Sastry et al [4] has proposed γ table approximation to decreasing complexity of the γ-δ Routh algorithm.

Yuri Dolgin suggested another generalization of direct routh table truncation method [5] for interval systems. He proved that the existing generalization of direct truncation of Routh fails to produce a stable system. But S-F.Yang commented [6] on Yuri Dolgin's method that it cannot guarantee the stability of lower order interval systems. To overcome all these problems Yuri Dolgin added some conditions [7] for getting the stability of reduced order through Routh approximation. To improve the effectiveness of model order reduction many mixed methods are proposed recently [8-11].

In the present paper, model order reduction of interval systems is carried out by using mixed method. The denominator of the reduced model is obtained by Mihailov Criterion and the numerator is obtained by Cauer second form. Thus the stability of the reduced order model is guaranteed, if the higher order interval system is asymptotically stable. The brief outline of this paper is as follows: Section 2 contains problem statement. Section 3 contains proposed method. Error analysis done in section 4. Numerical examples are presented in section 5 and t conclusion in section 6.

2. PROBLEM STATEMENT

Let the transfer function of a higher order interval systems be

$$G_n(s) = \frac{[p_0^-, p_0^+] + [p_1^-, p_1^+]s + \dots\dots + [p_{n-1}^-, p_{n-1}^+]s^{n-1}}{[q_0^-, q_0^+] + [q_1^-, q_1^+]s + \dots\dots + [q_n^-, q_n^+]s^n} = \frac{N(s)}{D(s)} \quad (1)$$

where $[p_i^-, p_i^+]$ for i = 0 to n-1 and $[q_i^-, q_i^+]$ for i = 0 to n are known as scalar constants.

The reduced order model of a transfer function be considered as

$$R_r(s) = \frac{[u_0^-, u_0^+] + [u_1^-, u_1^+]s + \dots\dots + [u_{r-1}^-, u_{r-1}^+]s^{r-1}}{[v_0^-, v_0^+] + [v_1^-, v_1^+]s + \dots\dots + [v_r^-, v_r^+]s^r} = \frac{N_r(s)}{D_r(s)} \quad (2)$$

where $[u_j^-, u_j^+]$ for j= 0 to r-1and $[v_j^-, v_j^+]$ for j = 0 to r are known as scalar constants.

The rules of the interval arithmetic have been defined in [12], as follows.

Let [e , f] and [g , h] be two intervals.

Addition:

[e , f] + [g , h] = [e + g , f + h]

Subtraction:

[e , f] – [g , h] = [e – h , f – g]

Multiplication:

[e , f] [g , h] = [Min (eg, eh, fg, fh), Max (eg, eh, fg, fh)]

Division:

$$\frac{[e, f]}{[g, h]} = [e, f]\left[\frac{1}{h}, \frac{1}{g}\right]$$

3. PROPOSED METHOD

The proposed method consists of the following steps for obtaining reduced order model.

Step 1: Determination of the denominator polynomial of the k^{th} order reduced model:

Substituting s = jω in D(s) and separating the denominator into real and imaginary parts,

$$D(jw) = [c_{11}^-, c_{11}^+] + [c_{12}^-, c_{12}^+](j\omega) + \dots + [c_{1,n+1}^-, c_{1,n+1}^+](j\omega)^n$$

$=[c_{11}^-, c_{11}^+] - [c_{13}^-, c_{13}^+]\, \omega^2 + \ldots\ldots) + j\omega\, ([c_{12}^-, c_{12}^+]\omega - [c_{14}^-, c_{14}^+]\omega^2 + \ldots\ldots)$

$= \xi(\omega) + j\omega\, \eta(\omega)$
(3)

where ω is the angular frequency, rad/sec.

$\xi(\omega) = 0$ and $\eta(\omega) = 0$, the frequencies which are intersecting $\omega_0 = 0, \pm[\omega_1^-, \omega_1^+], \ldots\ldots \pm[\omega_{n-1}^-, \omega_{n-1}^+]$ are obtained, where $|[\omega_1^-, \omega_1^+]| \prec |[\omega_2^-, \omega_2^+]| \prec \ldots\ldots \prec |[\omega_{n-1}^-, \omega_{n-1}^+]|$.
Similarly substituting s = jω in $D_k(s)$, then obtains

$D_k(j\omega) = \boldsymbol{\phi}(\omega) + j\omega\, \boldsymbol{\psi}(\omega)$
(4)

where

$\phi(\omega) = [d_{11}^-, d_{11}^+] - [d_{13}^-, d_{13}^+]\omega^2 + \ldots.$ and

$\psi(\omega) = [d_{12}^-, d_{12}^+] - [d_{14}^-, d_{14}^+]\omega^2 + \ldots.$

Put $\phi(\omega) = 0$ and $\psi(\omega) = 0$, then we get k number of roots and it must be positive and real and alternately distributed along the w axis. The first k numbers of frequencies are 0, $[\omega_1^-, \omega_1^+]$, $[\omega_2^-, \omega_2^+]$, ... , $[\omega_{k-1}^-, \omega_{k-1}^+]$ are kept unchanged and the roots of $\phi(\omega) = 0$ and $\psi(\omega) = 0$.

