Reliability of Component based Software System using Fuzzy AHP and Fuzzy TOPSIS

Jyoti Agarwal
MTECH CSE
Amity University, Noida

Renuka Nagpal
Assistant Professor
Amity University, Noida

Rajni Sehgal
Assistant Professor
Amity University, Noida

ABSTRACT
Component based software is a recent approach in the field of software engineering emphasizes on design and development of component based software system. It is based on reusability of code which let customer to have quality product by paying less amount of money and spending less time to produce. To enhance security there is need of reliable software. In this paper reliability study of windows operating system is done and aim of this study is to find out the most reliable windows operating system by use of fuzzy analytic hierarchy process and fuzzy technique for order preference by similarity to ideal solution (TOPSIS). Reliability factors for windows operating system is determined which forms criteria for selecting reliable windows operating system. After determining the criteria fuzzy multi-criteria decision making methods such as fuzzy analytic hierarchy process (AHP) and fuzzy technique for order preference by similarity to ideal solution (TOPSIS) are applied to reliable windows operating system selection problem and results are presented.

1. INTRODUCTION
Component-Based Software Engineering (CBSE) is a technology helps in development of complex system using reusable components. In order to make reliable software from reusable component quality need to measure at every phase of software development life cycle and at early phase so that at the time of testing no faults will occur as they have found at an early phase of software development. Reliability is the probability that software will provide failure-free operation in a fixed environment for a fixed interval of time. The system becomes unreliable due to system crashes, inconsistent output, and unavailability of external system, databases and networks which may cause failure of system. In this paper reliability study of windows operating system is done. As operating system provides link between the hardware and the software and ensure the different aspects of the computer work well together and can run together and also controls the execution of programs and devices by maximizing systems performance and ensuring the different aspects of the computer work well together.

Microsoft added advances to make the user experience more enjoyable and the development of software for the operating system easier. But after Windows vista failure because of security feature and slower processing which were not much better than previous version and incompatible with older PCs which limited number of user to upgrade from XP. So selecting reliable windows operating system is most important nowadays so that people upgrade their system to higher version operating system which is reliable and improves their work efficiency and data security. Reliability study of Windows 7, Windows 8 and Windows 8.1 is done and using fuzzy multi-criteria decision making methods reliable windows operating system is determined. In this paper reliability factors of windows operating system is determined which form selection criteria to apply fuzzy analytic hierarchy process (AHP) and fuzzy technique for order preference by similarity to ideal solution (TOPSIS) on selection of reliable windows operating system problem. In our study we have determined 9 selection criteria’s which are Interoperability, Ease of use, Security, System configuration, File management, Memory Management, Backup and Restore, System restore and On/Off transition experience. On the basis of 9 determined criteria multi-criteria decision approach such as fuzzy analytic hierarchy process (AHP) and fuzzy technique for order preference by similarity to ideal solution (TOPSIS) methods are applied on the problem and results are presented.

This paper is organized as follows: Section 2 describes fuzzy theory, Section 3 presented implementation and results, Section 4 summarizes the paper and Section 5 presented future work.

2. FUZZY THEORY
2.1 Fuzzy numbers
As triangular fuzzy number are useful in information processing and representing information in a fuzzy environment because of their computational simplicity. So in this study triangular fuzzy numbers are adopted in fuzzy AHP and fuzzy TOPSIS methods. Let \((l, m, u)\) be the tripled defined fuzzy number in triangular form where parameter \(l\), \(m\) and \(u\) respectively indicate smallest possible value, most promising value and largest possible value that describes fuzzy event. Let two positive fuzzy numbers in triangular form \((l_1, m_1, u_1)\) and \((l_2, m_2, u_2)\) and various operation performed is shown below:

\[
(l_1m_1u_1) + (l_2m_2u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \quad (1)
\]

\[
(l_1m_1u_1) \cdot (l_2m_2u_2) = (l_1l_2, m_1m_2, u_1u_2) \quad (2)
\]

\[
(l_1, m_1, u_1)^{-1} = \left(\frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1}\right) \quad (3)
\]

If \(k\) is a positive real number then:

\[
(l_2m_2u_2 k) = (l_2k, m_2k, u_2k) \quad (4)
\]

We can use vertex method to calculate distance between two triangular fuzzy number as:

\[
d_p(l, m, u) = \sqrt{\frac{1}{3}[(l_1 - l_2)^2 + (m_1 - m_2)^2 + (u_1 - u_2)^2]} \quad (5)
\]
2.2 Fuzzy Analytic Hierarchy process

It involves decision problem which consist of:[3]

1) Number of alternatives denoted as \( A_i \) (i = 1,2,… n)

2) A set of evaluation criteria \( C_j \) (j = 1,2 …m)

3) qualitative or quantitative assessments \( x_{ij} \) (i = 1,2,… n ; j = 1,2,…m) known as performance rating represent performance of each alternative \( A_i \) with respect to each criterion \( C_j \) leads to determination of decision matrix for the alternatives.

