Inventory Model for Decaying Items with Multivariate Demand and Variable Holding Cost under the Facility of Trade-Credit

Anupam Swami
Assistant Professor
Govt. P.G. College
Sambhal, U.P.

Sarla Pareek
Professor and HOD
Banasthali University
Jaipur, RAJ.

S.R. Singh, Ph.D.
Associate Professor
CCS University
Meerut, U.P.

Ajay Singh Yadav, Ph.D.
Assistant Professor
SRM University
NCR Campus, GZB

ABSTRACT
In this study, an inventory model for deteriorating items with multivariate demand and variable holding cost is developed. The facility of allowable delay in payment is also taken into consideration. During this period retailer can use the ensued money from sales of the supplied goods to earn interest. Demand rate is a function of on hand inventory and selling price of the item and it is considered that the stock affects the demand rate up to an assured time. Therefore, retailer will order more quantity to stimulate demand rate and to earn more money. Different cases of allowable delay in payment are discussed. The objective of this study is to maximize the retailer's total profit per unit time. The results are illustrated with some numerical examples and sensitivity analysis for each case is also carried out.

Keywords
Inventory Model, Sensitivity Analysis, and consumption rate

1. INTRODUCTION
The classical inventory models consider the demand rate to be either constant or time-dependent but independent of the available stock status. However, in many practical situations customers' purchasing manners may be affected by factors such as selling price, on hand inventory and so on. As deliberated by Levin et al. (1972) and Silver and Peterson (1985), sales at the retail point tend to be proportional to inventory displayed and a large piles of goods exhibited in a supermarket will escort the purchasers to buy more. Many marketing researchers and practitioners have paying attention to investigate the modelling aspects of this occurrence. Gupta and Vrat (1986) first designed a model for consumption environment to minimize the cost with the assumption that demand rate is a function of the initial stock level. Mandal and Phaujdar (1989), Datta and Pal (1990), Padmanabhan and Vrat (1995) further developed inventory models under stock-dependent consumption rate with some different assumptions. Since a firm may use a pricing strategy to stimulate demand for its seasonal goods, the inventory problems with selling price and stock dependent demand cannot be disregarded. Urban and Baker (1997) investigated a deterministic inventory problem in with multivariate demand rate. Datta and Paul (2001) analyzed an inventory system where the consumption rate of the goods is affected by both displayed stock level and selling price. You (2005) investigated an inventory model by considering price and time dependent demand rate. Some inspiring research articles associated to this research environment are, You and Hsieh (2007), Chang et al. (2010), Lee and Dye (2012).

In today's business communications, it is more and more frequent to see that the supplier allows the retailer a fixed time (trade-credit) period. This provides an advantage to the retailer, as he/she does not has to pay the supplier immediately after receiving the items; in contrast during the delay period an interest can be earned by the retailer on the accumulated revenue. On the other hand, the strategy of allowing a permissible delay period is also beneficial for the supplier as it attracts new retailers/purchasers who consider this policy to be a type of purchasing cost reduction. Based on this phenomenon, Goyal (1985) analyzed the effect of trade credit on the optimal inventory policy. Later on, Aggarwal and Jaggi (1995) extended Goyal (1985) model with an exponential deterioration rate under the policy of allowable delay in payments. Jamal et al. (1997), Chang and Dye (2001) put forwarded inventory models with trade-credit policy by considering shortages. Teng (2002) proposed an inventory model by estimating the difference between unit selling price and unit cost and recognized an easy analytical closed-form solution to the problem. Sana and Chaudhuri (2008) analyzed optimal trade-credit policies when a price discount is offered. Khanra et al. (2011) proposed an EOQ (Economic Order Quantity) model for a deteriorating item having time dependent demand rate when delay in payment is permitted. Teng et al. (2012) deliberated an inventory model with non-decreasing demand and trade-credit financing.

The above mentioned literature reveals that inventory models for decaying items under the condition of allowable delay in payment with variable holding cost, stock and price dependent demand rate, while the on hand inventory affects the demand rate up to a assured period are not discussed so far. Therefore, in this study, an inventory model for deteriorating items with multivariate rate is developed. It is assumed that a trade credit period is offered by the supplier to the retailer. The time dependent holding cost is also taken into consideration. Three cases are discussed according to the situation of the delay period. To validate the concept of this study numerical examples are provided and sensitivity analysis for different cases is also discussed. The concavity of the profit function in each case is disclosed graphically.