Therefore,

$\phi(\omega) = [\lambda_1^-, \lambda_1^+]\,(\omega^2 - [\omega_1^2, \omega_1^2])\,(\omega^2 - [\omega_3^2, \omega_3^2])\,(\omega^2 - [\omega_5^2, \omega_5^2]) \ldots$
(5)

$\psi(\omega) = [\lambda_2^-, \lambda_2^+]\,(\omega^2 - [\omega_2^2, \omega_2^2])\,(\omega^2 - [\omega_4^2, \omega_4^2])\,(\omega^2 - [\omega_6^2, \omega_6^2]) \ldots$
(6)

For finding the coefficient values of $[\lambda_1^-, \lambda_1^+]$ and $[\lambda_2^-, \lambda_2^+]$ are calculated from $\xi(0) = \phi(0)$ and $\eta([\omega_1^-, \omega_1^+]) = \psi([\omega_1^-, \omega_1^+])$. keeping these values of $[\lambda_1^-, \lambda_1^+]$ and $[\lambda_2^-, \lambda_2^+]$ in equations (5) and (6), respectively, $\phi(\omega)$ and $\psi(\omega)$ are obtained and $D_k(j\omega)$ is obtained as

$D_k(j\omega) = \boldsymbol{\phi}(\omega) + j\omega\, \boldsymbol{\psi}(\omega)$
(7)

Now replace jω by s, then the k^{th} order reduced denominator $D_k(s)$ is obtained as

$D_k(j\omega) = [d_{11}^-, d_{11}^+] + [d_{12}^-, d_{12}^+](s) + \ldots. + [d_{1,k+1}^-, d_{1,k+1}^+]\,(s)^k$
(8)

Step 2: Determination of the numerator coefficients of the k^{th} order reduced model by using Cauer second form:

Coefficient values from Cauer second form $[h_p^-, h_p^+]$ (p =1 , 2 , 3 . . . . k) are evaluated by forming Routh array as

$$[h_1^-, h_1^+] = \frac{[c_{11}^-, c_{11}^+]}{[c_{21}^-, c_{21}^+]} \begin{cases} [c_{11}^-, c_{11}^+]\ [c_{12}^-, c_{12}^+]\ \ldots\ldots \\ [c_{21}^-, c_{21}^+]\ [c_{22}^-, c_{22}^+]\ \ldots\ldots \end{cases}$$

$$[h_2^-, h_2^+] = \frac{[c_{21}^-, c_{21}^+]}{[c_{31}^-, c_{31}^+]} \begin{cases} [c_{21}^-, c_{21}^+]\ [c_{22}^-, c_{22}^+]\ \ldots\ldots \\ [c_{31}^-, c_{31}^+]\ [c_{32}^-, c_{32}^+]\ \ldots\ldots \end{cases}$$

$$[h_3^-, h_3^+] = \frac{[c_{31}^-, c_{31}^+]}{[c_{41}^-, c_{41}^+]} \begin{cases} [c_{31}^-, c_{31}^+]\ [c_{32}^-, c_{32}^+]\ \ldots\ldots \\ [c_{41}^-, c_{41}^+]\ [c_{42}^-, c_{42}^+]\ \ldots\ldots \end{cases}$$

....
....
(9)

The first two rows are copied from the original system numerator and denominator coefficients and rest of the elements are calculated by using well known Routh algorithm.

$[c_{ij}^-, c_{ij}^+] = [c_{(i-2,j+1)}^-, c_{(i-2,j+1)}^+] - [h_{i-2}^-, h_{i-2}^+][c_{(i-1,j+1)}^-, c_{(i-2,j+1)}^+]$
(10)

where i = 3,4,. . . . . and j = 1,2, ...