4) Weighting vector \( W = (W_1, W_2, \ldots, W_m) \) where \( W_1, W_2,\ldots, W_m \) are criteria weights represents relative importance of evaluation criteria with respect to overall objective of the problem.

In this project extension of AHP considered which was proposed by Hepu Deng[3]. The various steps of AHP are as follows : [3]

1) Formulate the decision problem and identify hierarchal structure of the problem.

2) Determine decision matrix based on fuzzy numbers –

   Table 1. Fuzzy numbers used for making qualitative assessments [3]

<table>
<thead>
<tr>
<th>Fuzzy number</th>
<th>Membership function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1, 1, 3)</td>
</tr>
<tr>
<td>2</td>
<td>(x-2, x, x+2) for x = 3 , 5</td>
</tr>
<tr>
<td>3</td>
<td>7, 9, 9</td>
</tr>
</tbody>
</table>

By using fuzzy number defined in above Table 1, fuzzy reciprocal judgment matrix for criteria performance (W) or alternative performance with respect to a specific criterion \( C_j \) can be determined as :

\[
C_j \text{ or } W = \begin{bmatrix}
\frac{a_{11}}{1} & \frac{a_{12}}{1} & \ldots & \frac{a_{1k}}{1} \\
\frac{a_{21}}{1} & \frac{a_{22}}{1} & \ldots & \frac{a_{2k}}{1} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{a_{k1}}{1} & \frac{a_{k2}}{1} & \ldots & \frac{a_{kk}}{1}
\end{bmatrix}
\]

(6)

Where, \( \frac{a_{pq}}{1} = \begin{cases} \frac{1}{p-q} , & p < q \\ 1 , & p = q ; p , q = 1,2,...k; \end{cases} \)

Alternative performance rating \( (x_{ij}) \) with respect to a specific criterion \( C_j \) can be determined as :

\[
x_{ij} \text{ or } W_j = \frac{\frac{x_{ij}}{1}}{\sum_{p=1}^{k} \frac{x_{ip}}{1}}
\]

(7)

Where, \( i = 1, 2, \ldots n \); \( j = 1, 2, \ldots m \) and \( k = m \) or \( n \) depends on whether performance rating is assessed by reciprocal judgment matrix or weights of the criteria involved. So decision matrix (X) and the weight vector (W) can be determined as:

\[
X = \begin{bmatrix}
x_{11} & x_{12} & \ldots & x_{1m} \\
x_{21} & x_{22} & \ldots & x_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
x_{n1} & x_{n2} & \ldots & x_{nm}
\end{bmatrix}
\]

\[
W = (W_1, W_2, \ldots, W_m)
\]

Where \( x_{ij} \) represents the resultant fuzzy performance assessment of alternative \( A_i \) (i = 1,2,…n) with respect to each criterion \( C_j \) and \( W_j \) is the resultant fuzzy weight of the criterion \( C_j \) (j=1,2…m) with respect to overall objective of the problem.

3) Determine the fuzzy performance matrix by multiplying the decision matrix obtained at step 2 by the weighting vector determined at step 2.

\[
Z = \begin{bmatrix}
w_{11}x_{11} & w_{12}x_{12} & \ldots & w_{1m}x_{1m} \\
w_{21}x_{21} & w_{22}x_{22} & \ldots & w_{2m}x_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
w_{n1}x_{n1} & w_{n2}x_{n2} & \ldots & w_{nm}x_{nm}
\end{bmatrix}
\]

(10)

4) Obtain the interval performance matrix by using \( \alpha \)-cut on the performance matrix.

\[
\alpha \text{ represents degree of confidence and its larger value indicates more confident decision matrix (} \alpha = 0.5). \]

\[
a_{i,j}^{\alpha} = [a - (m-l)] + l \\
a_{i,j}^{\alpha \text{right}} = u - [a + (u-m)]
\]

(11)

5) To incorporate with decision matrix attitude towards risk using an optimism index \( \lambda \) with value \( \lambda = 0.5 \), an overall crisp performance matrix is calculated as :

\[
Z^{\lambda}_{\alpha} = \begin{bmatrix}
z_{11}^{\lambda} & z_{12}^{\lambda} & \ldots & z_{1m}^{\lambda} \\
z_{21}^{\lambda} & z_{22}^{\lambda} & \ldots & z_{2m}^{\lambda} \\
\vdots & \vdots & \ddots & \vdots \\
z_{m1}^{\lambda} & z_{m2}^{\lambda} & \ldots & z_{mm}^{\lambda}
\end{bmatrix}
\]