2. NOTATIONS AND ASSUMPTIONS
The following notation is used throughout the paper:

$I(t)$ The inventory level at any time t, $t \geq 0$

T Cycle length (time units)

Q The replenishment/order size (units/cycle)

θ	Deterioration rate of on hand inventory during cycle time
T_1	Time up to which demand is affected by on hand stock
I_e	Interest earned (/$/cycle)
I_c	Interest charges per $ investment in inventory per cycle
M	The trade credit period length per cycle
p	Unit selling price per item ($)
$h+\delta t$	Holding cost per unit per unit time ($)
C_p	Purchasing cost per unit ($)
C_d	Purchasing cost per unit ($)
A	Ordering cost for placing an order ($/order)

3. ASSUMPTIONS

In developing the mathematical model, the following assumptions are made:

1. The demand rate, $D(t)$ is a function of stock and price and is define $D(t) = a + bI(t) - cp$, where a is positive constant, b is the stock-dependent consumption rate parameter, $0 \le b \le 1$, $c > 0$ is the selling price dependent factor and I(t) is the inventory level at time t.
2. The inventory system involves only one type of perishable item and the planning horizon is infinite.
3. The replenishment rate is infinite.
4. Holding cost per unit per unit time is $h + \delta t$ where $h > 0,\ 0 < \delta < 1$.
5. The deteriorating rate, $\theta(0 < \theta < 1)$, is constant and there is no replacement or repair of deteriorated units during the period under consideration.

4. MODEL FORMULATIONS

Here, the replenishment policy of a deteriorating item with variable demand rate is considered. The objective of the inventory system is to determine the optimal ordering quantity and the length of ordering cycle. The behavior of the inventory system at any time t is depicted in Figure 1. There may arise three cases according to the position of the delay period M as shown in Figure 1.

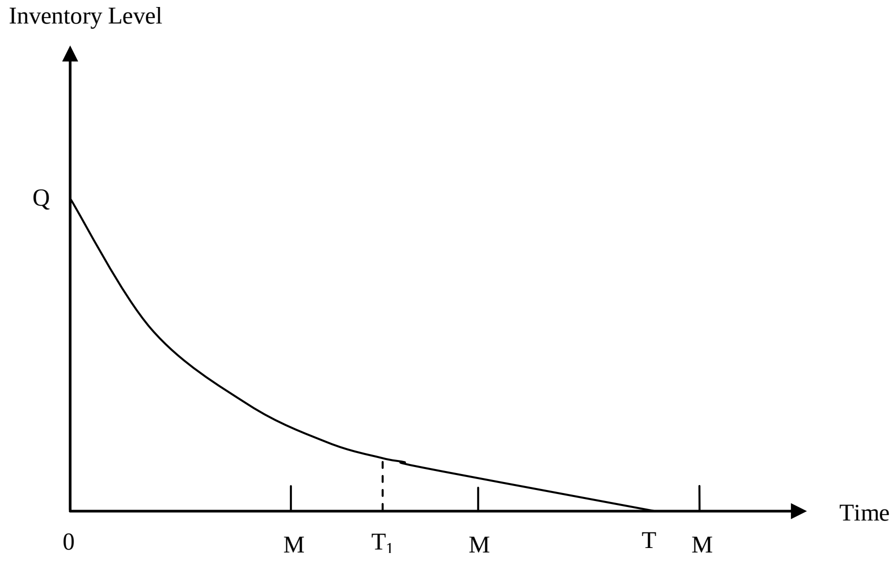

Figure 1: The graphical representation of inventory system

Replenishment is made at time $t = 0$ and the order size is Q. During the period $[0, T_1]$ the available stock and selling price of the item both affect the demand rate after that during the period $[T_1^{'}, T]$ only selling price influences the demand rate. The inventory level during the period $[0, T]$ decreases due to the combined effect of the demand rate and deterioration of the item and ultimately falls to zero at $t = T$. Hence, the differential equations representing the inventory status during the period $[0, T]$ are given by

$$I_1^{'}(t) + \theta I(t) = -\left(a + bI(t) - cp\right) \qquad 0 \le t \le T_1 \qquad (1)$$

$$I_2^{'}(t) + \theta I(t) = -\left(a - cp\right) \qquad T_1 \le t \le T \qquad (2)$$

With boundary conditions $I_1(0)=Q$, $I_2(T)=0$. Solving eq. (1) and (2) we get

$$I_1(t)=\left[\left(Q+\frac{(a-cp)}{(b+\theta)}\right)e^{-(b+\theta)t}-\frac{(a-cp)}{(b+\theta)}\right] \tag{3}$$

$$I_2(t)=\frac{(a-cp)}{\theta}\left(e^{\theta(T-t)}-1\right) \tag{4}$$

Since inventory level is continuous at $t=T_1$, so we have $I_1(T_1)=I_2(T_1)$

$$\Rightarrow Q=\left[\frac{(a-cp)}{\theta}\left(e^{\theta(T-T_1)}-1\right)-\frac{(a-cp)}{(b+\theta)}\left(e^{-(b+\theta)T_1}-1\right)\right]e^{(b+\theta)T_1} \tag{5}$$