$[h_i^-, h_i^+] = \frac{[c_{i,1}^-, c_{i,1}^+]}{[c_{(i+1,1)}^-, c_{(i+1,1)}^+]}$; i = 1,2,3,.........k
(11)

The coefficient values of $[d_{i,j}^-, d_{i,j}^+]$(j = 1,2,.....(k+1))]

Of the equation (8) and Cauer quotients $[h_p^-, h_p^+]$ (p = 1. 2, . . . k) of the the equation (9) are matched for finding the coefficients of numerator of the reduced model $R_k(s)$. The inverse routh array is constructed as

$[d_{(i+1,1)}^-, d_{(i+1,1)}^+] = \frac{[d_{i,1}^-, d_{i,1}^+]}{[h_i^-, h_i^+]}$
(12)

i = 1,2, . . .,k and k ≤ n

$[d_{(i+1,j+1)}^-, d_{(i+1,j+1)}^+] = \frac{([d_{(i,j+1)}^-, d_{(i,j+1)}^+] - [d_{(i+2,j)}^-, d_{(i+2,j)}^+]}{[h_i^-, h_i^+]}$
(13)

where i = 1. 2, ... , (k-j) and j = 1,2 ,. . . . , (k-1)

Using the above equations, the numerator coefficients of the reduced model are obtained and numerator is written as

$N_k(s) = [d_{21}^-, d_{21}^+] + [d_{22}^-, d_{22}^+]s + \cdots + [d_{2k}^-, d_{2k}^+]s^{k-1}$ (14)

4. Error Analysis

The integral square error (ISE) in between the transient responses of higher order system (HOS) and Lower order system (LOS) and is given by:

$$\text{ISE} = \int_0^\infty [y(t) - y_r(t)]^2 \, dt \qquad (15)$$

where, y (t) and y_r(t) are the unit step responses of original system $G_n(s)$ and reduced order system $R_k(s)$.

5. Numerical Examples

Example 1: Consider a third order system described by the transfer function [13]

$$G_3(s) = \frac{[2,3]s^2 + [17.5,18.5]s + [15,16]}{[2,3]s^3 + [17,18]s^2 + [35,36]s + [20.5,21.5]} \qquad (16)$$

Step 1: Put s = jω in the denominator D (s)

D (jω) = ([20.5, 21.5] – [17, 18]ω^2) + jω ([35, 36] – [2, 3]ω^2)

Step 2: The intersecting frequencies are

$[\omega_i^-, \omega_i^+] = 0, [1.0929, 1.0981], [3.4641, 4.1833]$

Step 3: Following the procedure given in section 3, the denominator of the second order model is taken as

$D_2(j\omega) = [\lambda_1^-, \lambda_1^+](\omega^2 - [1.0929, 1.9081]^2) + j\omega\, [\lambda_2^-, \lambda_2^+],$

Here$[\lambda_1^-, \lambda_1^+] = -[17.0011, 18.0007]$ and $[\lambda_2^-, \lambda_2^+] = [31.3826, 33.6111]$

Step 4: Substitute the values of $[\lambda_1^-, \lambda_1^+]$ and $[\lambda_2^-, \lambda_2^+]$ in step 3 and also substitute jω = s.

Hence, the denominator D(s) is given by

$$D_2(s) = [17..0011, 18.0007]s^2 + [31.3826, 33.6111]s + [20.3061, 21.7052]$$

Step 5: Using the Cauer second form as described in the section 3, it is obtained

$[h_1^-, h_1^+] = [1.2812, 1.4333]$
$[h_2^-, h_2^+] = [1.1046, 1.8859]$

$[d_{21}^-, d_{21}^+] = [14.1674, 16.9413]$
$[d_{22}^-, d_{22}^+] = [11.1949, 20.3706]$ are obtained.

Step 6: Numerator of second order system is written as

$N_2(s) = [11.1949, 20.3706]s + [14.1674, 16.9413]$

Step 7: The reduced order model $R_2(s)$ is given as

$$R_2(s) = \frac{[11.1949, 20.3706]s + [14.1674, 16.9413]}{[17..0011, 18.0007]s^2 + [31.3826, 33.6111]s + [20.3061, 21.7052]}$$

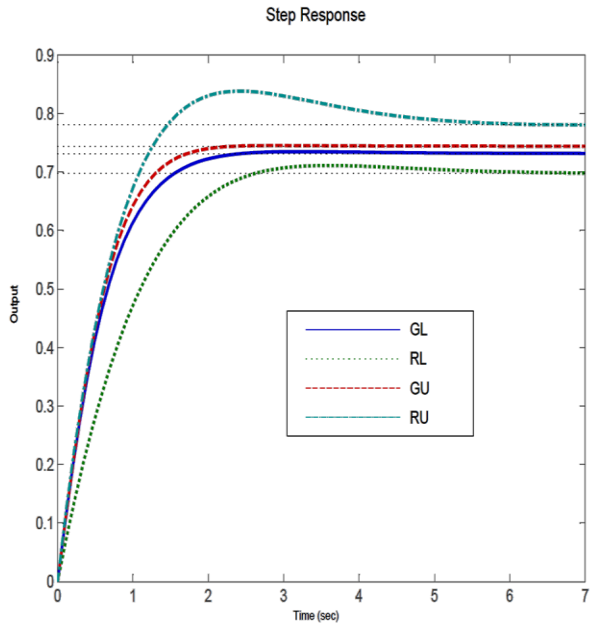

Fig. 1 : Step Response of original model and ROM

Table 1. Comparison of Reduced Order Models

Method of Order reduction	*ISE for lower limit errL*	*ISE for upper limit errU*
Proposed Algorithm	0.0089	0.0113
G.V.S.S Sastry [3]	0.2256	0.0095

The step response of second order model is obtained by the proposed method and compared with second order model.