(12)

Where, \( Z_{ij}^{\lambda} = \lambda Z_{ij}^{\alpha} + (1-\lambda)Z_{ij}^{\alpha \text{right}} ; \lambda \in [0,1] \)

6) Normalized performance matrix is calculated to each criterion as :

\[
Z_{ij}^{\lambda} = \frac{Z_{ij}^{\lambda}}{\sqrt{\sum_{j=1}^{m} Z_{ij}^{\lambda}}}
\]

(13)

\[
Z_{i}^{\lambda} = \begin{bmatrix}
z_{i1}^{\lambda} & z_{i2}^{\lambda} & \ldots & z_{im}^{\lambda}
\end{bmatrix}
\]

(14)

7) Determine positive ideal solution and negative ideal solution –

Positive ideal solution \( A_{i}^{+}\) and negative ideal solution \( A_{i}^{-}\) can be determined by selecting maximum and minimum value across all alternatives with respect to each criteria which respectively represent the best possible and worst possible results among the alternatives across all criteria.
Degree of similarity between each alternative and the positive ideal solution and the negative ideal solution can be calculated respectively by:

\[ S_{k}^{+} = \frac{d_{k}^{+}}{\max(d_{k}^{+})} \]

\[ S_{k}^{-} = \frac{d_{k}^{-}}{\max(d_{k}^{-})} \]

Where, \( d_{k}^{+} = \max(2Z_{1k}, 2Z_{2k}, ..., 2Z_{mk}) \) and \( d_{k}^{-} = \min(2Z_{1k}, 2Z_{2k}, ..., 2Z_{mk}) \)

8) Degree of similarity between each alternative and the positive ideal solution and the negative ideal solution can be calculated respectively by:

\[ S_{k}^{+} = \frac{d_{k}^{+}}{\max(d_{k}^{+})} \]

\[ S_{k}^{-} = \frac{d_{k}^{-}}{\max(d_{k}^{-})} \]

Where, \( d_{k}^{+} = \max(2Z_{1k}, 2Z_{2k}, ..., 2Z_{mk}) \) is \( k \)th row of overall performance matrix. So, larger value of \( S_{k}^{+} \) and \( S_{k}^{-} \) represents higher degree of similarity between each alternative and positive and negative ideal solution respectively.

9) Determine overall performance of each alternative

\[ \bar{P}_{ki} = \frac{S_{k}^{+}}{S_{k}^{-}} \]

An alternative with higher degree of similarity to the positive ideal solution and at the same time a low degree of similarity to the negative ideal solution is preferred.

10) Arrange performance index values in descending order and give rank to the alternatives.

2.3 Fuzzy TOPSIS method

The basic concept of fuzzy TOPSIS method is that shortest distance must exist between alternative chosen and positive ideal solution and longest distance from the negative ideal solution with minimum cost criteria and maximum benefit criteria for positive ideal solution and minimum benefit criteria with maximum cost criteria for negative ideal solution.

In this paper TOPSIS method proposed by Chen CT [4] is used. The algorithm of this method is described as follows:[4]

1) Form decision makers committee – Let \( k \) be decision makers and \( D_{k} \) be fuzzy rating of each decision maker i.e. \( D_{k} = (k = 1, 2, \ldots k) \) can be represent as triangular fuzzy number \( \tilde{F}_{k} = (k = 1, 2, \ldots k) \) with membership function \( \mu_{k}(x) \)

2) Identify the evaluation criteria

3) Decide linguistic variables for alternatives and evaluation criteria.

4) Aggregate the weight criteria – Aggregate the fuzzy rating of all decision maker represented as triangular fuzzy number \( \tilde{F}_{k} = (\tilde{A}_{k}, \tilde{B}_{k}, \tilde{C}_{k}) \) where \( k = 1, 2, \ldots k \) can be determined as \( \tilde{F} = (a, b, c) \) where \( k = 1, 2, \ldots k \). Here:

\[ a = \min \{ A_{k} \}, b = \frac{1}{k} \sum_{k=1}^{k} b_{k}, c = \max \{ C_{k} \} \]

Aggregate fuzzy rating of alternative with respect to each criterion when fuzzy rating and importance weight of the \( k \)th decision maker are:

\[ \tilde{X}_{ij} = (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}) \]

Where, \( a_{ij} = \min \{ A_{ij} \} \), \( b_{ij} = \frac{1}{k} \sum_{k=1}^{k} b_{ij} \), \( c_{ij} = \max \{ C_{ij} \} \)

