Exact Traveling Wave Solutions for Modified Liouville Equation Arising in Mathematical Physics and Biology

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ABSTRACT

In this paper, we employ extended tanh-function method and the $(\frac{G'}{G})$-expansion method to find the exact traveling wave solutions involving parameters of nonlinear evolution equation Modified Liouville equation and comparison between this two method and another method which have been solved it. When these parameters are taken to be special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the proposed methods provides a more powerful mathematical tool for constructing exact traveling wave solutions for many other nonlinear evolution equations.

Keywords:

Extended tanh-function method; The $\left(\frac{G'}{G}\right)$-expansion method; Modified Liouville equation; Traveling wave solutions; Solitary wave solutions.

1. INTRODUCTION

The investigation of exact traveling wave solutions of nonlinear partial differential equations (NLPDEs) plays an important role in the analysis of complex physical phenomena. The NLPDEs appear in physical sciences, various scientific and engineering problems, such as, fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, chemistry and many others. In recent years, to obtain exact traveling wave solutions of NLPDEs, many effective and powerful methods have been presented in the literature, such as the Backlund transformation [1], The modified simple equation method [2], the $exp\,(\phi(\xi))$-expansion Method [3], Extended Jacobian Elliptic Function Expansion Method [4], the Adomian decomposition method [5], [6], the homotopy perturbation method [7], the F-expansion method [8], the Hirota's bilinear method [9], the exp-function method [10], the Cole-Hopf transformation [11], the general projective Riccati equation method [12] and others [13]-[20] and extended tanh - method [21]-[23], the extended tanh method, developed by Wazwaz [24]-[25] and $(\frac{G'}{G})$- expansion method [26]-[28] and so on.

Nowaday, searching analytical solutions of the NLPDEs has become more lucrative partly due to the accessibility computer symbolic systems, like Maple, Mathematica, and Matlab, which help us to calculate the complicated and wearisome algebraic calculations on computer.

The objective of this article is to apply the extended tanh-function method and the $\left(\frac{G'}{G}\right)$-expansion method for finding the exact traveling wave solution of Modified Liouville equation which play an important role in biology and mathematical physics.

The rest of this paper is organized as follows: In Section 2, we give the description of the extended tanh-function method and the $\left(\frac{G'}{G}\right)$-expansion method. In Section 3, we use this methods to find the exact solutions of the nonlinear evolution equations and some figures of it and make the comparison between two method. In Section 4, conclusions are given.

2. DESCRIPTION OF METHOD

2.1 The extended tanh-function method

Consider the following nonlinear evolution equation

$$F(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0, \tag{1}$$

where F is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following,we give the main steps of this method [21]-[25]:

Step 1. We use the wave transformation

$$u(x,t) = u(\xi), \qquad \xi = x - ct, \tag{2}$$

where c is a constant, to reduce Eq. (1)to the following ODE:

$$P(u, u', u'', u''', ...) = 0, \tag{3}$$

where P is a polynomial in $u(\xi)$ and its total derivatives,while $\left\{' = \frac{d}{d\xi}'\right\}$.

Step 2. Suppose that the solution of Eq. (3) has the form:

$$u(\xi) = a_0 + \sum_{i=1}^{m} (a_i\, \phi^i + b_i\, \phi^{-i}), \tag{4}$$

where a_i, b_i are constants to be determined, such that $a_m \neq 0$ or $b_m \neq 0$ and ϕ satisfies the Riccati equation

$$\phi' = b + \phi^2, \tag{5}$$

where b is a constant. Eq. (5) admits several types of solutions according to :
Case 1. If $b < 0$, then

$$\phi = -\sqrt{-b}\, tanh(\sqrt{-b}\,\xi), \quad or \;\; \phi = -\sqrt{-b}\, coth(\sqrt{-b}\,\xi). \quad (6)$$

Case 2. If $b > 0$, then

$$\phi = \sqrt{b}\, tan(\sqrt{b}\,\xi), \quad or \quad \phi = -\sqrt{b}\, cot(\sqrt{b}\,\xi). \quad (7)$$

Case 3. If $b = 0$, then

$$\phi = -\frac{1}{\xi}. \quad (8)$$

Step 3. Determine the positive integer m in Eq. (4) by balancing the highest order derivatives and the nonlinear terms.
Step 4. Substitute Eq. (4) along Eq. (5) into Eq. (3) and collecting all the terms of the same power ϕ^i, $i = 0, \pm1, \pm2, \pm3, ...$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_i and b_i.
Step 5. substituting these values and the solutions of Eq. (5) into Eq. (4) we obtain the exact solutions of Eq. (1).