Now, ordering cost per unit time is OC and is given by

$$OC=\frac{A}{T} \tag{6}$$

Purchasing cost per unit time is PC and is given by

$$PC=\frac{C_pQ}{T} \tag{7}$$

Inventory carrying cost per unit time is HC and is given by

$$HC=\frac{1}{T}\left[\int_0^{T_1}(h+\delta t)I_1(t)dt+\int_{T_1}^{T_2}(h+\delta t)I_2(t)dt\right]$$

$$\begin{aligned}\Rightarrow HC=\frac{1}{T}\Bigg[&\left(Q+\frac{(a-cp)}{(b+\theta)}\right)\left\{\frac{h}{(b+\theta)}+\frac{\delta}{(b+\theta)^2}-\left(\frac{(h+\delta T_1)}{(b+\theta)}+\frac{\delta}{(b+\theta)^2}\right)e^{-(b+\theta)T_1}\right\}+\frac{b(a-cp)}{\theta(b+\theta)}\left(hT_1+\frac{\delta T_1^2}{2}\right)\\&+\frac{(a-cp)}{\theta}\left\{\left(\frac{(h+\delta T_1)}{\theta}+\frac{\delta}{\theta^2}\right)e^{\theta(T-T_1)}-\left(\frac{(h+\delta T)}{\theta}+\frac{\delta}{\theta^2}\right)\right\}-\frac{(a-cp)}{\theta}\left(hT+\frac{\delta T^2}{2}\right)\Bigg]\end{aligned} \tag{8}$$

Deterioration cost per unit time is DC and is given by

$$DC=\frac{C_d}{T}\left[\int_0^{T_1}\theta I_1(t)dt+\int_{T_1}^{T_2}\theta I_2(t)dt\right]$$

$$\Rightarrow DC=\frac{C_d}{T}\left[\frac{\left(1-e^{-(b+\theta)T_1}\right)}{(b+\theta)}\left(Q+\frac{(a-cp)}{(b+\theta)}\right)-\frac{(a-cp)T_1}{(b+\theta)}+\frac{(a-cp)}{\theta}\left\{\frac{1}{\theta}\left(e^{\theta(T-T_1)}-1\right)-(T-T_1)\right\}\right] \tag{9}$$

Sales revenue per unit time is SR and is given by

$$SR=p\left[\int_0^{T_1}(a+bI_1(t)-cp)\,dt+\int_{T_1}^{T}(a-cp)\,dt\right]$$

$$\Rightarrow SR=p\left[(a-cp)T+b\left\{\left(Q+\frac{(a-cp)}{(b+\theta)}\right)\frac{\left(1-e^{-(b+\theta)T_1}\right)}{(b+\theta)}-\frac{(a-cp)T_1}{(b+\theta)}\right\}\right] \tag{10}$$

Now, according to the position of delay period M there arise three cases:

Case (1) $M \le T_1 < T$ Case (2) $T_1 \le M < T$ Case (3) $M \ge T$

We shall discuss these three cases one by one.

Case (1): $M \le T_1 < T$

During the credit period the buyer can use the sale revenue to earn an interest and can collect more capital. So, interest earned per unit time is IE_1 and is given by

$$IE_1=\frac{pI_e}{T}\left[\int_0^{M}(a+bI_1(t)-cp)\,tdt\right]$$

$$\Rightarrow IE_1 = \frac{pI_e}{T}\left[\frac{\theta(a-cp)M^2}{2(b+\theta)} + b\left(Q + \frac{(a-cp)}{(b+\theta)}\right)\left\{\frac{1}{(b+\theta)^2} - \left(\frac{M}{(b+\theta)} + \frac{1}{(b+\theta)^2}\right)e^{-(b+\theta)M}\right\}\right] \quad (11)$$

After, the credit period the buyer/retailer has to pay the interest for the goods still in stock with annual rate I_c .

Therefore, interest charged per unit time is IC_1 and is given by

$$IC_1 = \frac{C_p I_c}{T}\left[\int_M^{T_1} I_1(t)dt + \int_{T_1}^{T} I_2(t)dt\right]$$

$$\Rightarrow IC_1 = \frac{C_p I_c}{T}\left[\left(Q + \frac{(a-cp)}{(b+\theta)}\right)\frac{\left(e^{-(b+\theta)M} - e^{-(b+\theta)T_1}\right)}{(b+\theta)} - \frac{(a-cp)}{(b+\theta)}(T_1 - M) + \frac{(a-cp)}{\theta}\left\{\frac{\left(e^{\theta(T-T_1)} - 1\right)}{\theta} - (T - T_1)\right\}\right]. \quad (12)$$

The total profit per unit time for this case is AP_1 and is given by

$$AP_1(T, p) = \left[(SR + IE_1) - (OC + PC + HC + DC + IC_1)\right] \quad (13)$$

The objective of this study is to maximize the total profit per unit time $AP_1(T,p)$. The necessary conditions for maximizing the profit are

$$\frac{\partial AP_1(T, p)}{\partial T} = 0 \quad (14)$$

$$\frac{\partial AP_1(T, p)}{\partial p} = 0 \quad (15)$$

From equation (14) and (15) with the help of software Mathematica-8.0, we can determine the optimum values of T^* and p^* simultaneously and optimal order size (Q^*) can be observed from (5). Also, the optimal value $AP_1(T^*, p^*)$ of the average profit can be determined by (13) provided they satisfy the sufficiency conditions that are given as follows:

$$\frac{\partial^2 AP_1(T, p)}{\partial T^2} < 0,\ \frac{\partial^2 AP_1(T, p)}{\partial p^2} < 0 \, and \, \frac{\partial^2 AP_1(T, p)}{\partial T^2}\frac{\partial^2 AP_1(T, p)}{\partial p^2} - \left(\frac{\partial^2 AP_1(T, p)}{\partial T \partial p}\right)^2 > 0 \quad (16)$$