So aggregated fuzzy weights \( \tilde{W}_{ij} \) of each criterion are calculated as:

\[ \tilde{W}_{ij} = (\tilde{W}_{1j}, \tilde{W}_{2j}, \tilde{W}_{ij}) \]

where, \( \tilde{W}_{1j} = \min \{ W_{1j} \} \). \( \tilde{W}_{2j} = \frac{1}{k} \sum_{k=1}^{k} W_{kj} \), \( \tilde{W}_{3j} = \max \{ W_{kj} \} \)

5) Construct fuzzy decision matrix as:

\[ \tilde{D} = \begin{bmatrix} \tilde{C}_{11} & \tilde{C}_{12} & \ldots & \tilde{C}_{1n} \\ \tilde{C}_{21} & \tilde{C}_{22} & \ldots & \tilde{C}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C}_{m1} & \tilde{C}_{m2} & \ldots & \tilde{C}_{mn} \end{bmatrix} \]

\[ W = [\tilde{W}_{1}, \tilde{W}_{2}, \ldots \tilde{W}_{m}] \]

Where, \( \tilde{C}_{ij} = (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}) \) and \( \tilde{W}_{ij} = (\tilde{W}_{1j}, \tilde{W}_{2j}, \tilde{W}_{3j}) \); \( i = 1, 2, \ldots m \) and \( j = 1, 2, \ldots n \)

6) Normalize the fuzzy decision matrix – Normalized fuzzy decision matrix \( \tilde{D} \) can be determined as:

\[ \tilde{Z}_{ij} = \left( \frac{\tilde{a}_{ij}}{\tilde{c}_{ij}}, \frac{\tilde{b}_{ij}}{\tilde{c}_{ij}}, \frac{\tilde{c}_{ij}}{\tilde{c}_{ij}} \right) \]  

where \( \tilde{c}_{ij} = \max \{ c_{ij} \} \)

7) Construct weighted normalized fuzzy decision matrix – Weighted normalized fuzzy decision matrix is computed by multiplying the importance weight of evaluation criteria and values in the normalized fuzzy decision matrix. It is represented as \( \tilde{D} \)

\[ \tilde{D} = [\tilde{D}_{ij}]_{m \times n} \; i = 1, 2, \ldots m \; \text{and} \; j = 1, 2, \ldots n \]

where,

\[ \tilde{D}_{ij} = \tilde{Z}_{ij} \tilde{W}_{ij} \]

Here, \( \tilde{D}_{ij} \) represents importance weight of criterion \( \tilde{C}_{j} \)

8) Determine fuzzy positive ideal solution and fuzzy negative ideal solution –

Fuzzy positive ideal solution (FPIS, \( \tilde{A}^{+} \)) is determined as:

\[ \tilde{A}^{+} = (\tilde{A}_{1}^{+}, \tilde{A}_{2}^{+}, \ldots, \tilde{A}_{n}^{+}) \]

Fuzzy negative ideal solution (FPIS, \( \tilde{A}^{-} \)) is determined as:

\[ \tilde{A}^{-} = (\tilde{A}_{1}^{-}, \tilde{A}_{2}^{-}, \ldots, \tilde{A}_{n}^{-}) \]

Where, \( \tilde{A}_{ij}^{+} = \max \{ V_{ij} \} \) and \( \tilde{A}_{ij}^{-} = \min \{ V_{ij} \} \); \( i = 1, 2, \ldots m \) and \( j = 1, 2, \ldots n \)

9) Calculate distance of each alternative from FPIS and FNIS – The distance of each alternative from FPIS and FNIS are calculated as:

\[ d_{ij}^{+} = \sum_{j=1}^{n} d_{ij}^{+} (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}) \]

\[ d_{ij}^{-} = \sum_{j=1}^{n} d_{ij}^{-} (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}) \]  

\( i = 1, 2, \ldots m \)
Here, \( d_{ij} \) is the distance measurement between two fuzzy numbers.

10) Calculate the closeness coefficient of each alternative –
A closeness coefficient \( (C_i) \) is defined to rank all possible alternatives. It represents the distance to the FPIS \( (A^+) \) and FNIS \( (A^-) \) simultaneously \( C_i \) can be calculated as :
\[
C_i = \frac{d^+ - d^-}{d^+ + d^-}; i = 1,2,\ldots,m
\] (31)

11) According to closeness coefficient give ranking to alternative in descending order.

