Distributed Cross-layer Power Control in Pre-equalized Dwnlink of a MC-CDMA System

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ABSTRACT
The main purpose of this elaboration is to determine the optimal powers of mobile users in a multi-carrier CDMA (MC-CDMA) system receiving data from a base station (BS). The downlink transmission is pre-equalized and the utility function is a generalized energy efficiency (EE) based on a cross-layer approach and taking into account the presence of a queue. We rely on game theory as a mathematical tool by considering a non-cooperative game whose equilibrium point is called Nash equilibrium (NE) defining the optimal powers. Accordingly, we developed a distributed algorithm aiming at determining this equilibrium. When the subcarriers number increases, important interference is then introduced. In such case, we showed that the social welfare (sum of utilities of all users) decreases while powers at NE increase. In addition, we deduced that a good choice of the consumed power leads to minimize the individual power and maximize the utility at the NE. Moreover, we noticed that when the packet arrival rate increases, the considered generalized EE can be maximized compared to the conventional EE approach while the power at the NE can be minimized. Therefore, we contribute to improve globally the system performances in terms of social welfare, individual utility function and individual power.

General Terms:
power control, game theory, non-cooperative game

Keywords:
distributed algorithm, cross-layer energy efficiency (EE), Nash equilibrium (NE), MC-CDMA system.

1. INTRODUCTION
The fast evolution of wireless technologies led to spectrum congestion where a huge number of mobile users are transmitting and receiving data. Ultra-wide band and cognitive radio introduce solutions to the under-exploitation of the limited spectrum and energy management of wireless systems contributes to optimize the energy consumption over wireless networks. Designing communications systems with efficient power control has been then a critical issue aiming at determining optimal powers and improving the global systems performances.

Different works were interested in power control by referring either to game theory which studies interaction between players (users) or to optimization problems consisting generally in maximizing a utility function under certain constraints. In [1], a distributed network comprising an interference channel in parallel with an interference relay channel is considered. Authors were interested in determining how each source node is going to use its power over the two frequency bands in an optimal way by referring to game theory. A non-cooperative game of power control was considered where each user seeks to maximize its Shannon transmission rate. Existence and uniqueness of the Nash equilibrium (NE) have been proved. In [2], the study consisted in maximizing the ergodic capacity of a secondary user to determine its optimal power over fading states subject to a new proposed constraint in order to protect the primary user transmission.

In this work, we opted to the multi-carrier CDMA (MC-CDMA) system as it is a promising technique enabling downlink transmissions synchronization [3]. The considered system is described in [4] where multiple mobile users receive data from a base station (BS) and the downlink transmission is pre-equalized. Authors in [4] studied the power control by maximizing the Goodman energy efficiency (EE) developed in [5] and the sum-rate as well. Therefore, closed form expressions of pre-equalization factors are established for both maximization problems (utility maximization and sum-rate maximization). These optimal solutions are distributed according to channel state information (CSI) and data rates, but centralized regarding the spreading sequences of all active users [4].

Here, we are interested in exploiting game theory, a powerful mathematical tool which has been recently used to study power control in wireless communications. Contrary to the maximization problem studied in [4], our elaboration refers to game theory since a situation of interaction between the users can be defined as they are able to observe each other and react accordingly [1]. Indeed, game theory focuses on the multiuser competitiveness and interaction by modeling a power control problem as a game where the users are the players, the strategies are the powers and the utility function is a metric to be optimized (energy efficiency (EE), Shannon capacity, etc) [6]. Our work is focused on a new generalized EE function recently developed in [7][8]. This EE considers the whole terminal power and not only the radiated power and takes into account the presence of queue of arriving packets transmitted with a success rate determined according to the transmit power and the channel conditions [7]. Authors in [8] developed a distributed algorithm for...
an interference channel system operating in a single carrier. In our contribution, we studied a non-cooperative power control game in a multi-carrier CDMA system where all the players (users) seek to maximize selfishly their EE. The resulting optimal solutions set in terms of users powers is called Nash equilibrium (NE) from which no player has interest to deviate unilaterally. The optimal powers of the mobile users are resulting from the determination of the optimal pre-equalization factors.

