Analysis of Frequency Division Spectrum Sensing in Cognitive Radio Network for Nakagami Fading Channel

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ABSTRACT
In this paper we have analyzed and simulated Frequency Division Spectrum Sensing (FDSS) in Cognitive Radio Network (CRN) for Nakagami fading channel. The simulation results shows that for different values of Nakagami Parameters \( m \) the performance of the FDSS is different. As the value of Nakagami parameter \( m \) increases probability of false alarm decreases, probability of detection increases and secondary user’s throughput also increases. The simulation results show that for the optimum spectrum sensing bandwidth \( W_s \) 0.5 MHz, the false alarm probability decreases by 49.6% and 96.6% for \( m=2 \) and \( m=5 \) respectively as compare to \( m=1 \), the probability of detection increases by 6% and 17.7% for \( m=2 \) and \( m=5 \) respectively as compare to \( m=1 \), the secondary user throughput increases by 14% and 29.1% for \( m=2 \) and \( m=5 \) respectively as compare to \( m=1 \).

Keywords
Cognitive radio, spectrum sensing framework, Nakagami fading channel.

1. INTRODUCTION
Cognitive Radio Network allows opportunistic access of licensed spectrum to unlicensed users (Secondary user) in such a way that it must not cause any interference to the licensee (Primary user) [1-2]. The different types of spectrum sensing methods like energy detector, Cyclostationary detector, matched filter based detector etc. [3-5] are used to detect absence of the primary signal. In absence of the primary signal the Secondary User (SU) is allowed to access the licensed spectrum. As soon as the SU detects the primary signal it needs to stop its data transmission or switch to other available frequency spectrum. For spectrum sensing different types of spectrum sensing frameworks are used like time division based, frequency based, external spectrum sensing etc. [6-11]. In time division based spectrum sensing the SU senses the spectrum periodically and transmits its data when the primary signal is detected absent. The periodic sensing interrupts data transmission therefore the SU’s throughput remains low. To address the periodic interruption in data transmission using time division based spectrum sensing framework, a frequency division based continuous spectrum sensing framework may be used. In Frequency Division Spectrum Sensing Framework (FDSSF) the SU senses a part of the spectrum continuously while in remaining part of the spectrum it transmits its own data in absence of primary signal [12-16]. The SU’s throughput [17] remains low due to the partial use of the available spectrum. To ensure full utilization of the available spectrum by the SU, an external spectrum sensing framework may be used. In the external spectrum sensing separate modules are required for spectrum sensing and data transmission. The performance of wireless communication systems is affected by fading. The fading effect is analyzed using some fading channel models like Recian, Rayleigh and Nakagami etc. The Rayleigh fading is most applicable and suitable when there is no dominant propagation along a line of sight between transmitter and receiver. If there is a dominant line of sight signal, Rician fading model is more applicable. The Nakagami model behaves as Rayleigh fading channel for Nakagami parameter \( m = 1 \) and Rician model for their equivalent mean values. The Nakagami fading model is more suitable for multipath scattering with relatively large time-delay spread.

In this paper we have analyzed fading effect on FDSSF for Nakagami channel [18-20]. The simulation results show that for different values of Nakagami Parameters \( m \) the performance of the FDSSF is different. As the Nakagami parameter \( m \) increases probability of false alarm decreases, probability of detection increases and secondary user’s throughput also increases. The simulation results show that for the optimum spectrum sensing bandwidth \( W_s \) 0.5 MHz [7], the false alarm probability decreases by 49.6% and 96.6% for \( m=2 \) and \( m=5 \) respectively as compare to \( m=1 \), the probability of detection increases by 6% and 17.7% for \( m=2 \) and \( m=5 \) respectively as compare to \( m=1 \), the secondary user throughput increases by 14% and 29.1% for \( m=2 \) and \( m=5 \) respectively as compare to \( m=1 \).

2. MATHEMATICAL MODEL OF SPECTRUM SENSING
In CRN the SUs can access the licensed band using two main approaches: (i) the SUs are allowed to access a frequency band only when it is detected idle, and (ii) the SUs coexist with the PUs under the condition of protecting the latter from harmful interference. We assume that energy-detection-based spectrum sensing is used by the CR user and noise is independent, identically distributed (i.i.d.) and the transmitted signal is a complex valued Phase shift Keying (PSK) signal. For mathematical analysis of the energy detector [3], let us assume that the received signal has the following form

\[
y(n) = s(n) + w(n)
\]

(1)

Where \( y(n) \) is the received signal, \( s(n) \) is the signal to be detected, \( w(n) \) is the noise sample and \( n \) is sample index. The decision metric for energy detector can be written as

\[
D = \sum_{n=1}^{N} |y(n)|^2
\]

(2)

By comparing the decision metric \( D \) with a threshold \( \lambda \) the spectrum occupancy decision is made. The decision is made by distinguishing between following hypotheses:

\[
H_0 : \quad y(n) = w(n)
\]