3. IMPLEMENTATION & RESULTS
Our application is related with the selection of reliable windows operating system which is needed after failure of windows vista and changed people thinking that higher version is more reliable than previous version. So in this study three windows version i.e. Windows 7, Windows 8, Windows 8.1 are considered as alternative and denoted as \( A_1, A_2, A_3 \) respectively.
Then evaluation criteria are determined as Interoperability\( (C_1) \), Ease of use\( (C_2) \), System configuration\( (C_3) \), File management\( (C_4) \), Memory Management\( (C_5) \), Backup and Restore\( (C_6) \), System restore\( (C_7) \), On/Off transition experience\( (C_8) \).

3.1 APPLICATION WITH FUZZY AHP
Based on the comprehensive discussion about considered criterion and using Saaty’s 1-9 scale shown in Table 2, fuzzy reciprocal judgment matrix with respect to alternatives in regard to each criterion is given below:
\[
C_1 = \begin{bmatrix}
\frac{5}{7} & \frac{7}{5} & \frac{7}{9} \\
\frac{7}{9} & \frac{1}{9} & \frac{1}{7} \\
\frac{7}{9} & \frac{7}{1} & \frac{7}{1}
\end{bmatrix}, C_2 = \begin{bmatrix}
\frac{3}{7} & \frac{7}{3} & \frac{7}{3} \\
\frac{7}{3} & \frac{1}{3} & \frac{1}{7} \\
\frac{7}{3} & \frac{7}{7} & \frac{7}{7}
\end{bmatrix},
C_3 = \begin{bmatrix}
\frac{7}{5} & \frac{7}{5} & \frac{7}{5} \\
\frac{5}{7} & \frac{1}{5} & \frac{1}{5} \\
\frac{5}{7} & \frac{1}{5} & \frac{1}{5}
\end{bmatrix},
C_4 = \begin{bmatrix}
\frac{1}{5} & \frac{5}{7} & \frac{5}{7} \\
\frac{7}{7} & \frac{1}{7} & \frac{1}{7} \\
\frac{7}{7} & \frac{7}{1} & \frac{7}{1}
\end{bmatrix},
C_5 = \begin{bmatrix}
\frac{1}{5} & \frac{5}{7} & \frac{5}{7} \\
\frac{7}{7} & \frac{1}{7} & \frac{1}{7} \\
\frac{7}{7} & \frac{7}{1} & \frac{7}{1}
\end{bmatrix}, C_6 = \begin{bmatrix}
\frac{7}{5} & \frac{7}{5} & \frac{7}{5} \\
\frac{5}{7} & \frac{1}{5} & \frac{1}{5} \\
\frac{5}{7} & \frac{1}{5} & \frac{1}{5}
\end{bmatrix},
C_7 = \begin{bmatrix}
\frac{7}{5} & \frac{7}{5} & \frac{7}{5} \\
\frac{5}{7} & \frac{1}{5} & \frac{1}{5} \\
\frac{5}{7} & \frac{1}{5} & \frac{1}{5}
\end{bmatrix},
C_8 = \begin{bmatrix}
\frac{7}{5} & \frac{7}{5} & \frac{7}{5} \\
\frac{5}{7} & \frac{1}{5} & \frac{1}{5} \\
\frac{5}{7} & \frac{1}{5} & \frac{1}{5}
\end{bmatrix}
\] (32)

Table 2. Saaty’s 1-9 scale [12]

<table>
<thead>
<tr>
<th>Linguistic variables</th>
<th>fuzzy y number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equally important</td>
<td>1</td>
</tr>
<tr>
<td>Somewhat more important</td>
<td>3</td>
</tr>
<tr>
<td>Much more important</td>
<td>5</td>
</tr>
<tr>
<td>Very Much more important</td>
<td>7</td>
</tr>
<tr>
<td>Absolutely more important</td>
<td>9</td>
</tr>
<tr>
<td>Intermediate values</td>
<td>2,4,6,8</td>
</tr>
</tbody>
</table>

Fuzzy decision matrix shown in equation (33) is calculated by considering fuzzy reciprocal judgment matrix shown in equation (32) and using equation (7) alternative performance rating with respect to each criterion is calculated. To determine relative importance of the selection criteria, fuzzy reciprocal judgment matrix shown in equation (34) is used and Weighting vectors is calculated shown in equation (35).
\[
W = \begin{bmatrix}
1 & 3 & \frac{1}{9} & \frac{1}{4} & \frac{1}{7} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{3} & 1 & \frac{1}{7} & \frac{1}{5} & \frac{1}{5} & 3 & 3 & 7 \\
9 & 7 & 1 & 9 & 5 & 5 & 3 & 3 \\
\frac{1}{3} & \frac{1}{3} & 1 & 9 & 1 & 3 & 5 & 9 \\
4 & 5 & \frac{1}{5} & \frac{1}{3} & 1 & \frac{1}{7} & \frac{1}{3} & \frac{1}{7} \\
7 & 5 & \frac{1}{5} & \frac{1}{5} & 7 & 1 & 5 & 3 \\
4 & \frac{1}{3} & \frac{1}{3} & 1 & 9 & 3 & 1 & \frac{1}{5} \\
4 & \frac{1}{7} & \frac{1}{4} & \frac{1}{3} & \frac{1}{3} & \frac{1}{5} & \frac{1}{5} & 7 \\
\end{bmatrix}
\] (34)

