Application of Fuzzy Topological relation in Flood Prediction

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ABSTRACT
Now a day in GIS application fuzzy spatial objects have become extremely important. There have been many research developments on the conceptual description of topological relation between spatial objects. In this paper a formal definition of the computational fuzzy topology is shown which is based on the interior operator and closure operators. In spatial object modeling the interior and exterior boundary are computed based on computational fuzzy topology. An example for determining interior boundary and exterior boundary of flood affected areas of upper Assam based on data collected from Govt. of Assam GOI Directory Assam Tourism NIC ASHA Districts of India.

Keywords
Fuzzy topology; Fuzzy spatial objects; closure operator; interior operator

1. INTRODUCTION
Topological relation between spatial objects is used in geographic information system with positional and attributes information. Information on topological relations can be used for spatial queries, spatial analysis data quality control (e.g. checking for topological consistency) and others. Topology relations can be crisp or fuzzy depending on the certainty or uncertainty of spatial objects and the nature of their relations. Originally in the modeling of spatial objects, such as rivers, roads, trees, and building in GIS, it was normally assumed that the measurement on the spatial objects were free of errors. But in reality the description of the spatial objects in

GIS contain some uncertainties, such as random errors in measuring spatial objects or vagueness in interpreting boundaries of nature. For example vagueness or fuzziness in the boundary between states (Blakemore 1984) or between urban and rural areas is difficult to describe by traditional GIS. Therefore there is a need to enhance existing GIS's by further copying with the uncertainties in spatial objects and the topological relation between uncertain spatial objects. Thus the classical set theory (Gaal, 1964, A postol 1974) which is based on a crisp boundary, may not be fully suitable for handling such problem of uncertainty (Wang et al, 1990). On the other hand fuzzy sets provide a useful tool to describe uncertainty of single object in GIS.

Zadeh (1965) introduced the concept of the fuzzy set. Fuzzy theory has been developed since 1996 and the theory of fuzzy topology (Zadeh 1965;Chang 1968 ;Wong 1974; Wu and Zheng 1991; Liu and Luo 1997) has been developed based on the fuzzy set. Fuzzy topology theory can potentially be applied to the modeling of fuzzy topological relations among spatial objects.

There are two stages for modeling fuzzy topological relation among spatial objects (a) to define and describe spatial relations qualitatively and (b) to compute the fuzzy topological relations quantitatively. For the first stage of modeling fuzzy spatial relations a number of models have been developed (Egenhofer 1993; Winter 2000; Cohn and Gotts 1996; Clementini and Di Felice 1996; Smith 1996; Tang and Kainz 2002; Tang et al 2003; Tang 2004) which

can provide a conceptual definition of uncertain topological relation between spatial objects based on descriptions of the interior boundary and exterior of spatial objects in GIS. And for the second stage of the modeling of uncertain topological relations, for instance to compute the membership values of interior boundary and exterior of a spatial object based on fuzzy membership function.

In this paper by collecting the real world data set we apply the developed method to GIS to compute the interior, closure and boundary of spatial objects and analysis of the model to derive the topological relation between spatial object.