2.2 The $\left(\frac{G'}{G}\right)$-expansion method

In $\left(\frac{G'}{G}\right)$-expansion method, the solution $u(\xi)$ of Eq. (1) can be expressed as following:

$$u(\xi) = a_0 + \sum_{j=1}^{M} a_j \left(\frac{G'}{G}\right)^j, \quad a_m \neq 0, \quad (9)$$

where a_0 and a_j, for $(j = 1, 2, 3, ..., M)$, are constants to be determined later, $G(\xi)$ satisfies a second order linear ordinary differential equation (LODE):

$$G'' + \lambda G' + \mu G = 0, \quad (10)$$

where λ and μ are arbitrary constants.The positive integer M can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. (3).
Step 1. Substitute Eq. (9) along Eq. (10) into Eq. (3) and collecting all the terms of the same power $\left(\frac{G'}{G}\right)^j$, $j = 0, \pm1, \pm2, \pm3, ...$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_j, since $a_m \neq 0$.
The solution of Eq. (10) depending on whether $\lambda^2 - 4\mu > 0$, $\lambda^2 - 4\mu < 0$, $\lambda^2 - 4\mu = 0$ are given as
Case 1. When $\lambda^2 - 4\mu > 0$

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\lambda^2 - 4\mu}}{2}(M_1) - \frac{\lambda}{2}. \quad (11)$$

Since,

$$M_1 = \left(\frac{A_1 sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + A_2 cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}{A_1 cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi + A_2 sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}\right)$$

Case 2. When $\lambda^2 - 4\mu < 0$

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{4\mu - \lambda^2}}{2}(M_2) - \frac{\lambda}{2}. \quad (12)$$

Since,

$$M_2 = \left(\frac{-A_1 sin(\frac{1}{2}\sqrt{4\mu - \lambda^2})\xi + A_2 cos(\frac{1}{2}\sqrt{4\mu - \lambda^2})\xi}{A_1 cos(\frac{1}{2}\sqrt{4\mu - \lambda^2})\xi + A_2 sin(\frac{1}{2}\sqrt{4\mu - \lambda^2})\xi}\right)$$

Case 3. When $\lambda^2 - 4\mu = 0$

$$\left(\frac{G'}{G}\right) = \frac{A_2}{A_1 + A_2\xi} - \frac{\lambda}{2}. \quad (13)$$

Step 2. Substituting these values and the solutions of Eq. (10) into Eq. (9) we obtain the exact solutions of Eq. (1).

3. APPLICATION

Here, we will apply the extended tanh-function method and the $\left(\frac{G'}{G}\right)$-expansion method described in Sec.2 to find the exact traveling wave solutions and then the solitary wave solutions for the following nonlinear evolution equations.

3.1 Example 1: Modified Liouville equation using the extended tanh-function method.

Now, let us consider the Modified Liouville equation [29].

$$a^2 u_{xx} - u_{tt} + be^{\beta u} = 0, \quad (14)$$

respectively, where a, β and b are non zero and arbitrary coefficients. Using the wave transformation $u(x,t) = u(\xi), \xi = kx + \omega t$, $v = e^{\beta u}$, to reduce Eq. (14) to be in the form:

$$(\frac{k^2a^2}{\beta} - \frac{\omega^2}{\beta})v''v - (\frac{k^2a^2}{\beta} - \frac{\omega^2}{\beta})v'^2 + bv^3 = 0. \quad (15)$$

Balancing $v''v$ and v^3 in Eq. (15) yields, $N + 2 + N = 3N \Longrightarrow N = 2$. Consequently, we have the formal solution:

$$v = a_0 + a_1\Phi + a_2\Phi^2 + \frac{b_1}{\Phi} + \frac{b_2}{\Phi^2}, \quad (16)$$

$$v' = a_1 b + a_1\Phi^2 + 2\,a_2\Phi\, b + 2\,a_2\Phi^3 - \frac{b_1 b}{\Phi^2} - b_1 - 2\,\frac{b_2 b}{\Phi^3} - 2\,\frac{b_2}{\Phi}, \quad (17)$$

$$\begin{aligned} v'' = {} & 2\,a_1\Phi\, b + 2\,a_1\Phi^3 + 2\,a_2 b^2 + 8\,a_2 b\Phi^2 + 6\,a_2\Phi^4 \\ & + 2\,\frac{b_1 b^2}{\Phi^3} + 2\,\frac{b_1 b}{\Phi} + 6\,\frac{b_2 b^2}{\Phi^4} + 8\,\frac{b_2 b}{\Phi^2} + 2\,b_2. \end{aligned} \quad (18)$$