\[
W = \begin{bmatrix}
.013, .039, .174 \\
.032, .087, .226 \\
.010, .226, .471 \\
.068, .151, .315 \\
.026, .069, .185 \\
.071, .164, .374 \\
.026, .070, .186 \\
.056, .123, .291 \\
.032, .066, .167
\end{bmatrix}
\] (35)

Based on criteria weights fuzzy performance matrix shown in equation (36) of the problem is calculated using equation (10). Using equation (11) α-cut on the performance matrix is calculated and interval performance matrix is formed shown in equation (37). To calculate crisp performance matrix shown in equation (38) use equation (12) and then normalized performance matrix shown in equation (39) is calculated using equation (13).
\[
Z^\alpha_1 = \begin{bmatrix}
.135 .010 .031 .032 .014 .024 .008 .015 .007 \\
.084 .047 .259 .127 .014 .179 .04 .120 .036 \\
.053 .088 .108 .127 .071 .082 .072 .069 .066
\end{bmatrix}
\] (38)

\[
Z^\beta_1 = \begin{bmatrix}
.808 .01 .109 .175 .191 .121 .097 .107 .093 \\
.502 .47 .918 .697 .191 .904 .487 .863 .48 \\
.317 .08 .382 .191 .972 .414 .878 .496 .88
\end{bmatrix}
\] (39)

Positive and negative ideal solution is calculated using equation (15),(16),(17) and (18).
\[
\left( A_1^B \right)^+ = (.808, .088, .918, .697, .972, .904, .878, .863, .88)
\]
\[
\left( A_1^B \right)^- = (.317, .01, .109, .175, .191, .121, .097, .107, .093)
\]

Then calculate using equation (19) and (20) degree of similarity between each alternative and the positive ideal solution and the negative ideal solution respectively and using equation (21) overall performance of each alternative is...
calculated and in Table 5 performance index and ranking of windows is shown. From Table 3 it can be seen that performance index of alternative \( A_3 \) (=0.806) is greater than alternatives \( A_2 \) (=0.780) and \( A_1 \) (=0.309) . So we prefer alternatives in order \( A_3 \), \( A_2 \) and \( A_1 \) (\( A_3 \approx A_2 \approx A_1 \)). Hence we get Windows 8.1(\( A_3 \)) as the most reliable windows version in comparison with Windows 8(\( A_2 \)) and Windows 7 (\( A_1 \)).So reliability order is: Windows 8.1>Windows 8 >Windows 7

Table 3. Performance index and ranking of windows

<table>
<thead>
<tr>
<th>Windows</th>
<th>Performance index</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>0.309</td>
<td>3</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.780</td>
<td>2</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>0.806</td>
<td>1</td>
</tr>
</tbody>
</table>

3.2 APPLICATION WITH FUZZY TOPSIS

In this section fuzzy TOPSIS is proposed for the reliable windows operating system selection problem. Firstly, importance of criteria by using linguistic variables is evaluated by three decision makers \( D_1 \), \( D_2 \), \( D_3 \) shown in Table 4 and importance weights of the criteria determined by these three decision makers are shown in Table 5. To evaluate the rating of alternatives with respect to each criterion these decision makers use linguistic variables shown in Table 6 and aggregate fuzzy rating of alternative with respect to each criterion is calculated by using equation (23). And aggregated fuzzy weights of each criterion are calculated using equation (24). The normalized fuzzy decision matrix is formed as in Table 9.

After a weighted normalized fuzzy decision matrix is formed, fuzzy positive ideal solution and fuzzy negative ideal solution are determined as in the following:

\[
\begin{align*}
A^+ &= ((0.9,0.9,0.9) (1,1,1) (1.1,1) (0.8,0.8,0.8) (0.6,0.6,0.6) (0.5,0.5,0.5)) \\
A^- &= ((0.35,0.35,0.35) (0.49,0.49,0.49) (0.4,0.4,0.4) (0.2,0.2,0.2) (0.1,0.1,0.1)) \\
\end{align*}
\]