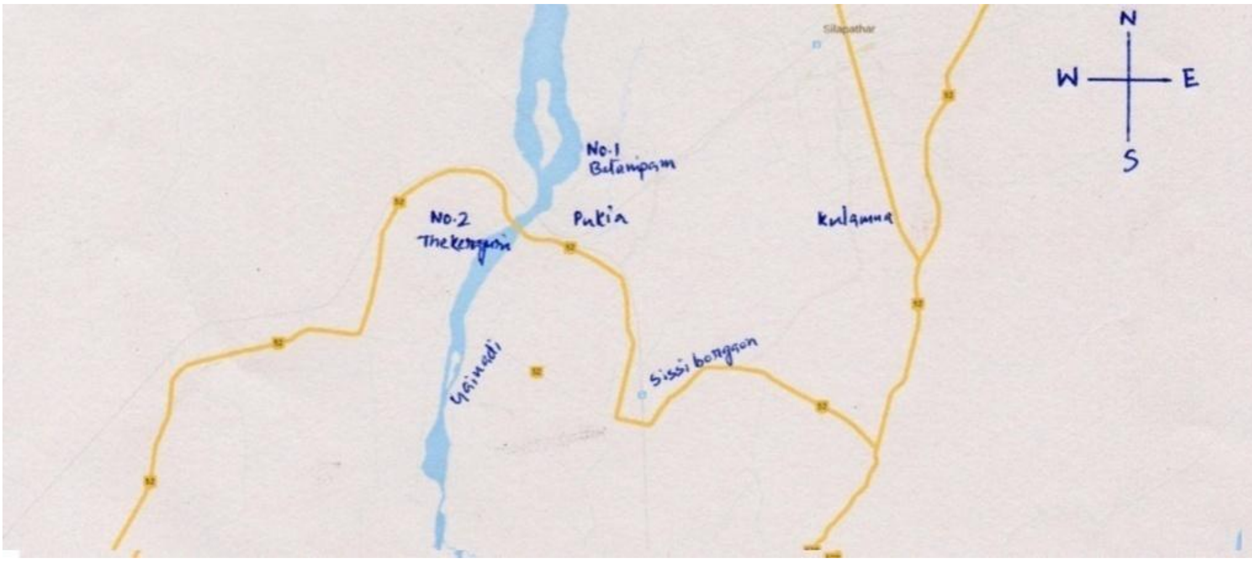


Figure - I : Gainodi River basin Study Area

STUDY AREA

The river Gainadi is a north bank tributary of river Brahmaputra and its total river basin is within the longitude $94^0$40'03"E and 94040'06"E and latitudes $27^0$3'39" and $27^0$40'06"N. the Gainadi sub-basin falls in the West Siang District of Arunachal Pradesh and Dhemaji district of Assam. It is bounded by the Moridhal sub basin on its west and by chimen sub basin on its east. The southern side of the sub basin is bounded by the amguri a channel of Brahmaputra. Total catchment area of the sub basin is about 1320 sq.km out of which 525 sq. km are in Arunashal Pradesh and 795 sq.km are in plains of Assam. The hill carchment comprise of nearly30% of the overall catchment of the basin. The geographic location of the Gainodi river basin and data collection spots are indicated in Figure (I)

The river Gainodi is primarily fed by ground water covering mostly hilly regions. The river has minimal flow during off season but it creates havoc during the monsoon. This necessitates flood forecast during the monsoon to save life and property in the lower part of the basin. The data used in this study are obtained from Govt. of Assam GOI Directory Assam Tourism NIC ASHA Districts of India.

2. FUZZY TOPOLOGY

Fuzzy sets and fuzzy topology

A fuzzy set in X is a function with domain X and value in I, that is an element of I^x. Let $A \in I^X$. The subset of X in which A assumes non-zero values in known as the support of A. For every $x \in X$, A(x) is called the grade of membership of x in A. X is called the carrier of the fuzzy set A. If A takes only the values 0 and 1, then A is called a crisp set in X.

Let $A,B \in I^X$ we define the following fuzzy sets.

1. $A=B$ if $\mu_A(x)=\mu_B(x)$ for all x in X.

2. $A \leq B$ if $\mu_A(x) \leq \mu_B(x)$ for all x in X.

3. $C=A \vee B$ if $\mu_C(x)=$ max $\{ \mu_A(x), \mu_B(x)$ for all x$\}$

4. $A \wedge B \in I^X$ by $(A \wedge B)(x)=$ min$\{A(x), B(x)$, for every $x \in X$ (intersection)$\}$.

3. Fuzzy sets, open fuzzy sets and closed fuzzy sets are the basic elements of fuzzy topology. In the following ,we introduce the concept of fuzzy topology.