Substituting Eq. (16) and its derivatives in Eq. (15) and equating the coefficient of different power's of ϕ^i where $i = 0,\ \pm1,\ \pm2,\ \pm3, ...$ to zero, we get

$$2\left(\frac{k^2a^2}{\beta} - \frac{\omega^2}{\beta}\right){a_2}^2 + b{a_2}^3 = 0, \quad (19)$$

$$4\left(\frac{k^2a^2}{\beta} - \frac{\omega^2}{\beta}\right)a_1a_2 + 3\,b{a_1}{a_2}^2 = 0, \quad (20)$$

$$\begin{aligned} & 8\left(\tfrac{k^2a^2}{\beta} - \tfrac{\omega^2}{\beta}\right){a_2}^2 b - \left(\tfrac{k^2a^2}{\beta} - \tfrac{\omega^2}{\beta}\right)(8\,{a_2}^2 b + {a_1}^2) \\ & + b\left(3\,a_0{a_2}^2 + 3\,{a_1}^2 a_2\right) = 0, \end{aligned} \quad (21)$$

$$\begin{aligned} & \left(\tfrac{k^2a^2}{\beta} - \tfrac{\omega^2}{\beta}\right)(10\,a_1 b a_2 + 2\,a_1 a_0 + 6\,a_2 b_1) \\ & - \left(\frac{k^2a^2}{\beta} - \frac{\omega^2}{\beta}\right)(8\,a_1 b a_2 - 4\,a_2 b_1) \\ & + b\left(6\,a_0 a_1 a_2 + 3\,{a_2}^2 b_1 + {a_1}^2\right) = 0, \end{aligned} \quad (22)$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(8\,a_2ba_0+2\,{a_1}^2b+8\,a_2b_2+2\,{a_2}^2b^2+2\,a_1b_1\right)-\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(2\,{a_1}^2b-8\,a_2b_2+4\,{a_2}^2b^2-2\,a_1b_1\right)+b\left(6\,a_1a_2b_1+3\,{a_0}^2a_2+3\,a_0{a_1}^2+3\,{a_2}^2b_2\right)=0, \tag{23}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(2\,a_1ba_0+2\,a_2b^2a_1+10\,a_2bb_1+4\,a_1b_2\right)-\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(4\,a_2b^2a_1-8\,a_2bb_1-4\,a_1b_2\right)+b\left(6\,a_0a_2b_1+6\,a_1a_2b_2+3\,{a_0}^2a_1+3\,{a_1}^2b_1\right)=0, \tag{24}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(4\,a_1bb_1+16\,a_2bb_2+2\,a_2b^2a_0+2\,b_2a_0\right)-\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(-4\,a_1bb_1-16\,a_2bb_2+{a_1}^2b^2+{b_1}^2\right)+b\left(6\,a_0a_2b_2+3\,{a_1}^2b_2+6\,a_0a_1b_1+3\,a_2{b_1}^2+{a_0}^3\right)=0, \tag{25}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(10\,a_1bb_2+4\,a_2b^2b_1+2\,b_1ba_0+2\,b_2b_1\right)-\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(-8\,a_1bb_2-4\,a_2b^2b_1+4\,b_2b_1\right)+b\left(6\,a_0a_1b_2+6\,a_2b_1b_2+3\,a_1{b_1}^2+3\,{a_0}^2b_1\right)=0, \tag{26}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(8\,a_2b^2b_2+2\,b_1b^2a_1+8\,b_2ba_0+2\,{b_1}^2b+2\,{b_2}^2\right)-\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(-8\,a_2b^2b_2-2\,b_1b^2a_1+2\,{b_1}^2b+4\,{b_2}^2\right)+b\left(6\,a_1b_1b_2+3\,{a_0}^2b_2+3\,a_0{b_1}^2+3\,a_2{b_2}^2\right)=0, \tag{27}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(2\,b_1b^2a_0+10\,b_1bb_2+6\,b_2b^2a_1\right)-\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(-4\,b_2b^2a_1+8\,b_1bb_2\right)+b\left(6\,a_0b_1b_2+3\,a_1{b_2}^2+{b_1}^3\right)=0, \tag{28}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(6\,b_2b^2a_0+8\,{b_2}^2b+2\,{b_1}^2b^2\right)-\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(8\,{b_2}^2b+{b_1}^2b^2\right)+b\left(3\,{b_1}^2b_2+3\,a_0{b_2}^2\right)=0, \tag{29}$$

$$4\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)b_1b^2b_2+3\,bb_1{b_2}^2=0, \tag{30}$$

$$2\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right){b_2}^2b^2+b{b_2}^3=0. \tag{31}$$