(3)

\[
H_1 : \quad y(n) = s(n) + w(n).\]

(4)

Probability of detection \( (p_d) \) and probability of false alarm \( (p_f) \) are used to analyze performance of the spectrum detector. The
\( p_d \) is probability of detecting a primary user signal on considered frequency spectrum when the signal is truly present. Thus a large detection probability is desired. It can be formulated as

\[
p_d = Pr(D > \lambda | H_1)
\]

(5)

\( p_f \) is the false detection probability which indicates that the test incorrectly decides that the primary user signal is present in the considered frequency spectrum. The \( p_f \) should be kept as small as possible in order to prevent underutilization of transmission opportunities. The decision threshold \( \lambda \) can be selected in such a way that gives optimum values of \( p_d \) and \( p_f \).

Assume that the CR users experience independent Nakagami fading channel with the same average SNR (\( \gamma \)) then the power spectral density of the instantaneous SNR (\( \gamma \)) is given by

\[
f_\gamma(y, m) = \frac{\gamma^m}{\gamma^m \Gamma(m)} e^{-\frac{\gamma}{\gamma}} , \gamma \geq 0
\]

(7)

Here \( m \) is the Nakagami parameter. For \( m=1 \), the Nakagami fading channel behaves like Rayleigh fading channel. The Nakagami fading channel is more suitable for analysis of both line of sight and non-line of sight communication.

3. FREQUENCY DIVISION SPECTRUM SENSING FRAMEWORK

Since periodic spectrum sensing over the entire PU spectrum interrupts the SU data transmission in time division based spectrum sensing, which degrades throughput of the SU. The continuous sensing of the PU’s spectrum improves spectrum detection probability. Therefore, to alleviate the SU interruption problem during data transmission and to improve spectrum detection probability, the PU frequency band is divided into two subbands, one for opportunistic SU data transmission, and the other for continuous spectrum sensing. Thus, the average SU transmission delay is removed by selecting the proper bandwidth for spectrum sensing within each frame. Since different SUs may have different requirements on their quality of services, so the achievable average SU throughput is maximized by choosing the optimal sensing bandwidth within multiple adjacent frames.

![Fig 1: Frequency division framework for data transmission and spectrum sensing simultaneously](image)

The SU carries on spectrum sensing in sensing subband \( W_s \) continuously and transmits its data over transmission subband \( W-W_s \). The probabilities of false alarm and detection for the system model given in equation (3), (4), (5) and (6) for AWGN channel are formulated as function of sensing bandwidth \( W_s \) and frame duration \( T \) as given below. For throughput analysis assuming \( N_a \) and \( N_r \) are noise power spectral density of AWGN and power spectral density of evenly distributed PU signal.

\[
P_d(W_s, \lambda, \gamma) = Q\left(\frac{\lambda}{\gamma^{1/2}}\left(1 + \frac{\gamma}{\gamma}\right) - \sqrt{\frac{2T W_s}{\gamma}}\right)
\]

(8)

\[
P_f(W_s, \lambda, \gamma) = Q\left(\frac{\lambda}{\gamma^{1/2}}\left(1 + \frac{\gamma}{\gamma}\right) - \sqrt{\frac{2T W_s}{\gamma}}\right)
\]

(9)

Here \( Q(x) = \frac{1}{\sqrt{2\pi}}\int_{x}^{\infty} e^{-\frac{u^2}{2}} du \) The quantity \( \gamma \) is the received PU signal-to-noise ratio (SNR) at the CR. \( f_s \) is the sampling frequency used at CR to detect the PU signal, and \( N_0 \) noise power at sensing receiver of CR.

The probabilities of detection (\( P_d \)) and false alarm (\( P_f \)) for target probabilities of false alarm (\( \bar{P}_f \)) and detection (\( \bar{P}_d \)) are given respectively as:

\[
P_d(W_s) = Q\left(\frac{1}{\gamma^{1/2}}\left(1 + \frac{\gamma}{\gamma}\right) - \sqrt{\frac{2T W_s}{\gamma}}\right)
\]

(10)

\[
P_f(W_s) = Q\left(1 + \gamma\right)Q\left(\frac{1}{\gamma^{1/2}}\left(1 + \frac{\gamma}{\gamma}\right) - \sqrt{\frac{2T W_s}{\gamma}}\right)
\]

(11)

Where \( \gamma \) is the signal to noise ratio (SNR) of the PU signal observed at the SU receiver and \( Q \) is error function given in previous section.