Case (2): $T_1 \le M \le T$

In this case, interest earned per unit time is IE_2 and is given by

$$IE_2 = \frac{pI_e}{T}\left[\int_0^{T_1}(a + bI_1(t) - cp)tdt + (M - T_1)\int_0^{T_1}(a + bI_1(t) - cp)dt + \int_{T_1}^{M}(a - cp)tdt\right]$$

$$\Rightarrow IE_2 = \frac{pI_e}{T}\left[\frac{\theta(a-cp)T_1^2}{2(b+\theta)} - b\left(Q + \frac{(a-cp)}{(b+\theta)}\right)\left\{\frac{T_1 e^{-(b+\theta)T_1}}{(b+\theta)} + \frac{\left(e^{-(b+\theta)T_1} - 1\right)}{(b+\theta)^2}\right\} + \frac{(a-cp)(M^2 - T_1^2)}{2} + (M - T_1)\left\{\frac{\theta(a-cp)T_1}{(b+\theta)} - \frac{b}{(b+\theta)}\left(Q + \frac{(a-cp)}{(b+\theta)}\right)\left(e^{-(b+\theta)T_1} - 1\right)\right\}\right] \quad .(17)$$

For this case, interest charged per unit time is IC_2 and is given by

$$IC_2 = \frac{C_p I_c}{T}\left[\int_M^T I_2(t)dt\right] = \frac{(a-cp)C_p I_c}{\theta T}\left[\frac{\left(e^{\theta(T-M)} - 1\right)}{\theta} - (T - M)\right] \quad (18)$$

The total profit per unit time for this case is AP_2 and is given by

$$AP_2(T, p) = \left[(SR + IE_2) - (OC + PC + HC + DC + IC_2)\right] \quad (19)$$

Again, the objective of this study is to determine the optimal values of T and p, when $T_1 < M \le T$, in order to maximize the total profit per unit time $AP_2(T, p)$. The necessary conditions for maximizing the profit are

$$\frac{\partial AP_2(T, p)}{\partial T} = 0 \quad (20)$$

$$\frac{\partial AP_2(T,p)}{\partial p}=0 \qquad (21)$$

From equation (20) and (21) with the help of software Mathematica-8.0, we can determine the optimum values of T^* and p^* simultaneously and optimal order size (Q^*) can be observed from (5). Also, the optimal value $AP_2(T^*, p^*)$ of the average profit can be determined by (19) provided they satisfy the sufficiency conditions that are given as follows

$$\frac{\partial^2 AP_2(T,p)}{\partial T^2}<0,\ \frac{\partial^2 AP_2(T,p)}{\partial p^2}<0\ and\ \frac{\partial^2 AP_2(T,p)}{\partial T^2}\frac{\partial^2 AP_2(T,p)}{\partial p^2}-\left(\frac{\partial^2 AP_2(T,p)}{\partial T\partial p}\right)^2>0 \qquad (22)$$

Case (3): T < M

In this case, interest earned per unit time is IE_3 and is given by

$$IE_3=\frac{pI_e}{T}\left[\int_0^{T_1}(a+bI_1(t)-cp)t\,dt+(M-T_1)\int_0^{T_1}(a+bI_1(t)-cp)\,dt+\int_{T_1}^{T}(a-cp)t\,dt+(M-T)\int_{T_1}^{T}(a-cp)\,dt\right]$$

$$IE_3=\frac{pI_e}{T}\left[\frac{\theta(a-cp)T_1^2}{2(b+\theta)}-b\left(Q+\frac{(a-cp)}{(b+\theta)}\right)\left\{\frac{T_1e^{-(b+\theta)T_1}}{(b+\theta)}+\frac{\left(e^{-(b+\theta)T_1}-1\right)}{(b+\theta)^2}\right\}+\frac{(a-cp)(T^2-T_1^2)}{2}\right.$$
$$\left.+(M-T_1)\left\{\frac{\theta(a-cp)T_1}{(b+\theta)}-\frac{b}{(b+\theta)}\left(Q+\frac{(a-cp)}{(b+\theta)}\right)\left(e^{-(b+\theta)T_1}-1\right)\right\}+(M-T)(a-cp)(T-T_1)\right] \qquad .(23)$$

Interest charged per unit time is IC_3 and is given by

$$IC_3=0 \qquad (24)$$

The total profit per unit time for this case is AP_3 and is given by

$$AP_3(T,p)=\left[(SR+IE_3)-(OC+PC+HC+DC+IC_3)\right] \qquad (25)$$

The purpose of this study is to determine the optimal values of T and p, when M > T, in order to maximize the total profit per unit time $AP_3(T, p)$. The necessary conditions for maximizing the profit are

$$\frac{\partial AP_3(T,p)}{\partial T}=0 \qquad (26)$$

$$\frac{\partial AP_3(T,p)}{\partial p}=0 \qquad (27)$$

From equation (26) and (27) with the help of software Mathematica-8.0, we can determine the optimum values of T^* and p^* simultaneously and optimal order size (Q^*) can be observed from (5).