Eqs. (19)- (31) yields

$$a=a,\ b=b,\ k=k,\ \omega=\omega,\ a_1=a_2=b_1=0,$$

$$b_2=\frac{-2b\left(k^2a^2-\omega^2\right)}{\beta},\ a_0=\frac{-2\left(k^2a^2-\omega^2\right)}{\beta}.$$

So that the solution be in the form:

$$v=\frac{-2\left(k^2a^2-\omega^2\right)}{\beta}-\frac{2b\left(k^2a^2-\omega^2\right)}{\beta}\frac{1}{\phi^2}. \tag{32}$$

Let us now discuss the following case:
Case 1. If $b<0$, then

$$v=\frac{-2\left(k^2a^2-\omega^2\right)}{\beta}-\frac{2b\left(k^2a^2-\omega^2\right)}{\beta\left(-\sqrt{-b}\,tanh(\sqrt{-b}\,\xi)\right)^2}, \tag{33}$$

for this

$$u=\frac{1}{\beta}\,ln(v). \tag{34}$$

Case 2. If $b>0$, then

$$v=\frac{-2\left(k^2a^2-\omega^2\right)}{\beta}, \tag{35}$$

for this

$$u=\frac{1}{\beta}\,ln(v). \tag{36}$$

Case 3. If $b=0$, then

$$v=\frac{-2\left(k^2a^2-\omega^2\right)}{\beta}-\frac{2b\left(k^2a^2-\omega^2\right)\xi^2}{\beta}, \tag{37}$$

for this

$$u=\frac{1}{\beta}\,ln(v). \tag{38}$$

3.2 Example 2. Modified Liouville equation using the $\left(\frac{G'}{G}\right)$-expansion method.

Using the $\left(\frac{G'}{G}\right)$-expansion method, we have the formal solution of Eq. (15):

$$v=a_0+a_1\left(\frac{G'}{G}\right)+a_2\left(\frac{G'}{G}\right)^2, \tag{39}$$

$$v'=-a_1\mu-\left(-a_1\lambda+2a_2\mu\right)\left(\frac{G'}{G}\right)-\left(a_1+2a_2\lambda\right)\left(\frac{G'}{G}\right)^2-2a_2\left(\frac{G'}{G}\right)^3, \tag{40}$$

$$v''=a_1\lambda\mu+2a_2\mu^2+\left(a_1\lambda^2+2a_1\mu+6a_2\lambda\mu\right)\left(\frac{G'}{G}\right)+\left(3a_1\lambda+4a_2\lambda^2+8a_2\mu\right)\left(\frac{G'}{G}\right)^2+\left(2a_1+10a_2\lambda\right)\left(\frac{G'}{G}\right)^3+6a_2\left(\frac{G'}{G}\right)^4. \tag{41}$$

Substituting Eq. (39) and its derivatives in Eq. (2) and equating the coefficient of different power's of $\left(\frac{G'}{G}\right)$ to zero, we get

$$2\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right){a_2}^2+b{a_2}^3=0, \tag{42}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(4\,a_1a_2+2\,{a_2}^2\lambda\right)+3\,ba_1{a_2}^2=0, \tag{43}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)\left(5\,a_1\lambda\,a_2+6\,a_2a_0+{a_1}^2\right)+b\left(3\,a_0{a_2}^2+3\,{a_1}^2a_2\right)=0, \tag{44}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)(2\,a_1\mu\,a_2+a_1\lambda^2a_2+10\,a_2\lambda\,a_0-2\,{a_2}^2\mu\,\lambda +{a_1}^2\lambda+2\,a_0a_1)+b\left(6\,a_0a_1a_2+{a_1}^3\right)=0, \tag{45}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)(3\,a_1\lambda\,a_0+8\,a_2\mu\,a_0+4\,a_2\lambda^2a_0 -a_1\lambda\,\mu\,a_2-2\,{a_2}^2\mu^2) +b\left(3\,{a_0}^2a_2+3\,a_0{a_1}^2\right)=0, \tag{46}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)(2\,a_1\mu\,a_0-{a_1}^2\lambda\,\mu+a_1\lambda^2a_0-2\,a_2\mu^2a_1 +6\,a_2\mu\,\lambda\,a_0)+3\,b{a_0}^2a_1=0, \tag{47}$$

$$\left(\frac{k^2a^2}{\beta}-\frac{\omega^2}{\beta}\right)(a_1\lambda\,\mu\,a_0+2\,a_2\mu^2a_0-{a_1}^2\mu^2)+b{a_0}^3=0. \tag{48}$$