The study showed relevant conclusions. Indeed, we highlight that the social welfare at the NE increases with the number of users while it decreases with the subcarriers number. This is due to the multiple-access interference introduced by the diversity induced by the MC-CDMA. Accordingly, the individual power at the NE increases with the number of subcarriers as the user needs sufficient power to receive data from the BS when the interference increases. Moreover, results showed that as much as the consumed power is reduced, the minimum of optimal power is required and the maximum of EE is reached. We can deduce therefore that a good choice of the consumed power contributes to a global improvement of the system performances. Finally, we studied the effect of the packet arrival rate on the system performance. Numerical results are presented in section 4 and finally several conclusions are mentioned.

2. SYSTEM MODEL

The system model under study is the one considered in [4]. We consider the downlink transmission of a MC-CDMA wireless system composed of users as shown in Fig.1. At an instant \( k \), all data are transmitted from the BS to the users with a processing gain \( N \) on orthogonal subcarriers whose number is assumed equal to \( N \) (same as the processing gain). We denote \( b_j[k] \in \mathbb{C} \) the data corresponding to the \( j \)th user, \( c_{j}[1, j, 2, \ldots, j, N]^{T} \in \mathbb{R}^{N} \) the spreading sequence with \( c_{j}[l] = \pm 1/\sqrt{N} \) is the spreading factor of the \( j \)th user on the \( l \)th subcarrier and \( p_{j,k} \) the transmitted power of the \( j \)th user on the \( N \) subcarriers with \( \lvert \cdot \rvert \) refers to the transpose operator. Therefore, the transmitted signal \( x[k] \) is given by:

\[
x[k] = \sum_{j=1}^{M} x_j[k] = \sum_{j=1}^{M} \begin{bmatrix} p_{1,j,k}c_{1,j} \\ \vdots \\ p_{N,j,k}c_{N,j} \end{bmatrix} b_j[k].
\]

When assuming that the transmitted data symbols have zero mean and normalized power, the transmitted power of the \( j \)th user at instant \( k \) is:

\[
\mathbb{E}[\lVert x_{j}[k] \rVert^2] = \mathbb{E} \left[ \sum_{l=1}^{N} \lvert p_{j,l,k}c_{j,l}b_{j,k} \rvert^2 \right] = \frac{1}{N} \lVert p_{j}[k] \rVert^2. \tag{2}
\]

Since the pre-equalization vector has a maximum power denoted as \( P_{\text{max}} \) supposed identical for all the users, the user power is then limited as well. Let \( b_{j}[k] = [b_{j,1}[k], b_{j,2}[k], \ldots, b_{j,N}[k]]^{T} \in \mathbb{C}^{N} \) denotes the channel gains vector of the \( N \) subcarriers corresponding to the \( j \)th user and \( n[k] \) is an additive white Gaussian noise with zero mean and covariance \( \sigma^2 \mathbf{I} \). Accordingly, the received signal \( y[k] \) at instant \( k \) is given by:

\[
y[k] = \sum_{j=1}^{M} \begin{bmatrix} p_{1,j,k}c_{1,j}h_{1,j} \\ \vdots \\ p_{N,j,k}c_{N,j}h_{N,j} \end{bmatrix} b_j[k] + n[k].
\]

At each user, a matched filter detector is used depending only on the spreading sequence \( c_{j} \). Consequently, the SINR corresponding to the \( j \)th user is expressed as follows:

\[
\gamma_{j}[k] = \frac{R_{0}}{R} \frac{\mathbf{p}_{j}^{H} \mathbf{g}_{s,j} \mathbf{g}_{s,j}^{H} \mathbf{p}_{j}}{\mathbf{p}_{j}^{H} \mathbf{g}_{s,j} \mathbf{g}_{s,j}^{H} \mathbf{p}_{j} + \sigma^2},
\]

with \( R_{0} \) and \( R \) are the bandwidth and the throughput respectively, \( g_{s,j}[k] = [c_{j,1}h_{1,j}^{*}[k], \ldots, c_{j,N}h_{j,j}^{*}[k]]^{T} \in \mathbb{C}^{N} \) and \( \lvert \cdot \rvert^{H} \) refers to the complex conjugate-transpose operator. In this paper, we are interested in studying how each user is going to control its power optimally according to a cross-layer approach. In what follows, we define the EE function considering, identify the static game formulation and present our distributed algorithm determining the NE solution.

3. GAME THEORY AND NASH EQUILIBRIUM

Each mobile user has to optimize its power. Such an optimization problem can be solved by resorting either to maximization problem since each mobile user seeks to maximize its profit [4] or to game theory. This latter is a powerful mathematical tool aiming at studying interaction between players and maximizing an utility function which is considered as the EE.