2.1 Fuzzy topological space

Let X be a non empty ordinary set and I= [0,1], $\partial \subset I^X$,∂ is called a I-fuzzy topology on X, and (I^X, ∂), is called I- fuzzy topological space (I-fts), if ∂ satisfies the following conditions:

(I) $0,1 \in \partial$;

(II) If $A,B \in \delta$, then $A \wedge B \in \partial$.

(III) Let $\{ A_i : i \in I \} \subset \partial$, where I is an index set, then $V_{i \in I} A_i \in \partial$.

Where $0 \in \partial$ means the empty set and $1 \in \partial$ means the whole set X.

The elements in ∂ are called open elements and the elements in the component of ∂ are called closed elements and the set of the complement of open set is denoted by ∂'.

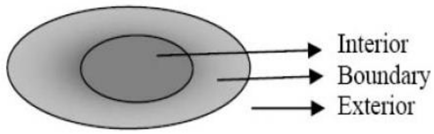
2.2 Definition

For any Fuzzy set A, we define

The interior of A the joining of all the open subsets contained in A, denoted by A^0, i.e. $A^0=V\{B \in \partial; B \leq A\}$

(II) The closure of A as the meeting of all the closed subsets containing A, denoted by $\bar{A}$, i.e. $\bar{A}=\Lambda\{B \in \partial' : B \geq A\}$

(III)The exterior of A is the complement of the closure of A. Figure is given below .



The figure indicates the concept of interior, exterior and boundary of a set.

2.3 Definition

For any fuzzy set A. We define the complements of A by $A'(x)=1-A(x)$

2.4 Definition

The boundary of a fuzzy se A is defined $\partial A = \bar{A} \wedge \bar{A}^{C}$

The above are the commonly used definition of fuzzy topological space, fuzzy interior, fuzzy closure, fuzzy complement and fuzzy boundary (Liu and Luo, 1997).

3. FUZZY TOPOLOGY INDUCED BY THE INTERIOR AND CLOSURE OPERATORS.

We know that each interior operator corresponds to one fuzzy topology and each closure operator corresponds to one topology (Liu and Luo, 1997). In general if we define two operators, interior and closure, separately they will define to fuzzy topologies, respectively. These two topologies may not cohere to each other.

The following two definitions are about the interior and closure operators which are coherent with each other in defining fuzzy topology.

3.1. Definition:

Let A be a fuzzy set in $[0, 1]^X = I^X$ for any fixed $\propto \in [0,1]$ define the interior and closure operators on$[0,1]^X = I^X$ as

$$A \overset{\underline{\propto}}{\rightarrow} A_{\propto} \in I^X \text{ And}$$

$$A \overset{\overline{\propto}}{\rightarrow} A^{\propto} \in I^X \text{ Respectively}$$

Where the fuzzy sets $A_{\propto}$ and $A^{\propto}$ in X are defined by

$$A_{\propto}(X) = \begin{cases} A(X) \; if \; A(X) > \propto \\ 0 \quad if \; A(X) \leq \propto \end{cases}$$

$$A^{\propto}(X) = \begin{cases} 1 \quad if \; A(X) \geq \propto \\ A(X) \; if \; A(X) \leq \propto \end{cases}$$

Remark: Pascali and Ajmal (1997) also defined two similar operators, interior and closure operators. In their definitions these two operators may not further define a coherent fuzzy topology and they give a necessary and sufficient condition for these two operators to be coherent with each other. Those are the necessary and sufficient conditions for these two operators to define an identical fuzzy topology.

4. FUZZY BOUNDARY AND INTERSECTION THEORY

In ordinary topology when we define a topology the boundary of a set A is defined as the intersection of the closure of A with the closure of the complement of A. That is, $\partial A = \overline{A} \Lambda \left(\overline{A^C}\right)$. On the other hand it has a equivalent definition that is $\partial A = \overline{A} - A^{\circ}$. Uniformly, the later is no longer true in fuzzy topology (Liu and Luo,1997; Tang and Kainz 2002; Wu and Zheng 1991). However to be consistent with the previous studies, we adopt the former as the definition of fuzzy boundary.