Also, the optimal value $AP_3(T^*, p^*)$ of the average profit can be determined by (25) provided they satisfy the sufficiency conditions that are given as follows:

$$\frac{\partial^2 AP_3(T,p)}{\partial T^2}<0,\ \frac{\partial^2 AP_3(T,p)}{\partial p^2}<0\ and\ \frac{\partial^2 AP_3(T,p)}{\partial T^2}\frac{\partial^2 AP_3(T,p)}{\partial p^2}-\left(\frac{\partial^2 AP_3(T,p)}{\partial T\partial p}\right)^2>0 \qquad (28)$$

5. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

Example (1): (For case (1) $M \le T_1 < T$) To illustrate the model we consider the following data on the basis of previous study:

a=200, b=0.5, c=1.8, θ=0.3, I_e=0.15, I_c=0.17, M=0.25, h=6, δ=0.1, C_d=3, A=130, C_p=40, T_1=0.32.

Then, the optimal solution is T^*=0.479925, p^*=77.7625, Q^*=34.5945

and $AP_1(T^*, p^*)$=1995.04.

The concavity of the profit function for case (1) is shown graphically in Figure 2.

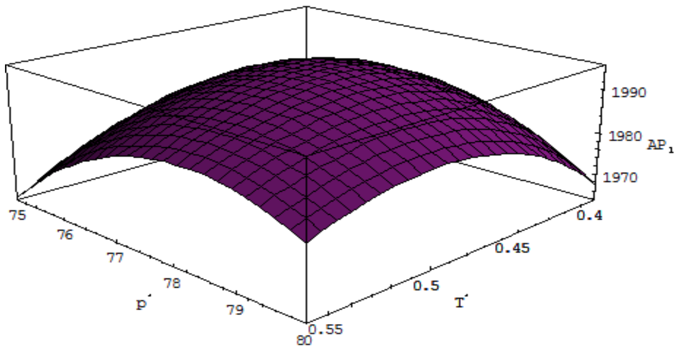

Figure 2. The concavity of the profit function (for case (1)) w.r.t. selling price and cycle length

Table 1: The sensitivity analysis for case (1) is performed by changing the value of each of the parameters by $\pm 10\%$ and $\pm 20\%$, taking one parameter at a time and keeping the remaining parameters unchanged

Parameter	% Change	T^*	p^*	Q^*	$AP_1(T^*, p^*)$
A	-20	0.537022	66.9978	25.7843	740.169
	-10	0.500892	72.3341	30.1190	1303.56
	+10	0.468168	83.2468	39.3156	2813.14
	+20	0.462501	88.7680	49.3158	3706.31
B	-20	0.459121	77.6623	32.2959	1940.28
	-10	0.469450	77.7127	33.4238	1967.29
	+10	0.490551	77.8117	35.8098	2023.55
	+20	0.501336	77.8602	37.0721	2052.82
C	-20	0.492831	91.7296	40.3270	3366.50
	-10	0.483872	83.9593	37.2182	2593.31
	+10	0.480230	72.7139	32.3118	1523.84
	+20	0.484433	68.5306	30.2708	1147.95
Θ	-20	0.507888	77.5893	36.4813	2037.60
	-10	0.493404	77.6772	35.5070	2016.08
	+10	0.467339	77.8454	33.7373	1974.44
	+20	0.455552	77.9260	32.9301	1954.27
I_e	-20	0.485451	77.8408	34.9621	1984.38
	-10	0.482697	77.8018	34.7790	1989.70
	+10	0.477135	77.7230	34.4083	2000.41
	+20	0.474326	77.6833	34.2204	2005.81
I_c	-20	0.489664	77.7852	35.3638	1999.83
	-10	0.484687	77.7738	34.9703	1997.40
	+10	0.475363	77.7514	34.2350	1992.75
	+20	0.470989	77.7405	33.8908	1990.54
H	-20	0.492194	77.6913	35.6704	2014.62
	-10	0.485943	77.7271	35.1217	2004.76
	+10	0.474124	77.7975	34.0871	1985.45
	+20	0.468528	77.8322	33.5985	1976.00
C_d	-20	0.481707	77.7519	34.7505	1997.94

	-10	0.480813	77.7572	34.6722	1996.49
	+10	0.479041	77.7678	34.5173	1993.59
	+20	0.478162	77.7731	34.4401	1992.15
A	-20	0.443718	77.5431	31.8732	2051.34
	-10	0.462178	77.6550	33.2614	2022.64
	+10	0.497035	77.8663	35.8780	1968.43
	+20	0.513573	77.9666	37.1171	1942.70
C_p	-20	0.511087	73.6465	41.7256	2615.13
	-10	0.493806	75.7018	37.9337	2295.32
	+10	0.469056	79.8307	31.6210	1713.90
	+20	0.460971	81.9087	28.9470	1451.60