3.1 Review of the Goodman energy-efficiency

EE concept has been developed by Goodman in [5] to measure a tradeoff between a high throughput of transmission and a low energy consumption [9]. Thus, the EE was defined as the ratio of the user throughput to its transmit power level as follows [6, 10]:

\[
\eta = \frac{T}{P}, \quad \text{[bits/Joule]},
\]

with \( T \) and \( P \) are the throughput and the user power, respectively. When considering the transmission of information bits in packets of \( B \) bits at a rate \( R \) (bits/s) with a Frame Success Rate \( P_{c} \) (probability of correct reception of a frame at the receiver), the throughput is expressed as:

\[
T = \frac{LR}{B} P_{c}, \quad \text{[bits/s]},
\]

\[
\]
where $P_e$ is a function of the Bit Error Rate denoted as $P_e$ such that [5,11]:

$$P_e = (1 - P_e)^B \text{ [bits/s]},$$

(7)

Since the substitution of $P_e$ in the utility function $u$ causes a mathematical problem when $p^u = 0$, an efficiency function was defined in [5] to approximate $P_e$ as follows:

$$f(\gamma) = (1 - 2P_e)^B.$$ 

(8)

This function is sigmoid, increasing, continuous and identical for all the users such that $f : [0, +\infty) \to [0, 1]$, $f(0) = 0$ and $\lim_{x \to +\infty} f(x) = 1$ [5,10]. Accordingly, the Goodman EE expresses as:

$$\eta_j(p^u) = \frac{Rf(\gamma_j)}{b + p_j},$$

(9)

where $p^u = (p_1^u, p_2^u, \ldots, p_N^u)$ describes the power vector of all users.

### 3.2 Generalized energy-efficiency

In [12], authors consider a total transmission cost of the type $\text{radiated power} (p^r) + \text{consumed power} (b)$ to design distributed power control strategies for multiple access channels [7,8]. Thus, a new EE form of EE than the classic one introduced by Goodman in [5] has been considered and is given by:

$$\eta_j(p^u) = \frac{Rf(\gamma_j)}{b + p_j}.$$ 

(10)

A new EE metric has been developed in [7,8] by considering the previous EE form of equation (10) where not only a consumed power $b \neq 0$ is added to the denominator of (9) but also a packet arrival rate $q$ and a finite memory buffer of size $K$, into which packets arrive from an upper layer, are introduced such that:

$$\eta_j(p^u) = \frac{Rq(1 - \Phi(\gamma_j(p^u)))}{b + qp_j(1 - \Phi(\gamma_j(p^u)))}.$$ 

(11)

The expression of the function $\Phi$ is:

$$\Phi(\gamma_j) = 1 - f(\gamma_j)\Pi_K(\gamma_j).$$ 

(12)

It defines the packet loss due to both bad channel conditions and the finiteness of the packet buffer while $\Pi_K(\gamma_j)$ is the stationary probability that the buffer is full and is expressed as follows:

$$\Pi_K(\gamma_j) = \frac{\rho^K(\gamma_j)}{1 + \rho(\gamma_j) + \ldots + \rho^K(\gamma_j)},$$

(13)

with:

$$\rho(\gamma_j) = q(1 - f(\gamma_j)) \frac{1}{1 - qf(\gamma_j)}.$$ 

(14)

When considering a system with an interference channel operating in a single carrier, a non-cooperative game has been studied in [7,8] and a distributed algorithm determining the NE was developed. Here, we are interested to establish a distributed algorithm seeking to determine the NE according to the pre-equalized downlink transmission of a MC-CDMA wireless system.

### 3.3 Static power control game formulation

As the users are able to observe each other and react accordingly, a situation of interaction can be defined [1] and a static power control game can be identified as well.

**DEFINITION 1.** The game is defined by the ordered triplet $G = (M, (A_j)_{j \in M}, (u_j)_{j \in M})$ where:

$-M$ is the players set (the $M$ transmitters)
$-A_1, \ldots, A_M$ are the strategies sets with $A_j = [0, P_{\text{max}}/N]$
$-u_1, \ldots, u_M$ are the utility functions given by:

$$u_j(P) = \eta_j(P) = \frac{Rq(1 - \Phi(\gamma_j(P)))}{b + qp_j(1 - \Phi(\gamma_j(P)))}.$$ 

(15)

with $P = [p_1, p_2, \ldots, p_M]$ is a matrix composed of the $M$ pre-equalization vectors.
3.4 Distributed algorithm for Nash equilibrium

In a non-cooperative static game, each user aims at selfishly maximizing its individual EE. This game with complete information and rational players is quasiconcave in the sense of Rosen and has a unique pure NE from which no player has interest to deviate unilaterally [13] [14]. Indeed, authors in [7] proved that the NE exists and is unique. It results by setting $\partial u_j / \partial p^*_j$ to zero. However, it is impossible to solve analytically such equation due to the mathematical form of the new utility function. For this reason, a distributed algorithm was developed in [8] to determine the NE such that each user relies on its SINR corresponding to his chosen power level.

