A New Approach to the Theory of Soft Sets

Tridiv Jyoti Neog
Research Scholar,
Department of Mathematics
CMJ University, Shillong, Meghalaya.

Dusmanta Kumar Sut
Assistant Professor,
Department of Mathematics, N.N.Saikia College,
Jorhat, Assam.

ABSTRACT
Molodtsov initiated the concept of soft set as a new mathematical tool for dealing with uncertainties. In 2003, Maji put forward several notions on Soft Set Theory. However, the axioms of exclusion and contradiction are not valid under the definition of complement of a soft set initiated by Maji. In this paper, we reintroduce the concept of complement of a soft set and show that the laws of exclusion and contradiction, Involution, De Morgan Inclusions and De Morgan laws are valid for soft sets with respect to our new definition of complement. We justify our claim with proof and examples.

Keywords
Soft set, Union, Intersection, Complement.

1. INTRODUCTION
Most of our traditional tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. However, there are many complicated problems in economics, engineering, environment, social science, medical science etc. that involve uncertainties. The Theory of Probability, Theory of Fuzzy Sets,Theory of Intuitionistic Fuzzy Sets,Theory of Vague Sets, Theory of Interval Mathematics and Theory of Rough Sets are considered as mathematical tools for dealing with uncertainties. In 1999, Molodstov [7] pointed out that these theories which are considered as mathematical tools for dealing with uncertainties, have certain limitations. He further pointed out that the reason for these limitations is, possibly, the inadequacy of the parameterization tool of the theory. The Soft Set Theory introduced by Molodstov [7] is quite different from these theories in this context. The absence of any restrictions on the approximate description in Soft Set Theory makes this theory very convenient and easily applicable. Fuzzy set theory proposed by Professor L. A. Zadeh [9] in 1965 is considered as a special case of the soft sets. Fuzzy set theory, being generalization of crisp sets, should satisfy the axioms of exclusion and contradiction. The Zadehian definition of complement does not meet these requirements but it has been proved recently by H. K. Baruah [3, 4] that the fuzzy sets, too, follow the set theoretic axioms of exclusion and contradiction. In 2003, P.K.Maji, R.Biswas and A.R.Roy [6] studied the theory of soft sets initiated by Molodstov [7] and developed several basic notions of Soft Set Theory. At present, researchers are contributing a lot on the extension of soft set theory. In 2005, Pei and Miao [8] and Chen et al. [5] studied and improved the findings of Maji et al [6]. In 2009, M.Irfan Ali, Feng Feng, Xiaoyan Liu,Won Keun Min, M.Shabir [2] gave some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets along with a new notion of relative complement of a soft set. We claim that the definition of relative complement of a soft set proposed by M. Irfan Ali et al [2] should actually be the definition of complement of a soft set. Since soft set is a generalization of fuzzy sets, so the same problem mentioned above regarding complement of soft set arises here. *i.e.* in soft set also, we should have the union of a soft set and its complement must be absolute soft set and intersection between a soft set with its complement must be null soft set, which is not the case under the current definition of complement. Accordingly we reintroduce the concept of complement of a soft set so that the fundamental properties related to complement are satisfied also by the complement of a soft set. Our work is an attempt to generalize the notion of relative complement introduced by M. Irfan Ali [2].

2. PRELIMINARIES
We first recall some basic notions in soft set theory. Let U be an initial universe, and E be the set of all possible parameters under consideration with respect to U. The set of all subsets of U, *i.e.* the power set of U is denoted by $P(U)$ and A is a subset of E. Molodstov [7] defined the notion of a soft set in the following way –

Definition 2.1 [7] A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.
In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (F, E), or as the set of ε - approximate elements of the soft set.

Definition 2.2 [6] The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$, where $F^c: \rceil A \rightarrow P(U)$ is a mapping given by $F^c(\sigma) = U - F(\neg\sigma)$ for all $\sigma \in \rceil A$.