From sensitivity Table 1 the following inferences are drawn:

1. From Table 1 it is clear that the total profit per unit time $AP_1(T^*,p^*)$ increases or decreases with increase or decrease in the values of model parameters a, b and I_e, while $AP_1(T^*,p^*)$ decreases or increases with increase or decrease in the values of c, θ, I_c, h, C_d, A and C_p . The obtained results show that $AP_1(T^*,p^*)$ is highly sensitive to changes in a, c and C_p. It is moderately sensitive to changes in b, I_e, θ, I_c, h, C_d and A.

2. From Table 1 it is clear that Q^* increases or decreases with increase or decrease in the values of model parameters a, b and A, while Q^* decreases or increases with increase or decrease in the values of model parameters I_e, c, θ, I_c, h, C_d and C_p. The obtained results show that Q^* is highly sensitive to changes in a and C_p. It is fairly sensitive to changes in b, c, θ, I_e, I_c, h and C_d.

Example (2): (For case (2) $T_1 < M \le T$) We consider the following data in appropriate units: a=200, b=0.5, c=1.8, θ=0.3, I_e=0.15, I_c=0.17, M=0.4, h=6, δ=0.1, C_d=3, A=130, C_p=40, T_1=0.32. Then, the optimal solution is T^*=0.437287, p^*=76.8656, Q^*=31.9856 and $AP_2(T^*, P)$= 2137.46. The concavity of the profit function for case (2) is shown graphically in Figure 3.

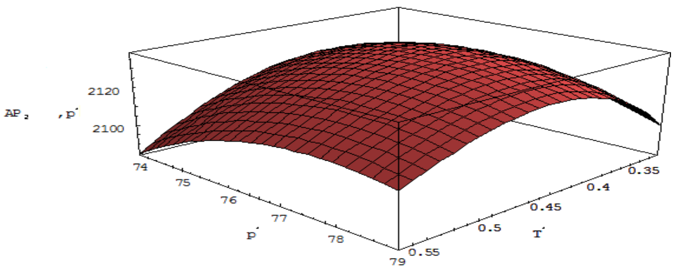

Figure 3. The concavity of the profit function (for case (2)) w.r.t. selling price and cycle length

Table 2: The sensitivity analysis for case (2) is performed by changing the value of each of the parameters by $\pm 10\%$ and $\pm 20\%$, taking one parameter at a time and keeping the remaining parameters unchanged.

Parameter	% Change	T^*	p^*	Q^*	$AP_2(T^*, p^*)$
A	-20	0.505166	66.1738	24.9677	819.376
	-10	0.46407	71.4760	28.4890	1412.65
	+10	0.419408	82.3099	35.5644	2992.16
	+20	0.407566	87.7915	39.2910	3975.75
B	-20	0.4137	76.7314	29.5532	2084.12
	-10	0.425441	76.7991	30.7480	2110.36
	+10	0.449248	76.9309	33.4628	2165.35
	+20	0.461332	76.9951	34.5976	2194.30

C	-20	0.439337	90.7672	36.1470	3550.83
	-10	0.436382	83.0328	33.9014	2755.15
	+10	0.441502	71.8413	30.2942	1649.25
	+20	0.448791	67.6777	28.7547	1258.28
Θ	-20	0.463019	76.7210	33.7641	2176.99
	-10	0.449706	76.7946	32.8466	2157.00
	+10	0.425666	76.9341	31.1754	2118.34
	+20	0.414758	77.0003	30.4108	2099.63
I_e	-20	0.457965	77.1366	33.4297	2100.62
	-10	0.447758	77.0029	32.7189	2118.84
	+10	0.426532	76.7242	31.2284	2156.50
	+20	0.415468	76.5783	30.4452	2176.00
I_c	-20	0.439318	76.8772	32.1423	2137.60
	-10	0.438276	76.8712	32.0619	2137.53
	+10	0.436349	76.8602	31.9133	2137.40
	+20	0.435456	76.8551	31.8444	2137.33
H	-20	0.448864	76.8075	32.9980	2155.71
	-10	0.442965	76.8367	32.4842	2146.52
	+10	0.431817	76.8941	31.5086	2128.53
	+20	0.426542	76.9222	31.0494	2119.72
C_d	-20	0.438968	76.8570	32.1324	2140.17
	-10	0.438125	76.8613	32.0588	2138.81
	+10	0.436454	76.8699	31.9129	2136.11
	+20	0.435625	76.8742	31.8406	2134.77
A	-20	0.397256	76.6158	28.9131	2199.77
	-10	0.417757	76.7440	30.4874	2167.87
	+10	0.455973	76.9814	33.4177	2108.35
	+20	0.473915	77.0923	34.7914	2080.39
C_p	-20	0.463193	72.8865	38.0963	2759.07
	-10	0.448716	74.8732	36.7689	2430.82
	+10	0.428556	78.8655	29.4405	1853.74
	+20	0.422311	80.8751	27.1526	1587.74