Eqs. (42)- (48) yields

$$a_0=-2\,\frac{\mu\,(k^2a^2-\omega^2)}{b\beta},\ a_1=\left(-2\,\frac{\lambda\,(k^2a^2-\omega^2)}{b\beta}\right),$$

$$a_2=-2\,\frac{k^2a^2-\omega^2}{b\beta}.$$

Let us now discuss the following case:
Case 1. When $\lambda^2-4\mu>0$

$$v=-2\,\frac{\mu\left(k^2a^2-\omega^2\right)}{b\beta}-2\,\frac{\lambda\left(k^2a^2-\omega^2\right)}{b\beta}\,(M_1) -2\,\frac{(k^2a^2-\omega^2)}{b\beta}\,(M_1)^2, \tag{49}$$

for this

$$u=\frac{1}{\beta}\,ln(v) \tag{50}$$

Case 2. When $\lambda^2-4\mu<0$

$$v=-2\,\frac{\mu\left(k^2a^2-\omega^2\right)}{b\beta}-\left(2\,\frac{\lambda\left(k^2a^2-\omega^2\right)}{b\beta}\right)(M_2) -2\,\frac{k^2a^2-\omega^2}{b\beta}\,(M_2)^2, \tag{51}$$

for this

$$u=\frac{1}{\beta}\,ln(v). \tag{52}$$

Case 3. When $\lambda^2-4\mu=0$

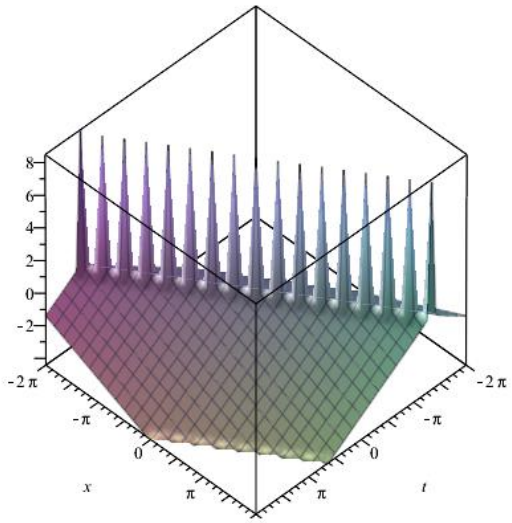
$$v=-2\,\frac{\mu\left(k^2a^2-\omega^2\right)}{b\beta}-\left(2\,\frac{\lambda\left(k^2a^2-\omega^2\right)}{b\beta}\right)\left(\frac{A_2}{A_1+A_2\xi}-\frac{\lambda}{2}\right) -2\,\frac{k^2a^2-\omega^2}{b\beta}\left(\frac{A_2}{A_1+A_2\xi}-\frac{\lambda}{2}\right)^2, \tag{53}$$

for this

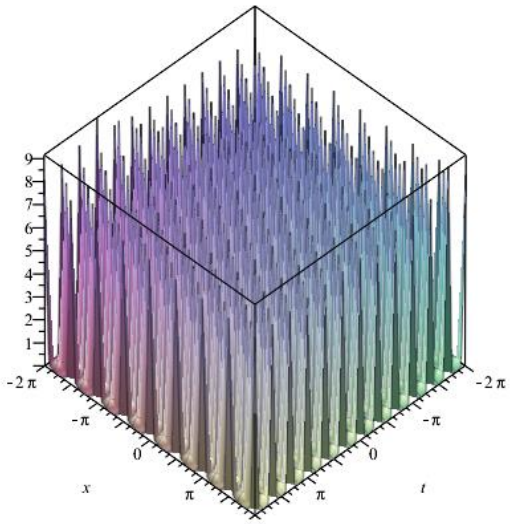
$$u=\frac{1}{\beta}\,ln(v). \tag{54}$$

3.3 Comparison

The extended tanh-function method is reliable and effective and gives more solutions .The applied method will be used in further works to establish more entirely new solutions for other kinds of nonlinear equations.