Definition 2.3 [6] A soft set (F, A) over U is said to be null soft set denoted by $\tilde{\varphi}$ if $\forall \varepsilon \in A, F(\varepsilon) = \varphi$ (Null set)

Definition 2.4 [6] A soft set (F, A) over U is said to be absolute soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A, F(\varepsilon) = U$.

Definition 2.5 [6] For two soft sets (F, A) and (G, B) over the universe U, we say that (F, A) is a soft subset of (G, B), if
(i) $A \subset B$,
(ii) $\forall \varepsilon \in A$, $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations and is written as $(F, A) \tilde{\subseteq} (G, B)$.

Pei and Miao [8] modified this definition of soft subset in the following way –

Definition 2.6 [8] For two soft sets (*F*, *A*) and (*G*, *B*) over the universe *U*, we say that (*F*, *A*) is a soft subset of (*G*, *B*), if

(*i*) $A \subseteq B$,

(*ii*) $\forall \varepsilon \in A, F(\varepsilon) \subseteq G(\varepsilon)$ and is written as (*F* , *A*) $\tilde{\subseteq}$ (*G*, *B*).

(*F*, *A*) is said to be soft superset of (*G*, *B*) if (*G*, *B*) is a soft subset of (*F*, *A*) and we write (*F* , *A*) $\tilde{\supseteq}$ (*G*, *B*).

Definition 2.7 [6] Union of two soft sets (*F*, *A*) and (*G*, *B*) over a common universe *U*, is the soft set (*H*, *C*), where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } x \in A - B \\ G(\varepsilon), & \text{if } x \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } x \in A \cap B \end{cases}$$

and is written as $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 2.8 [6] Intersection of two soft sets (*F*, *A*) and (*G*, *B*) over a common universe *U*, is the soft set (*H*, *C*), where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon)$ or $G(\varepsilon)$ (as both are same set) and is written as $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Pei and Miao [8] pointed out that generally $F(\varepsilon)$ or $G(\varepsilon)$ may not be identical. Moreover in order to avoid the degenerate case, Ahmad and Kharal [1] proposed that $A \cap B$ must be non-empty and thus revised the above definition as follows.

Definition 2.9 [1]

Let (*F*, *A*) and (*G*, *B*) be two soft sets over a common universe *U* with $A \cap B \neq \phi$. Then Intersection of two soft sets (*F*, *A*) and (*G*, *B*) is a soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. We write $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Definition 2.10 [6] If (*F*, *A*) and (*G*, *B*) be two soft sets, then "(*F*, *A*) AND (*G*, *B*)" is a soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$, where $\cap$ is the operation intersection of two sets.

Definition 2.11 [6] If (*F*, *A*) and (*G*, *B*) be two soft sets, then "(*F*, *A*) OR (*G*, *B*)" is a soft set denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$, where $\cup$ is the operation union of two sets.

M. Irfan Ali et al. [2] proposed the definition of relative complement of a soft set as -

Definition 2.12 [2] The relative complement of a soft set (*F*, *A*) is denoted by (*F*, *A*) ^{r} and is defined by $(F, A)^r$ = (F^r, *A*), where $F^r: A \longrightarrow P(U)$ is a mapping given by $F^r(\varepsilon) = U - F(\varepsilon)$ for all $\varepsilon \in A$.

3. ILLUSTRATING EXAMPLE

We take an example below -

Example 3.1 Let $U = \{c_1, c_2, c_3, c_4\}$ be the set of four cars under consideration and $E = \{e_1(\text{costly}), e_2(\text{Beautiful}), e_3(\text{Fuel Efficient}), e_4(\text{Modern Technology}), e_5(\text{Luxurious})\}$ be the set of parameters and $A = \{e_1, e_2, e_3\} \subseteq E$. Then

$(F, A) = \{F(e_1) = \{c_1, c_4\}, F(e_2) = \{c_1, c_2, c_4\}, F(e_3) = \{c_3\}\}$

is the soft set representing the 'attractiveness of the car' which Mr. X is going to buy. Complement of this soft set (*F*, *A*) is given by the soft set

$(F, A)^c = (F^c, \rceil A)$
$= \{F^c(\neg e_1) = \{c_2, c_3\}, F^c(\neg e_2) = \{c_3\}, F^c(\neg e_3) = \{c_1, c_2, c_4\}\}$

Let us now see what happens when we try to find out $(F, A) \tilde{\cup} (F, A)^c$ and $(F, A) \tilde{\cap} (F, A)^c$.