From sensitivity Table 2 the following inferences are drawn:

1. From Table 2 it can be shown that the total profit per unit time $AP_2(T^*,p^*)$ increases or decreases with increase or decrease in the values of model parameters a, b and I_e, while $AP_2(T^*,p^*)$ decreases or increases with increase or decrease in the values of c, θ, I_c, h, C_d, A and C_p . The obtained results show that $AP_2(T^*,p^*)$ is highly sensitive to changes in a, c and C_p. It is moderately sensitive to changes in b, I_e, θ, I_c, h, C_d and A.

2. From Table 2 it is clear that Q^* increases or decreases with increase or decrease in the values of model parameter a, b and A, while Q^* decreases or increases with increase or decrease in the values of model parameters I_e, c, θ, I_c, h, C_d and C_p. The obtained outcomes show that Q^* is highly sensitive to changes in a and C_p. It is fairly sensitive to changes in b, c, θ, I_e, I_c, h and C_d.

Example (3): (For case case (3) M > T) We consider the following data in appropriate units: a=200, b=0.5, c=1.8, θ=0.3, I_e=0.15, I_c=0.17, M=0.6, h=6, δ=0.1, C_d=3, A=130, C_p=40, T_1=0.32. Then the optimal solution is T^*=0.533367, P^*=76.6228, Q^*=40.3092 and $AP_3(T^*, p^*)$= 2335.76. The concavity of the profit function for case (3) is shown graphically in Figure 4.

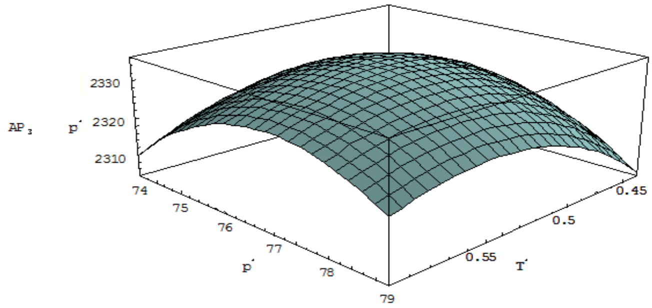

Figure 4. The concavity of the profit function (for case (3)) w.r.t. selling price and cycle length

Table 3: The sensitivity analysis for case (3) is performed by changing the value of each of the parameters by $\pm 10\%$ and $\pm 20\%$, taking one parameter at a time and keeping the remaining parameters unchanged.

Parameter	% Change	T^*	p^*	Q^*	$AP_3(T^*, p^*)$
A	-20	0.572862	65.699	29.3877	943.221
	-10	0.547164	71.1321	34.7295	1571.94
	+10	0.526602	82.1469	46.1678	3233.83
	+20	0.524254	87.6915	52.3155	4265.78
B	-20	0.515395	76.5623	37.9615	2271.33
	-10	0.524301	76.5925	39.1135	2303.17
	+10	0.542601	76.6531	41.5507	2369.13
	+20	0.552008	76.6834	42.8402	2403.27
C	-20	0.545945	90.5707	46.3861	3816.94
	-10	0.537996	82.8173	43.1686	2983.27
	+10	0.531747	71.5647	37.7266	1823.53
	+20	0.532992	67.3618	35.3589	1412.70
Θ	-20	0.559388	76.3880	42.0822	2384.37
	-10	0.546004	76.5070	41.1737	2359.86
	+10	0.521414	76.7358	39.4850	2312.07
	+20	0.510088	76.8461	38.6983	2288.76
I_e	-20	0.538529	76.9201	40.3995	2262.61
	-10	0.535847	76.7701	40.3482	2299.13

	+10	0.531069	76.4781	40.2811	2372.49
	+20	0.528932	76.3358	40.2625	2409.32
H	-20	0.545048	76.5255	41.4261	2358.32
	-10	0.539109	76.5745	40.8576	2346.97
	+10	0.527814	76.6706	39.7796	2324.70
	+20	0.522438	76.7179	39.2678	2313.76
C_d	-20	0.535070	76.6084	40.4717	2339.11
	-10	0.534216	76.6156	40.3902	2337.44
	+10	0.532523	76.6300	40.2286	2334.10
	+20	0.531682	76.6372	40.1484	2332.43
A	-20	0.507122	76.5016	38.2080	2385.74
	-10	0.520409	76.5626	39.2719	2360.44
	+10	0.546022	76.6822	41.3218	2311.68
	+20	0.558394	76.7408	42.3115	2288.14
C_p	-20	0.555727	72.6724	47.0640	2976.68
	-10	0.54362	74.6468	43.5469	2647.11
	+10	0.52486	78.6015	37.3118	2042.45
	+20	0.51804	80.5841	34.5221	1767.00

From sensitivity Table 3 the following inferences are drawn:

1. From Table 3 it is clear that the total profit per unit time $AP_3(T^*,p^*)$ increases or decreases with increase or decrease in the values of model parameters a, b and I_e, even as $AP_3(T^*,p^*)$ decreases or increases with increase or decrease in the values of c, θ, h, C_d, A and C_p . The attained results show that $AP_3(T^*,p^*)$ is highly sensitive to changes in a, c and C_p. It is moderately sensitive to changes in b, I_e, θ, h, C_d and A.