The average detection probability under the Nakagami fading channel can be obtained by following equation.

\[
\bar{P}_d_{\text{Naka}}(W_s) = \int_{0}^{\infty} P_d(W_s, \lambda, \gamma) \cdot f_\gamma(y, m) dy
\]

(12)

\[
\bar{P}_f_{\text{Naka}}(W_s) = \int_{0}^{\infty} P_f(W_s, \lambda, \gamma) \cdot f_\gamma(y, m) dy
\]

(13)

Once a SU decides that the PU is absent, it tries to access the PU band. Therefore the SU transmits data in two cases: Correct Detection of Spectrum Opportunity and Incorrect Detection of Spectrum Opportunity. In the case of correct detection of the spectrum, only the SU transmits its data. The achievable SU throughput is

\[
\bar{C}_o(W_s) = (W - W_s) \log_2(1 + \rho_1)
\]

(14)

Where \( \rho_1 \) is the Signal to Noise Ratio (SNR) observed by the SU receiver over its transmission band.

\[
\rho_1 = \frac{h_s |\mathcal{N}_a(W-W_s)|^2}{N_r (W-W_s)}
\]

(15)

Where, \( h_s \) is the channel gain for SU.

In the case of incorrect detection of the spectrum, the SU and PU transmit simultaneously. Thus, the SU receiver is interfered by the PU signal and the achievable throughput becomes

\[
\bar{C}_i(W_s) = (W - W_s) \log_2(1 + \rho_2)
\]

(16)

Where \( \rho_2 \) is the Signal to Noise and Interference Ratio (SINR) observed by the SU receiver over its transmission band.

\[
\rho_2 = \frac{h_s |\mathcal{N}_a(W-W_s)|^2}{N_r (W-W_s) + |\mathcal{N}_a(W-W_s)|^2}
\]

(17)

Where \( h_s \) is the block faded channel gain. The probabilities of the correct and incorrect detection of the spectrum are \( P(H_0)(1-P(W_s)) \) and \( P(H_1)(1-P(W_s)) \) respectively. The combined achievable throughput under the hypothesis of \( H_0 \) and \( H_1 \) can be obtained as

\[
\bar{C}(W_s) = P(H_0)(1 - P(W_s)) \bar{C}_o(W_s) + P(H_1)(1 - P(W_s)) \bar{C}_i(W_s)
\]

(18)
4. SIMULATION RESULT AND DISCUSSION

In this section, computer simulation results of FDSSF analyzed in previous section are presented. In Table 1 values of the parameters required in the simulation are given.

Table 1: Simulation variables and values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame duration (T)</td>
<td>100 ms</td>
</tr>
<tr>
<td>Sampling rate (µ)</td>
<td>1 KHz</td>
</tr>
<tr>
<td>Detection Probability (P_d)</td>
<td>0.9</td>
</tr>
<tr>
<td>Band width</td>
<td>5 MHz</td>
</tr>
<tr>
<td>P(H_0)</td>
<td>0.3</td>
</tr>
<tr>
<td>P(H_0)= 1- P(H_1)</td>
<td>0.7</td>
</tr>
</tbody>
</table>

To analyze fading effect on probability of false alarm of the spectrum sensing frameworks analyzed in the previous section, we first find out detection threshold for target probability of detection \( P_d = 0.9 \) and the detection threshold is used to determine the false alarm probability. Fig. 2 shows that probability of false alarm decreases rapidly and attains the minimum value as the sensing bandwidth (Ws) increases. As the value of Nakagami parameter (m) increases the false alarm probability decreases. The simulation results show that for the optimal spectrum sensing bandwidth (Ws) 0.5 MHz, the false alarm probability decreases by 49.6% and 96.6% for \( m=2 \) and \( m=5 \) respectively as compare to \( m=1 \).

To analyze probability of detection of the spectrum sensing frameworks, first we find out the detection threshold for target false alarm probability \( P_f = 0.05 \) and detection threshold is used to determine probability of detection. Fig. 3 shows that probability of detection increases rapidly and attains the maximum value as the sensing bandwidth (Ws) increases. As the value of Nakagami parameter (m) increases the false alarm probability also increases. The simulation results show that for the optimum spectrum sensing bandwidth (Ws) 0.5 MHz, the probability of detection increases by 6% and 17.7% for \( m=2 \) and \( m=5 \) respectively as compare to \( m=1 \).

As per the analysis of SU’s throughput for the FDSSF given in previous section, we assume target probability of detection \( P_d = 0.9 \) and corresponding false alarm probability is determined. The throughput of the analyzed spectrum sensing framework is calculated using the assumed and determined data in previous section. Fig. 4 shows that SU throughput increases as the sensing bandwidth (Ws) increases for constant PU SNR= -20dB and SU SNR= -4dB. The simulation results show that for the optimum spectrum sensing bandwidth (Ws) 0.5 MHz, the secondary user throughput increases by 14% and 29.1% for \( m=2 \) and \( m=5 \) respectively as compare to \( m=1 \).

5. CONCLUSION

In this paper we have analyzed and simulated Frequency Division Spectrum Sensing (FDSS) in Cognitive Radio Network (CRN) for Nakagami fading channel. The simulation results show that as the value of Nakagami parameter (m) increases probability of false alarm decreases, probability of detection increases and secondary user’s throughput also increases.

6. REFERENCES