We have,

$(F, A) \tilde{\cup} (F, A)^c$
$= (F, A) \tilde{\cup} (F^c, \rceil A)$
$= (H, C)$, where $C = \{e_1, e_2, e_3, \neg e_1, \neg e_2, \neg e_3\}$
$= \{H(e_1) = \{c_1, c_4\}, H(e_2) = \{c_1, c_2, c_4\}, H(e_3) = \{c_3\}, H(\neg e_1) = \{c_2, c_3\}, H(\neg e_2) = \{c_3\}, H(\neg e_3) = \{c_1, c_2, c_4\}\}$
$\neq \tilde{A}$

Again,

$(F, A) \tilde{\cap} (F, A)^c$
$= (F, A) \tilde{\cap} (F^c, \rceil A)$
$= (H, C)$, where $C = \varphi$

Thus we are arriving at a degenerate case here. In what follows, the axioms of exclusion and contradiction are not valid with respect to the definition of complement initiated by Maji et al. [6].

4. COMPLEMENT OF A SOFT SET

Our definition of complement of a soft set is as follows:

Definition 4.1 The complement of a soft set (*F*, *A*) is denoted by (*F*, *A*) ^{c} and is defined by $(F, A)^c$ = (F^c, *A*), where $F^c: A \longrightarrow P(U)$ is a mapping given by $F^c(\varepsilon) = [F(\varepsilon)]^c$ for all $\varepsilon \in A$.

Example 4.1 We take **Example 3.1**. Here

$(F, A) = \{F(e_1) = \{c_1, c_4\}, F(e_2) = \{c_1, c_2, c_4\}, F(e_3) = \{c_3\}\}$

is the soft set representing the 'attractiveness of the car' which Mr. X is going to buy. Complement of this soft set (*F*, *A*) is given by the soft set

$(F, A)^c = (F^c, A)$
$= \{F^c(e_1) = \{c_2, c_3\}, F^c(e_2) = \{c_3\}, F^c(e_3) = \{c_1, c_2, c_4\}\}$

Let us now see what happens when we try to find out $(F,A)\tilde{\cup}(F,A)^c$ and $(F,A)\tilde{\cap}(F,A)^c$.

We have,

$(F,A)\tilde{\cup}(F,A)^c$
$=(F,A)\tilde{\cup}(F^c, A)$
$=(H,A)$
$=\{H(e_1)=\{c_1,c_2,c_3,c_4\}, H(e_2)=\{c_1,c_2,c_3,c_4\}, H(e_3)=\{c_1,c_2,c_3,c_4\}\}$
$=\tilde{A}$

Again,

$(F,A)\tilde{\cap}(F,A)^c$
$=(F,A)\tilde{\cup}(F^c, A)$
$=(H,A)$
$=\{H(e_1)=\varphi, H(e_2)=\varphi, H(e_3)=\varphi\}$
$=\tilde{\varphi}$

We thus arrive at the following two propositions –

Proposition 4.1

For a soft set (*F*, *A*) over *U*, we have,

1. $(F,A)\tilde{\cup}(F,A)^c=\tilde{A}$ **(Exclusion)**
2. $(F,A)\tilde{\cap}(F,A)^c=\tilde{\varphi}$ **(Contradiction)**

Proof:

1. Let $(F,A)\tilde{\cup}(F,A)^c=(F,A)\tilde{\cup}(F^c,A)=(H,A)$,

Where $\forall\varepsilon\in A, H(\varepsilon)=F(\varepsilon)\cup F^c(\varepsilon)$
$=F(\varepsilon)\cup(F(\varepsilon))^c$
$=U$