In this elaboration, we aim at determining the NE by relying on the SINR corresponding to the chosen power level of the user and by determining all powers of the pre-equalization factors on all subcarriers corresponding to each user. Our idea is realized through the following distributed algorithm.

**Algorithm 1 Distributed NE determination**

- Initialize the difference in users powers denoted $\Delta$ to small value
- Let $\delta \leftarrow \Delta / 2$ define the threshold of $\Delta$ for the while loop
- $P = \begin{bmatrix} p_{1,1} & \cdots & p_{1,M} \\ \vdots & \ddots & \vdots \\ p_{N,1} & \cdots & p_{N,M} \end{bmatrix}$ Initialize the pre-equalization factors for all the $M$ users on all $N$ subcarriers such that $p_{i,j} \leq \sqrt{P_{\text{max}}}$ with $(i, j) \in [1, N] \times [1, M]$
- $t \leftarrow 0$
- While $\Delta \geq \delta$ do Control the difference between the powers levels
  - for $k = 1 : M$ Iterate over the users indices
  - Calculate $\gamma_k(P^t)$ for the $k^{\text{th}}$ user
  - $\Gamma_k = N\gamma_k(P^t)$ with $p_k^t$ is the $k^{\text{th}}$ column of $P^t$
  - presenting the pre-equalization factors of the $k^{\text{th}}$ user over the $M$ subcarriers
    - for $k = 1 : N$ Iterate over the subcarriers indices
    - $p^t \leftarrow Rq(1 - \Phi(p^t\Gamma_k))$
    - Determine the pre-equalization factor $p^t_{k,j}$ maximising the utility function on every subcarrier for every user
      - $|p^t_{k,j} \leftarrow \min(p^t_{k,j}, \sqrt{P_{\text{max}}})$
  - end for
  - $\Delta \leftarrow \max_k \left( \frac{1}{N} ||p_{k}^{t+1}||^2 - ||p_{k}^t||^2 \right)$
- $t \leftarrow t + 1$
- end while
- Determine the optimal power of each user from (2)

4. NUMERICAL RESULTS

We consider in our simulations the following settings:

- The efficiency function is $f(x) = e^{-c/x}$ with $c = 2 \pi / \sigma^2 - 1$.
- The throughput $R$ and the bandwidth $R_0$ are equal to $10^5$ bps and 1 MHz respectively.
- The consumed power $b$ is fixed to $10^{-4}$ Watt.
- The maximum power $P_{\text{max}}$ is set to 0.01 Watt.
- The noise variance $\sigma^2$ is set to $10^{-3}$ Watt.
- The buffer size $K$ is fixed to 10.

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The packet arrival rate $q$ is set to 0.5.

In Fig. 2, we consider a system of 2 mobile users receiving data on 2 subcarriers and the channel gains are assumed normalized. In this figure, we plot the achievable utilities region and the NE. We notice that the NE is close to the Pareto boundary which defines the outer frontier of the achievable utilities region.

![Fig. 2: Achievable utilities region plus the NE for $b = 10^{-4}$.](image)

We are interested to consider the global performance of the system by studying the variation of the social welfare (sum of utilities $\sum_{j=1}^M u_j$) versus the number of users $M$ and for different number of subcarriers $N$. Simulations are obtained by averaging over Rayleigh fading channels and the results are presented in Fig. 3. In this figure, we highlight that the social welfare is an increasing function of the number of users while it decreases when the number of subcarriers $N$ increases. In fact, since the social welfare defines the sum of the utilities of all users which are positive quantities, it is therefore an increasing function of the users number. However, when the number of subcarriers $N$ increases, this leads to increase the interference and then gains in terms of utilities become reduced.