Example 2: Consider a third order system described by the transfer function [16]

$$G_3(s) = \frac{[1,1]s^2 + [3.3,6.5]s + [2.7,10]}{[1,1]s^3 + [8.5,8.6]s^2 + [18,18.2]s + [10.25,10.76]} \qquad (17)$$

Step 1: Put s = jω in the denominator D (s)

D(jω)= ([10.25, 10.76]–[8.5, 8.6]ω^2) + jω([18, 18.2]–[1, 1]ω^2)

Step 2: The intersecting frequencies are

$$[\omega_i^-, \omega_i^+] = 0, [1.0981, 1.1185], [4.2426, 4.2661]$$

Step 3: Following the procedure given in section 3, the denominator of the second order model is taken as

$$D_2(j\omega) = [\lambda_1^-, \lambda_1^+](\omega^2 - [1.0929, 1.9081]^2) + j\omega\, [\lambda_2^-, \lambda_2^+],$$

Here $[\lambda_1^-, \lambda_1^+] = -[8.1934, 8.9235]$ and

$$[\lambda_2^-, \lambda_2^+] = [16.749, 16.9942]$$

Step 4: Substitute the values of $[\lambda_1^-, \lambda_1^+]$ and $[\lambda_2^-, \lambda_2^+]$ in step 3 and also substitute jω = s.

Hence, the denominator D(s) is given by

$$D_2(s) = [8.1934, 8.9235]s^2 + [16.749, 16.9942]s + [9.8796, 11.1633]$$

Step 5: Using the Cauer second form as described in the section 3, it is obtained

$$[h_1^-, h_1^+] = [1.025, 3.9852]$$
$$[h_2^-, h_2^+] = [-1.2652, 0.6749]$$

$[d_{21}^-, d_{21}^+] = [2.4791, 10.8910]$
$[d_{22}^-, d_{22}^+] = [0.1535, 24.9778]$ are obtained.

Step 6: Numerator of order system is written as

$$N_2(s) = [0.1535, 24.9778]s + [2.4791, 10.8910]$$

Step 7: The reduced second order model $R_2(s)$ is given as

$$R_2(s) = \frac{[0.1535, 24.9778]s + [2.4791, 10.8910]}{[8.1934, 8.9235]s^2 + [16.749, 16.9942]s + [9.8796, 11.1633]}$$

The step response of second order model is obtained by the proposed method and compared with second order model

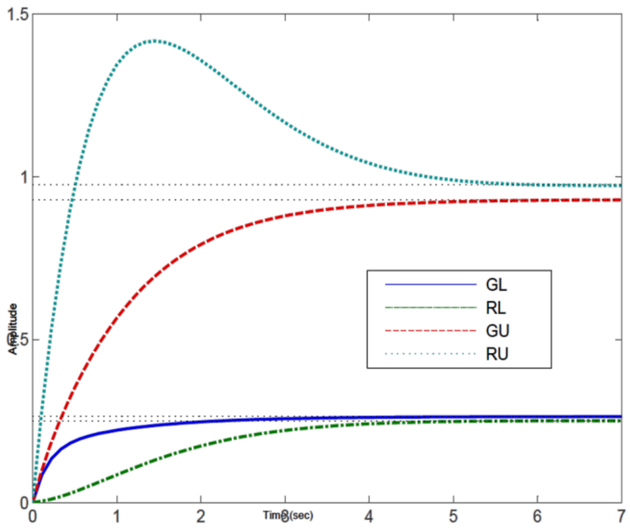

Fig.2.Step Response of original model and ROM

Table 2. Comparison of Reduced Order Models

Method of Order reduction	*ISE for lower limit errL*	*ISE for upper limit errU*
Proposed Algorithm	0.0264	1.0314
G.Saraswathi [8]	0.0034	0.0364

6. CONCLUSION

In this paper Mihailov criterion and Cauer second form is employed for order reduction. The reduced model of denominator polynomial is obtained by using Mihailov criterion and the numerator is determined by cauer second form. The proposed method guarantees the stability of reduced model if the original system is stable. This proposed method is conceptually simple and comparable with other well known methods. Two examples are taken from the literature and compared with other methods by using ISE.

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8. BIOGRAPHIES

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