Thus $(F,A)\tilde{\cup}(F,A)^c=\tilde{A}$

2. Let $(F,A)\tilde{\cap}(F,A)^c=(F,A)\tilde{\cap}(F^c,A)=(H,A)$,

Where $\forall\varepsilon\in A, H(\varepsilon)=F(\varepsilon)\cap F^c(\varepsilon)$
$=F(\varepsilon)\cap(F(\varepsilon))^c$
$=\varphi$

Thus $(F,A)\tilde{\cap}(F,A)^c=\tilde{\varphi}$

Proposition 4.2

1. $((F,A)^c)^c=(F,A)$ **(Involution)**
2. $\tilde{\varphi}^c=\tilde{A}$, $\tilde{A}^c=\tilde{\varphi}$

Proof:

1. $(F,A)^c=(F^c,A)$

Where $\forall\varepsilon\in A, F^c(\varepsilon)=[F(\varepsilon)]^c$

Also, $((F,A)^c)^c=(F^c,A)^c=((F^c)^c,A)$

Where $\forall\varepsilon\in A, (F^c)^c(\varepsilon)=[F^c(\varepsilon)]^c=[(F(\varepsilon))^c]^c=F(\varepsilon)$

It follows that $((F,A)^c)^c=(F,A)$

2. $\tilde{\varphi}=(F,A)$

Where $\forall\varepsilon\in A, F(\varepsilon)=\varphi$

Thus $\tilde{\varphi}^c=(F,A)^c=(F^c,A)$

Where $\forall\varepsilon\in A, F^c(\varepsilon)=(F(\varepsilon))^c=\varphi^c=U$

Thus $\tilde{\varphi}^c=\tilde{A}$

In a similar way, $\tilde{A}=(F,A)$,

Where $\forall\varepsilon\in A, F(\varepsilon)=U$

Thus $\tilde{A}^c=(F,A)^c=(F^c,A)$

Where $\forall\varepsilon\in A, F^c(\varepsilon)=(F(\varepsilon))^c=U^c=\varphi$

Thus $\tilde{A}^c=\tilde{\varphi}$

It is well known that De Morgan Laws inter-relate union and intersection via complements. Maji et al [6] gave the following propositions -

Proposition 4.3 [6]

1. $((F,A)\tilde{\cup}(G,B))^c=(F,A)^c\tilde{\cup}(G,B)^c$
2. $((F,A)\tilde{\cap}(G,B))^c=(F,A)^c\tilde{\cap}(G,B)^c$

M. Irfan Ali [2] proved by counter examples that these propositions are not valid. However we have the following inclusions –

Proposition 4.4

For soft sets (*F*, *A*) and (*G*, *B*) over the same universe *U*, we have the following -

1. $((F,A)\tilde{\cup}(G,B))^c\,\tilde{\subseteq}\,(F,A)^c\tilde{\cup}(G,B)^c$
2. $(F,A)^c\tilde{\cap}(G,B)^c\,\tilde{\subseteq}\,((F,A)\tilde{\cap}(G,B))^c$

Proof.

1. Let $(F,A)\tilde{\cup}(G,B)=(H,C)$, where $C=A\cup B$ and

$$\forall\varepsilon\in C, H(\varepsilon)=\begin{cases}F(\varepsilon) & \text{if } \varepsilon\in \text{A - B}\\ G(\varepsilon) & \text{if } \varepsilon\in \text{B - A}\\ F(\varepsilon)\cup G(\varepsilon) & \text{if } \varepsilon\in \text{A}\cap\text{B}\end{cases}$$

Thus

$((F,A)\tilde{\cup}(G,B))^c=(H,C)^c=(H^c,C)$, where $C=A\cup B$ and

$$\forall\varepsilon\in C, H^c(\varepsilon)=(H(\varepsilon))^c=\begin{cases}(F(\varepsilon))^c & \text{if } \varepsilon\in \text{A - B}\\ (G(\varepsilon))^c & \text{if } \varepsilon\in \text{B - A}\\ (F(\varepsilon)\cup G(\varepsilon))^c & \text{if } \varepsilon\in \text{A}\cap\text{B}\end{cases}$$

$$=\begin{cases}F^c(\varepsilon) & \text{if } \varepsilon\in \text{A - B}\\ G^c(\varepsilon) & \text{if } \varepsilon\in \text{B - A}\\ (F(\varepsilon))^c\cap(G(\varepsilon))^c & \text{if } \varepsilon\in \text{A}\cap\text{B}\end{cases}$$

$$=\begin{cases}F^c(\varepsilon) & \text{if } \varepsilon\in \text{A - B}\\ G^c(\varepsilon) & \text{if } \varepsilon\in \text{B - A}\\ F^c(\varepsilon)\cap G^c(\varepsilon) & \text{if } \varepsilon\in \text{A}\cap\text{B}\end{cases}$$