4.1 Case study

In this section a case study to compute the interior boundary, and exterior of spatial objects has been carried out. The aim of this case study is to determine the flood effected area of upper Assam. Each area affected by flood is recorded. The level of any area affected by flood is marked by a percentage level within the interval [0, 1]. For a particular value of $\propto$, we try to determine the interior, closure and boundary of each area effected by flood. For the interior, if the value is 1, this means the effect is high in relation to the overall effected area. If the value is closure zero, this means that the effect is low in relation to the overall affected areas.

The objective of this paper is to classify the fuzzy interior boundary and exterior of spatial objects for real GIS date of the area affected by flood using a new model.

4.2 Assumption

(I) Each land photo is showed as a fuzzy space.

(II) The size of each area affected by flood is used to calculate the fuzzy value of the fuzzy sets.

(III) The fuzzy value of each area affected by flood is defined as

$$\begin{cases} \dfrac{\log(Area\ of\ certain\ affected\ area)}{\log(Total\ area\ of\ affected\ area)} & \text{if } \dfrac{\log(\mathbf{0})}{\log(*)} > 0, \\ 0 & \text{otherwise,} \end{cases}$$

which is a well defined mapping from the interval $[1, \propto]$ to the interval $[0,1]$

(IV) The fuzzy interior and boundary will be computed for $\propto$ equal to 0.3,0.45 and 0.6 etc respectively.

Table no – I

I.D. NO	AREA(M.H)	I.D. NO	AREA(M.H)
1	1.24	22	10.24
2	1.53	23	10.262
3	2.23	24	10.56
4	2.31	25	12.29
5	2.74	26	12.40
6	3.06	27	12.87
7	4.3	28	12.91
8	4.57	29	14.19
9	4.88	30	15.11
10	6.73	31	15.17
11	6.9	32	15.30
12	7.23	33	15.95
13	7.26	34	29.00
14	7.53	35	29.00
15	7.58	36	38.2
16	8.16	37	38.64
17	8.72	38	39.01
18	8.78	39	39.78
19	9.66	40	49.82
20	9.97	41	68.85
21	10.01		

Total: 614.942

Average- 14.99858537

5. DISCUSSION ON THE NEWLY DEVELOPED FUZZY TOPOLOGY MODEL

Table no- 2: The values of fuzzy exterior, fuzzy interior and fuzzy boundary with different values of $\propto$