2. From Table 3 it is clear that Q^* increases or decreases with increase or decrease in the values of model parameters a, b and A, whereas Q^* decreases or increases with increase or decrease in the values of model parameters I_e, c, θ, h, C_d and C_p. The obtained results show that Q^* is highly sensitive to changes in a and C_p. It is fairly sensitive to changes in b, c, θ, I_e, h and C_d.

6. CONCLUSION

In this study, an inventory model is developed with the facility of allowable delay in payment. The consumption rate is considered as a function of on hand inventory and selling price of the products. During the development of the model it is assumed that the on hand inventory affects the demand rate only up to a certain time and after that only selling price affects the demand rate. According this theme three cases arise; all the cases are discussed and illustrated with the help of some numerical examples. Sensitivity analyses of system parameters clarify that a, c and C_p should be considered carefully.

There are several hopeful areas for further research. This model can be extended by considering probabilistic or inflation induced market demand rate. Another area for further research is that quantity discounted cash flow can be assumed.

7. REFERENCES

[1] Levin, R.I., McLaughlin, C.P., Lamone, R.P., and Kottas, J.F., (1972). Production/Operations Management: Contemporary Policy for Managing Operating Systems. McGraw-Hill, New York.

[2] Silver, E.A., and Peterson, R., (1985). Decision Systems for Inventory Management and Production Planning. Second ed. John Wiley, New York.

[3] Goyal, S.K., (1985). Economic order quantity under conditions of permissible delay in payments. Journal of the Operational Research Society, 36(4), 335–338.

[4] Gupta, R., and Vrat, P., (1986). Inventory model for stock-dependent consumption rate. Opsearch, 23, 19–24.

[5] Mandal, B.N., and Phaujdar, S., (1989). An inventory model for deteriorating with stock dependent consumption rate. Opsearch, 26, 43–46.

[6] Datta, T.K., and Pal, A.K., (1990). Deterministic inventory systems for deteriorating items with inventory-level-dependent demand rate and shortages. Opsearch, 27, 167–176.

[7] Padmanabhan, G., and Vrat, P., (1995). EOQ models for perishable items under stock dependent selling rate. European Journal of Operational Research, 86, 281–292.

[8] Aggarwal, S.P., and Jaggi, C.K., (1995). Ordering policies of deteriorating items under permissible delay in payments. Journal of the Operational Research Society, 46(5), 658–662.

[9] Jamal, A.M.M., Sarker, B.R., and Wang, S., (1997). An ordering policy for deteriorating items with allowable shortages and permissible delay in payment. Journal of the Operational Research Society, 48, 826–833.

[10] Urban, T.L., and Baker, R.C., (1997). Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns. European Journal of Operational Research, 103, 573–583.

[11] Datta, T.K., and Paul, K., (2001). An inventory system with stock-dependent, price-sensitive demand rate. Production Planning & Control, 12, 13–20.

[12] Chang, H.J., and Dye, C.Y., (2001). An inventory model for deteriorating items with partial backlogging and permissible delay in payments. International Journal of Systems Science, 32, 345–352.

[13] Teng, J.T., (2002). On the economic order quantity under conditions of permissible delay in payments. Journal of the Operational Research Society, 53, 915–918.

[14] You, P.S., (2005). Inventory policy for products with price and time dependent demand. Journal of the Operational Research Society, 56, 870–873.

[15] You, P.S., and Hsieh, Y.C., (2007). An EOQ model with stock and price sensitive demand. Mathematical and Computer Modelling, 45, 933–942.

[16] Sana, S.S., and Chaudhuri, K.S., (2008). A deterministic EOQ model with delays in payments and price-discount offers. European Journal of Operational Research, 184, 509–533.

[17] Chang, C.T., Chen, Y.J., Tsai, T.R. and Wu S.J., (2010). Inventory models with stock- and price dependent demand for deteriorating items based on limited shelf space. Yugoslav Journal of Operations Research, 20, 55–69.

[18] Khanra, S., Ghosh, S.K. and Chaudhuri, K.S., (2011). An EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment. Applied Mathematics and Computation, 218, 1, 1-9.

[19] Teng, J. T., Min, J., and Pan, Q., (2012). Economic order quantity model with trade credit financing for non-decreasing demand. Omega, 40(3), 328-335.

[20] Lee, Y. P., and Dye, C. Y., (2012). An inventory model for deteriorating it ems under stock-dependent demand and controllable deterioration rate. Computers & Industrial Engineering, 63(2) 474-482.