Again,

$(F,A)^c \tilde{\vee} (G,B)^c = (F^c, A) \tilde{\vee} (G^c, B) = (I, J)$, say

Where $J = A \cup B$

and $\forall \varepsilon \in J, I(\varepsilon) = \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \in A - B \\ G^c(\varepsilon) & \text{if } \varepsilon \in B - A \\ F^c(\varepsilon) \cup G^c(\varepsilon) & \text{if } \varepsilon \in A \cap B \end{cases}$

We see that $C = J$ and $\forall \varepsilon \in C, H^c(\varepsilon) \subseteq I(\varepsilon)$

Thus $((F,A) \tilde{\vee} (G,B))^c \tilde{\subseteq} (F,A)^c \tilde{\vee} (G,B)^c$

2. Let $(F,A) \tilde{\frown} (G,B) = (H,C)$,

Where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$

Thus $((F,A) \tilde{\frown} (G,B))^c = (H,C)^c = (H^c, C)$,

Where $C = A \cap B$ and

$$\begin{aligned} \forall \varepsilon \in C, H^c(\varepsilon) &= (F(\varepsilon) \cap G(\varepsilon))^c \\ &= (F(\varepsilon))^c \cup (G(\varepsilon))^c \\ &= F^c(\varepsilon) \cup G^c(\varepsilon) \end{aligned}$$

Again, $(F,A)^c \tilde{\frown} (G,B)^c = (F^c, A) \tilde{\frown} (G^c, B) = (I, J)$, say

Where $J = A \cap B$ and $\forall \varepsilon \in J, I(\varepsilon) = F^c(\varepsilon) \cap G^c(\varepsilon)$

We see that $C = J$ and $\forall \varepsilon \in C, I(\varepsilon) \subseteq H^c(\varepsilon)$

Thus $(F,A)^c \tilde{\frown} (G,B)^c \tilde{\subseteq} ((F,A) \tilde{\frown} (G,B))^c$

Proposition 4.5 (De Morgan Inclusions)

For soft sets (*F*, *A*) and (*G*, *B*) over the same universe *U*, we have the following -

1. $(F,A)^c \tilde{\frown} (G,B)^c \tilde{\subseteq} ((F,A) \tilde{\smile} (G,B))^c$
2. $((F,A) \tilde{\frown} (G,B))^c \tilde{\subseteq} (F,A)^c \tilde{\smile} (G,B)^c$

Proof.

1. Let $(F,A) \tilde{\smile} (G,B) = (H,C)$,

where $C = A \cup B$ and

$\forall \varepsilon \in C, H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B \\ G(\varepsilon) & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \in A \cap B \end{cases}$

Thus $((F,A) \tilde{\vee} (G,B))^c = (H,C)^c = (H^c, C)$,

where $C = A \cup B$ and

$$\forall \varepsilon \in C, H^c(\varepsilon) = (H(\varepsilon))^c = \begin{cases} (F(\varepsilon))^c & \text{if } \varepsilon \in A - B \\ (G(\varepsilon))^c & \text{if } \varepsilon \in B - A \\ (F(\varepsilon) \cup G(\varepsilon))^c & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$= \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \in A - B \\ G^c(\varepsilon) & \text{if } \varepsilon \in B - A \\ (F(\varepsilon))^c \cap (G(\varepsilon))^c & \text{if } \varepsilon \in A \cap B \end{cases}$$

Again, $(F,A)^c \tilde{\frown} (G,B)^c = (F^c, A) \tilde{\frown} (G^c, B) = (I, J)$, say