[Eq. (34)]



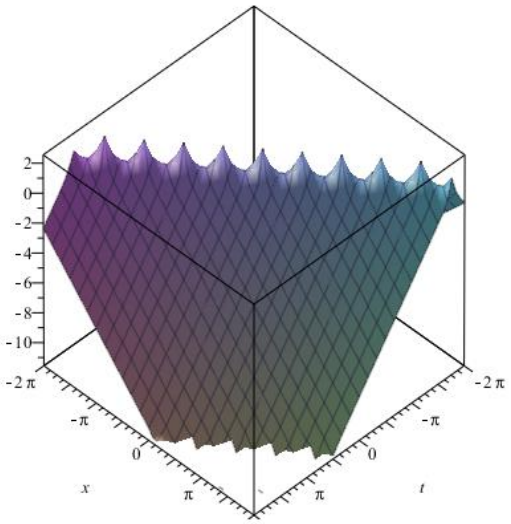
[Eq. (36)]

Fig. 1. The solitary wave solutions of Eqs. (34), (36)

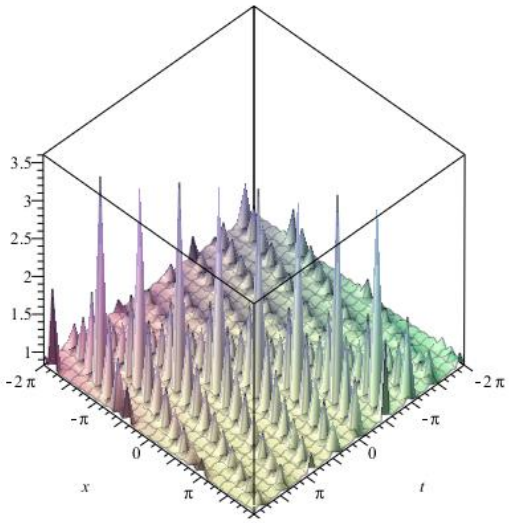
The $\left(\frac{G'}{G}\right)$-expansion method is direct, concise, powerful, effective and convenient technique and can be used for all integrable and non-integrable nonlinear models. Performance of this method is reliable, simple and gives many new solutions. It is also a standard and computerization method which allows us to solve complicated nonlinear evolution equations in diverse areas of science. Moreover, this method is capable of greatly minimizing the size of computational work compared to other existing techniques.
Comparison between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of Modified Liouville equation are new and different from those obtained in [29].
From the above it is possible concluded the following : we have arrived at the observation that these exact solutions are equivalent.

4. CONCLUSION

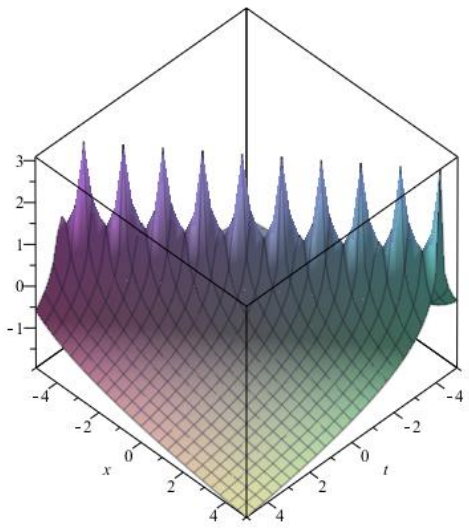
The extended tanh-function method and the $\left(\frac{G'}{G}\right)$-expansion method has been applied in this paper to find the exact traveling



[Eq. (50)]



[Eq. (52)]



[Eq. 54)]

Fig. 2. The solitary wave solutions of Eqs. (50), (52) and (54)

wave solutions and the solitary wave solutions of Modified Liouville equation and Fig. (1) and Fig. (2) show that. Making the comparison between two methods and others methods and clarify the advantages and disadvantages of each of these methods. The similarities and differences between both of them. To our knowledge, these solutions have not been reported in previous literature's. All of our results have been verified with Maple 16 by putting them back into the original equation. The transformation formula were used for every type of non linearity to show that our analysis is applicable to a variety of nonlinear problems. We have emphasized in this work that this relevant transformation is powerful and can be effectively used to discuss nonlinear evolution equations and related models in scientific fields. The availability of computer systems like Mathematica or Maple facilitates the tedious algebraic calculations.

5. ACKNOWLEDGMENT

The author thanks the referees for their suggestions and comments.

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