ID	AREA (HM)	FUZZY VALUE	$\propto = 0.3$			$\propto = 0.45$			$\propto = 0.6$		
			Exterior	Interior	Boundary	Exterior	Interior	Boundary	Exterior	Interior	Boundary
1	1.24	0.033	0.967	0	0.033	0.969	0	.033	.967	0	.033
2	1.53	0.066	0.934	0	0.066	.934	0	.066	.934	0	0.066
3	2.23	0.12	0.88	0	0.12	.88	0	.12	.88	0	.12
4	2.31	0.13	0.87	0	0.13	.87	0	.13	.87	0	.13
5	2.74	0.16	0.84	0	0.16	.84	0	.16	.84	0	0.16
6	3.06	0.17	0.83	0	0.17	.83	0	.17	.83	0	.17
7	4.3	0.23	0.77	0	0.23	.77	0	.23	.77	0	.23
8	4.57	0.24	0.76	0	0.24	.76	0	.24	.76	0	.24
9	4.88	0.25	0.75	0	0.25	.75	0	.25	.75	0	.25
10	6.73	0.30	0.70	0.30	0.30	.70	0	.20	.70	0	.30
11	9.6	0.30	0.70	0.30	0.30	.70	0	.30	.70	0	.30
12	7.23	0.31	0.69	0.31	0.31	.69	0	.31	.69	0	.31
13	7.26	0.31	0.69	0.31	0.31	.69	0	.31	.69	0	.31
14	7.53	0.31	0.69	0.31	0.31	.69	0	.31	.69	0	.31
15	7.58	0.32	0.68	0.32	0.32	.68	0	.32	.68	0	.32
16	8.16	0.33	0.67	0.33	0.33	.67	0	.33	.67	0	.33
17	8.72	0.34	0.66	0.33	0.34	.66	0	.34	.66	0	.34
18	8.78	0.34	0.66	0.33	0.34	.66	0	.34	.66	0	.34
19	9.66	0.35	0.65	0.35	0.35	.65	0	.35	.65	0	.35
20	9.97	0.36	0.64	.36	.36	.64	0	.36	.64	0	.36
21	10.01	0.36	0.64	.36	.36	.64	0	.36	.64	0	.36
22	10.24	0.36	0.64	.36	.36	.64	0	.36	.64	0	.36
23	10.262	0.36	0.64	.36	.36	.64	0	.36	.64	0	.36
24	10.56	0.37	0.63	.37	.37	.63	0	.37	.63	0	.37
25	12.29	0.39	0.61	0.39	0.39	.61	0	.39	.61	0	.39
26	12.40	0.39	0.61	0.39	0.39	.61	0	.39	.61	0	.39
27	12.87	0.40	0.60	0.40	0.40	.60	0	.40	.60	0	1.00
28	12.91	0.40	0.60	0.40	0.40	.60	0	.40	.60	0	1.00
29	14.19	0.41	.59	.41	.41	.59	0	0.41	.59	0	1.00
30	15.11	0.42	.58	.42	.42	.58	0	.42	.58	0	1.00
31	15.17	0.42	.58	.42	.42	.58	0	.42	.58	0	1.00
32	15.30	0.42	.58	.42	.42	.58	0	.42	.58	0	1.00
33	15.95	0.43	.57	.43	.43	.57	0	.43	.57	0	1.00
34	29.00	0.52	.48	.52	.48	.48	.52	.48	.48	0	1.00
35	29.00	0.52	.48	.52	.48	.48	.52	.48	.48	0	1.00
36	38.2	0.57	.43	.57	.43	.43	.57	.43	.43	0	1.00
37	38.64	0.57	.43	.57	.43	.43	.57	.43	.43	0	1.00
38	39.01	0.57	.43	.57	.43	.43	.57	.43	.43	0	1.00
39	39.78	0.57	.43	.57	.43	.43	.57	.43	.43	0	1.00
40	49.82	0.61	0.34	0.61	0.39	.39	.61	.39	.39	.61	.39
41	68.85	0.66	0.34	0.66	0.34	.34	.66	.34	.34	.66	.34

The table no 2 shows that for different value of $\propto$, the fuzzy exterior, interior, and fuzzy boundary are different. From the table we can say that for the larger value of$\propto$, the size of the interior will be smaller. For $\propto$ is equal to 0.3, ID no greater than 9 having nonzero value. Again for $\propto = 0.45$ ID no greater than 33 have nonzero value and for $\propto = 0.6$ ID no greater than 39 have nonzero values. This shows that the relation between $interior$ and size of closure are directly proportional to each other while the relation between exterior and the size of interior are inversely proportional. Clearly this new model can be used to simplify the fuzzy interior, boundary, and exterior of fuzzy spatial object. Simplifying the fuzzy interior, boundary and exterior of this areas affected by flood in Assam in this paper is a dynamic application in GIS.

6. CONCLUSION

In this paper an attempt has been made to provide a solution to compute the topological relations between spatial objects. The former approaches (Cohn and Gilts, 1996; Smith, 1996; Tang and Kainz, 2002) have introduced the concept of fuzzy topology into GIS. We use the application of fuzzy topology to compute the interior, exterior and boundary of fuzzy spatial objects. A case study for classifying the fuzzy interior, exterior, and boundary of the flood affected area of Assam has been carried out. It is found that not only we get information on fuzzy topological relations between two objects (Such as the value of the interior, boundary, and exterior) but a quantitative level of these topological relations between simple fuzzy regions can be obtained.

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