By using vertex method distance of each alternative from FPIS and FNIS with respect to each criterion is calculated as:

\[
d(A_i,A^+) = \sqrt{\frac{1}{3} [(0.9 - 0.35)^2 + (0.9 - 0.64)^2 + (0.9 - 0.9)^2]} = 0.351
\]

\[
d(A_i,A^-) = \sqrt{\frac{1}{3} [(0.35 - 0.35)^2 + (0.35 - 0.64)^2 + (0.35 - 0.9)^2]} = 0.358
\]

\[
C_{C_1} = \frac{d_{A_i}^-}{d_{A_i}^+ + d_{A_i}^-} = \frac{0.264}{0.283 + 0.264} = 0.450
\]

\[
C_{C_2} = \frac{d_{A_i}^-}{d_{A_i}^+ + d_{A_i}^-} = \frac{0.283}{0.283 + 0.253} = 0.520
\]

\[
C_{C_3} = \frac{d_{A_i}^-}{d_{A_i}^+ + d_{A_i}^-} = \frac{0.365}{0.264 + 0.253} = 0.450
\]

\[
d_{A_i}^+ \ and \ d_{A_i}^- \ of \ three \ alternatives \ are \ shown \ in \ Table \ 12. \ Using \ equation \ (31) \ closeness \ coefficient \ of \ three \ alternatives \ is \ calculated \ as:
\]

Ranking order of three alternatives according to their closeness coefficient is determined as: \( A_3 \approx A_2 \approx A_1 \). Hence we get Windows 8 is the most reliable Window operating system compared with Windows 8.1 and Windows 7. So reliability order is: Windows 8 > Windows 8.1 > Windows 7.

Table 4. Linguistic variables for importance weight of each criterion [4]

<table>
<thead>
<tr>
<th>Linguistic variables</th>
<th>Triangular fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low (VL)</td>
<td>(0.0,0.2)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>(0.1,0.2,0.3)</td>
</tr>
<tr>
<td>Medium low (ML)</td>
<td>(0.2,0.35,0.5)</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>(0.4,0.5,0.6)</td>
</tr>
<tr>
<td>Medium high (MH)</td>
<td>(0.5,0.65,0.8)</td>
</tr>
<tr>
<td>High(H)</td>
<td>(0.7,0.8,0.9)</td>
</tr>
<tr>
<td>Very high(VH)</td>
<td>(0.8,1,1)</td>
</tr>
</tbody>
</table>

Table 5. Importance weight of each criterion from 3 decision maker

<table>
<thead>
<tr>
<th>Criteria</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>H</td>
<td>VH</td>
<td>H</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>H</td>
<td>H</td>
<td>VH</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>M</td>
<td>MH</td>
<td>M</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>M</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>M</td>
<td>ML</td>
<td>M</td>
</tr>
<tr>
<td>( C_6 )</td>
<td>ML</td>
<td>ML</td>
<td>M</td>
</tr>
</tbody>
</table>

Table 6. Linguistic variables for ratings [4]

<table>
<thead>
<tr>
<th>Linguistic variables</th>
<th>Triangular fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very poor (VP)</td>
<td>(0.0,0.2)</td>
</tr>
<tr>
<td>Poor(P)</td>
<td>(1,2.3)</td>
</tr>
<tr>
<td>Medium poor (MP)</td>
<td>(2,0.3,5.5)</td>
</tr>
<tr>
<td>Fair(F)</td>
<td>(4,5.6)</td>
</tr>
<tr>
<td>Medium good (MG)</td>
<td>(5,6,5.8)</td>
</tr>
<tr>
<td>Good(G)</td>
<td>(7,8.9)</td>
</tr>
<tr>
<td>Very good (VG)</td>
<td>(8,10,10)</td>
</tr>
</tbody>
</table>
\[
X = 
\begin{bmatrix}
[0.03, 0.05, 0.31] & [0.03, 0.04, 0.16] & [0.03, 0.08, 0.19] & [0.04, 0.09, 0.32] & [0.03, 0.04, 0.16] & [0.03, 0.05, 0.21] & [0.03, 0.04, 0.16] & [0.03, 0.05, 0.17] & [0.03, 0.04, 0.16] \\
[0.25, 0.57, 1.68] & [0.16, 0.30, 0.60] & [0.37, 1.10, 1.14] & [0.16, 0.45, 1.15] & [0.16, 0.30, 0.60] & [0.31, 0.66, 1.28] & [0.16, 0.30, 0.60] & [0.30, 0.61, 1.09] & [0.16, 0.30, 0.60] \\
[0.14, 0.36, 1.08] & [0.35, 0.64, 1.03] & [0.11, 0.39, 0.55] & [0.16, 0.45, 1.15] & [0.35, 0.64, 1.03] & [0.11, 0.27, 0.62] & [0.35, 0.64, 1.03] & [0.19, 0.33, 0.63] & [0.35, 0.64, 1.03] 
\end{bmatrix}
\]