![Fig. 3: Variation of the social welfare (sum of utilities) as a function of the number of users $M$ for $b = 10^{-4}$.](image)
We present in Fig. 4 the variation of the social welfare as a function of the number of subcarriers $N$ for a system composed of 3 users and for different values of the consumed power $b$. We confirm accordingly that the social welfare is decreasing function of the number of subcarriers. Moreover, we stress that when $b$ increases, the social welfare decreases. This can be proved mathematically since the constant $b$ is in the denominator of the utility function (see equation (11)).

For the same scenario, we study the individual performance of an arbitrary user (user 1 for example). We plot then the corresponding utility and power at the NE as a function of the number of subcarriers. Results are given in figures 5 and 6. We confirm the previous conclusion since the individual utility is a decreasing function of $N$ and $b$. The individual power is however an increasing function of the subcarriers number $N$ and the constant $b$ as well. Indeed, since the subcarriers number increases, the network interference increases also and thus each user needs more power to receive the data from the BS.

Next, we are interested in studying the variations of the individual power and utility (for user 1) as a function of the consumed power $b$ for different values of the packet arrival rate $q$. Results are plotted in figures 7 and 8. According to Fig. 7, we confirm the decreasing aspect of the utility versus the constant $b$ as explained above. However, we highlight that the utility increases with the packet arrival rate $q$ and that the utilities values become closer for $q = 0.4$ and $q \approx 1$. In Fig. 8 we notice the increasing aspect of the power versus $b$ and versus the packet arrival rate $q$ as well.

In what follows, we study the variation of the social welfare and the individual power (of user 1) as a function of the consumed power $b$ for different users number and different subcarriers number. Results are shown in figures 9 and 10. In Fig. 9 we deduce that the social welfare increases with the packet arrival rate $q$ and the number of the users $M$ while it decreases with the number of the subcarriers $N$. This confirms results shown in figures 3 and 4. We highlight also that for any $(M, N)$, the social welfare values become closer when $q$ takes higher values. In addition, we stress...
in Fig. 10 that the individual power is an increasing function of \( q \) and of the subcarriers number \( N \) also, which confirms results in figure 9. In addition, when \( q \to 1 \), one can find the conventional EE expression given by equation (10). In this case, we stress from figures 9 and 10 that it exists \( q \neq 1 \) for which the social welfare reaches maximum values higher than when using the conventional EE (\( q \to 1 \)). For the same \( q \), the individual power takes lower values compared to the power when \( q \to 1 \). To illustrate this, when the number of subcarriers is \( N = 4 \), the social welfare curves as shown in Fig. 9 reach their maximum at \( q = 0.7 \) for which the powers curves in Fig. 10 show that powers take lower values (\( 0.85 \times 10^{-4} \) Watt) when comparing to \( q = 1 \) (power level is equal to \( 1.2 \times 10^{-4} \) Watt). Therefore, an important gain in terms of power (30\%) is achieved. This confirms what was proved in [8] concerning the arrival packet rate study when a system of interference channel with single carrier model is considered.

5. CONCLUSION

In this paper, we proposed a distributed algorithm targeting at determining the NE in a pre-equalized downlink of a MC-CDMA system according to a non-cooperative game. The considered EE has a generalized form based on a cross-layer approach and taking into account the presence of a queue of arriving packets transmitted with a success rate determined according to the transmit power and the channel conditions. We showed how the social welfare decreases when the number of subcarriers increases while the individual power increases accordingly. In fact, as the multi-carrier concept introduces a multiple-access interference, each user needs to increase its individual power at the equilibrium in order to ensure the data reception from the BS. Moreover, simulations showed that when the consumed power is decreasing, the individual power is reduced while the utility function is increased. Thus, a good choice of the consumed power can improve the global performance of the system. Finally, we studied the impact of the packet arrival rate on the social welfare and on the power. Numerical results proved that the considered EE can lead to better performances than the conventional approach of EE. Indeed, considerable gains (30\%) can be reached in terms of individual power and the social welfare can be maximized as well. Our elaboration can be extended by designing a Pareto efficient solution (located at the outer boundary of the achievable utilities region) which ensures considerable gains not only in terms of utilities but also in terms of powers. In this context, the investigation of cooperative games (defining the bargaining theory) and games with cooperative plans (known as repeated games) is an interesting direction to establish a distributed Pareto efficient solution. Indeed, in the bargaining theory, when the users cooperate, they are able to achieve utilities higher than the NE while in repeated games, efficient solutions can be defined when the players agree on a common cooperation plan.

6. REFERENCES