Where $J = A \cap B$ and $\forall \varepsilon \in J, I(\varepsilon) = F^c(\varepsilon) \cap G^c(\varepsilon)$

We see that $J \subseteq C$ and $\forall \varepsilon \in J, I(\varepsilon) \subseteq H^c(\varepsilon)$

Thus $(F,A)^c \tilde{\frown} (G,B)^c \tilde{\subseteq} ((F,A) \tilde{\smile} (G,B))^c$

2. Let $(F,A) \tilde{\frown} (G,B) = (H,C)$, where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$

Thus $((F,A) \tilde{\frown} (G,B))^c = (H,C)^c = (H^c, C)$,

Where $C = A \cap B$ and

$$\begin{aligned} \forall \varepsilon \in C, H^c(\varepsilon) &= (F(\varepsilon) \cap G(\varepsilon))^c \\ &= (F(\varepsilon))^c \cup (G(\varepsilon))^c \\ &= F^c(\varepsilon) \cup G^c(\varepsilon) \end{aligned}$$

Again, $(F,A)^c \tilde{\smile} (G,B)^c = (F^c, A) \tilde{\smile} (G^c, B) = (I, J)$, say

Where $J = A \cup B$ and

$\forall \varepsilon \in J, I(\varepsilon) = \begin{cases} F^c(\varepsilon) & \text{if } \varepsilon \in A - B \\ G^c(\varepsilon) & \text{if } \varepsilon \in B - A \\ F^c(\varepsilon) \cup G^c(\varepsilon) & \text{if } \varepsilon \in A \cap B \end{cases}$

We see that $C \subseteq J$ and $\forall \varepsilon \in C, H^c(\varepsilon) \subseteq I(\varepsilon)$

Thus $((F,A) \tilde{\frown} (G,B))^c \tilde{\subseteq} (F,A)^c \tilde{\smile} (G,B)^c$

Example 4.2 Let $U = \{c_1, c_2, c_3, c_4\}$ be the set of four cars under consideration and

$E = \{e_1(\text{costly}), e_2(\text{Beautiful}), e_3(\text{Fuel Efficient}), e_4(\text{Modern Technology}), e_5(\text{Luxurious})\}$ be the set of parameters and $A = \{ e_1, e_2, e_3\} \subseteq E$ and

$B = \{ e_1, e_2, e_3, e_5\} \subseteq E$. Then

(*F*, *A*) $= \{F(e_1) = \{c_1, c_4\}, F(e_2) = \{c_1, c_2, c_4\}, F(e_3) = \{c_3\}\}$

is the soft set representing the 'attractiveness of the car' which Mr. X is going to buy and

(*G*, *B*) $= \{G(e_1) = \{c_1, c_2, c_4\}, G(e_2) = \{c_1, c_3\}, G(e_3) = \{c_3\}, G(e_5) = \{c_2, c_3, c_4\}\}$

is the soft set representing the 'attractiveness of the car' which Mr. Y is going to buy. Now,

$(F, A)^c = (F^c, A)$
$= \{F^c(e_1) = \{c_2, c_3\}, F^c(e_2) = \{c_3\}, F^c(e_3) = \{c_1, c_2, c_4\}\}$

And

$(G, B)^c = (G^c, B)$
$= \{G^c(e_1) = \{c_3\}, G^c(e_2) = \{c_2, c_4\}, G^c(e_3) = \{c_1, c_2, c_4\}, G^c(e_5) = \{c_1\}\}$

$(F,A) \tilde{\frown} (G,B) = (H, A \cap B)$
$= \{H(e_1) = \{c_1, c_4\}, H(e_2) = \{c_1\}, H(e_3) = \{c_3\}\}$

$((F,A) \tilde{\frown} (G,B))^c = (H, A \cap B)^c = (H^c, A \cap B)$
$= \{H^c(e_1) = \{c_2, c_3\}, H^c(e_2) = \{c_2, c_3, c_4\}, H^c(e_3) = \{c_1, c_2, c_4\}\}$

$(F,A) \tilde{\smile} (G,B) = (I, A \cup B)$
$= \{I(e_1) = \{c_1, c_2, c_4\}, I(e_2) = \{c_1, c_2, c_3, c_4\}, I(e_3) = \{c_3\},$

$I(e_5) = \{c_2, c_3, c_4\}\}$

$\left((F,A)\tilde{\cup}(G,B)\right)^c = (I, A\cup B)^c = \left(I^c, A\cup B\right)$

$= \{I^c(e_1) = \{c_3\}, I^c(e_2) = \varphi,\ I^c(e_3) = \{c_1, c_2, c_4\},$
$I^c(e_5) = \{c_1\}\}$

$(F,A)^c\,\tilde{\cup}(G,B)^c = \left(F^c, A\right)\tilde{\cup}\left(G^c, B\right) = (J, A\cup B)$

$= \{J(e_1) = \{c_2, c_3\}, J(e_2) = \{c_2, c_3, c_4\}, J(e_3) = \{c_1, c_2, c_4\},$
$J(e_5) = \{c_1\}\}$

$(F,A)^c\,\tilde{\cap}(G,B)^c = \left(F^c, A\right)\tilde{\cap}\left(G^c, B\right) = (K, A\cap B)$

$= \{K(e_1) = \{c_3\}, K(e_2) = \varphi, K(e_3) = \{c_1, c_2, c_4\},\}$

It is clear that

$A\cap B \subseteq A\cup B$, and for all $\varepsilon \in A\cap B$, $K(\varepsilon) = I^c(\varepsilon)$. Thus

$(F,A)^c\,\tilde{\cap}(G,B)^c \,\tilde{\subseteq}\, \left((F,A)\tilde{\cup}(G,B)\right)^c$

Also

$A\cap B \subseteq A\cup B$, and for all $\varepsilon \in A\cap B$, $H^c(\varepsilon) = J(\varepsilon)$. Thus

$\left((F,A)\tilde{\cap}(G,B)\right)^c \,\tilde{\subseteq}\, (F,A)^c\,\tilde{\cup}(G,B)^c$

It is obvious from this example that the reverse inclusions in this case are not valid. However the De Morgan Laws are valid for soft sets with the same set of parameter, which is evident from the following proposition.

Proposition 4.6 (De Morgan Laws)

For soft sets (*F*, *A*) and (*G*, *A*) over the same universe *U*, we have the following -

1. $\left((F,A)\tilde{\cup}(G,A)\right)^c = (F,A)^c\,\tilde{\cap}(G,A)^c$
2. $\left((F,A)\tilde{\cap}(G,A)\right)^c = (F,A)^c\,\tilde{\cup}(G,A)^c$

Proof.

1. Let $(F,A)\tilde{\cup}(G,A) = (H,A)$,

Where $\forall\varepsilon \in A, H(\varepsilon) = F(\varepsilon)\cup G(\varepsilon)$

Thus $\left((F,A)\tilde{\cup}(G,B)\right)^c = (H,A)^c = \left(H^c, A\right)$,

Where $\forall\varepsilon \in A, H^c(\varepsilon) = (H(\varepsilon))^c = (F(\varepsilon)\cup G(\varepsilon))^c$

$= (F(\varepsilon))^c \cap (G(\varepsilon))^c = F^c(\varepsilon)\cap G^c(\varepsilon)$

Again, $(F,A)^c\,\tilde{\cap}(G,A)^c = \left(F^c, A\right)\tilde{\cap}\left(G^c, A\right) = (I, A)$, say

Where $\forall\varepsilon \in A, I(\varepsilon) = F^c(\varepsilon)\cap G^c(\varepsilon)$

Thus $\left((F,A)\tilde{\cup}(G,A)\right)^c = (F,A)^c\,\tilde{\cap}(G,A)^c$

2. Let $(F,A)\tilde{\cap}(G,A) = (H,A)$,

Where $\forall\varepsilon \in A, H(\varepsilon) = F(\varepsilon)\cap G(\varepsilon)$

Thus $\left((F,A)\tilde{\cap}(G,B)\right)^c = (H,A)^c = \left(H^c, A\right)$,

Where $\forall\varepsilon \in A, H^c(\varepsilon) = (H(\varepsilon))^c = (F(\varepsilon)\cap G(\varepsilon))^c$

$= (F(\varepsilon))^c \cup (G(\varepsilon))^c = F^c(\varepsilon)\cup G^c(\varepsilon)$

Again, $(F,A)^c\,\tilde{\cup}(G,A)^c = \left(F^c, A\right)\tilde{\cup}\left(G^c, A\right) = (I, A)$, say

Where $\forall\varepsilon \in A, I(\varepsilon) = F^c(\varepsilon)\cup G^c(\varepsilon)$

Thus $\left((F,A)\tilde{\cap}(G,A)\right)^c = (F,A)^c\,\tilde{\cup}(G,A)^c$

Maji et al [6] proved the following De Morgan Types of results for soft sets (*F*, *A*) and (*G*, *B*) over the same universe *U*. We can verify that these De Morgan types of results are valid under our new definition of complement of a soft set.

Proposition 4.7 For soft sets (*F*, *A*) and (*G*, *B*) over the same universe *U*, we have the following -

1. $\left((F,A)\wedge(G,B)\right)^c = (F,A)^c \vee (G,B)^c$
2. $\left((F,A)\vee(G,B)\right)^c = (F,A)^c \wedge (G,B)^c$

Proof.

1. Let $(F,A)\wedge(G,B) = (H, A\times B)$,

Where $H(\alpha,\beta) = F(\alpha)\cap G(\beta)$, $\forall\alpha\in A$ and $\forall\beta\in B$, where $\cap$ is the operation intersection of two sets.

Thus $\left((F,A)\wedge(G,B)\right)^c = (H, A\times B)^c = \left(H^c, A\times B\right)$

Where $\forall(\alpha,\beta)\in A\times B$,

$H^c(\alpha,\beta) = (H(\alpha,\beta))^c = (F(\alpha)\cap G(\beta))^c = (F(\alpha))^c \cup (G(\beta))^c = F^c(\alpha)\cup G^c(\beta)$

Let $(F,A)^c \vee (G,B)^c = \left(F^c, A\right)\vee\left(G^c, B\right) = (O, A\times B)$,

where $O(\alpha,\beta) = F^c(\alpha)\cup G^c(\beta)$, $\forall\alpha\in A$ and $\forall\beta\in B$, where $\cup$ is the operation union of two sets.

Thus $\left((F,A)\wedge(G,B)\right)^c = (F,A)^c \vee (G,B)^c$

2. Let $(F,A)\vee(G,B) = (H, A\times B)$,

Where $H(\alpha,\beta) = F(\alpha)\cup G(\beta)$, $\forall\alpha\in A$ and $\forall\beta\in B$, where $\cup$ is the operation union of two sets.

Thus $\left((F,A)\vee(G,B)\right)^c = (H, A\times B)^c = \left(H^c, A\times B\right)$

Where $\forall(\alpha,\beta)\in A\times B$,

$H^c(\alpha,\beta) = (H(\alpha,\beta))^c = (F(\alpha)\cup G(\beta))^c = (F(\alpha))^c \cap (G(\beta))^c = F^c(\alpha)\cap G^c(\beta)$

Let $(F,A)^c \wedge (G,B)^c = \left(F^c, A\right)\wedge\left(G^c, B\right) = (O, A\times B)$,

where $O(\alpha,\beta) = F^c(\alpha)\cap G^c(\beta)$, $\forall\alpha\in A$ and $\forall\beta\in B$, where $\cap$ is the operation intersection of two sets.

Thus $\left((F,A)\vee(G,B)\right)^c = (F,A)^c \wedge (G,B)^c$

5. CONCLUSION

We have seen that if we use the new definition of complement of a soft set, we arrive at the conclusion that the soft sets, too, follow the set theoretic axioms of exclusion and contradiction in addition to all those properties that complement of a set in classical sense really does. We have verified our claim with supporting examples and proof.

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