\[
Z = \begin{bmatrix}
[0.001, 0.270] & [0.002, 0.019] & [0.009, 0.053] & [0.008, 0.057] & [0.001, 0.28] & [0.005, 0.043] & [0.001, 0.016] & [0.003, 0.027] & [0.001, 0.014] \\
[0.012, 0.157] & [0.015, 0.090] & [0.126, 0.392] & [0.039, 0.215] & [0.012, 0.016] & [0.065, 0.293] & [0.012, 0.068] & [0.045, 0.196] & [0.012, 0.06] \\
[0.007, 0.100] & [0.033, 0.144] & [0.044, 0.173] & [0.039, 0.215] & [0.026, 0.117] & [0.026, 0.130] & [0.026, 0.118] & [0.027, 0.111] & [0.026, 0.107] 
\end{bmatrix}
\]

Table 8. Fuzzy decision matrix and fuzzy weights of three alternative

<table>
<thead>
<tr>
<th>Criteria</th>
<th>A₁</th>
<th>A₂</th>
<th>A₃</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>[5.7, 5.9]</td>
<td>[7.8, 9]</td>
<td>[5.7, 5.9]</td>
<td>[2.4, 6]</td>
</tr>
<tr>
<td>C₂</td>
<td>[5.7, 5.9]</td>
<td>[8.10, 10]</td>
<td>[7.93, 10]</td>
<td>[2.4, 6]</td>
</tr>
<tr>
<td>C₃</td>
<td>[5.6, 5.8]</td>
<td>[7.8, 9]</td>
<td>[5.7, 5.9]</td>
<td>[2.1, 3]</td>
</tr>
<tr>
<td>C₄</td>
<td>[5.7, 5.9]</td>
<td>[7.86, 10]</td>
<td>[7.93, 10]</td>
<td>[2.4, 6]</td>
</tr>
<tr>
<td>C₅</td>
<td>[5.7, 5.9]</td>
<td>[7.86, 10]</td>
<td>[7.93, 10]</td>
<td>[2.4, 6]</td>
</tr>
<tr>
<td>C₆</td>
<td>[5.7, 5.9]</td>
<td>[7.86, 10]</td>
<td>[7.93, 10]</td>
<td>[2.4, 6]</td>
</tr>
<tr>
<td>C₇</td>
<td>[5.7, 5.9]</td>
<td>[7.86, 10]</td>
<td>[7.93, 10]</td>
<td>[2.4, 6]</td>
</tr>
<tr>
<td>C₈</td>
<td>[5.7, 5.9]</td>
<td>[7.86, 10]</td>
<td>[7.93, 10]</td>
<td>[2.4, 6]</td>
</tr>
<tr>
<td>C₉</td>
<td>[5.7, 5.9]</td>
<td>[7.86, 10]</td>
<td>[7.93, 10]</td>
<td>[2.4, 6]</td>
</tr>
</tbody>
</table>

Table 10. Distances between Aᵢ (i = 1,2,3) and A⁺ with respect to each criterion

<table>
<thead>
<tr>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
<th>C₆</th>
<th>C₇</th>
<th>C₈</th>
<th>C₉</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.35</td>
<td>0.352</td>
<td>0.39</td>
<td>0.42</td>
<td>0.2</td>
<td>0.4</td>
<td>0.16</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>0.26</td>
<td>0.266</td>
<td>0.26</td>
<td>0.36</td>
<td>0.2</td>
<td>0.27</td>
<td>0.54</td>
<td>0.29</td>
<td>0.26</td>
</tr>
<tr>
<td>0.35</td>
<td>0.316</td>
<td>0.37</td>
<td>0.43</td>
<td>0.3</td>
<td>0.36</td>
<td>0.45</td>
<td>0.28</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 11. Distances between Aᵢ (i = 1,2,3) and A⁻ with respect to each criterion

\[
d(A₁, A⁺) = \begin{bmatrix}
0.35 \\
0.26 \\
0.35 
\end{bmatrix}
\]

\[
d(A₂, A⁺) = \begin{bmatrix}
0.36 \\
0.29 \\
0.37 
\end{bmatrix}
\]

\[
d(A₃, A⁺) = \begin{bmatrix}
0.36 \\
0.26 \\
0.31 
\end{bmatrix}
\]

Table 12. Computation of \(d_1 - d_3\), \(C₁\)

<table>
<thead>
<tr>
<th>A₁</th>
<th>A₂</th>
<th>A₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.213</td>
<td>3.216</td>
<td>3.075</td>
</tr>
<tr>
<td>2.634</td>
<td>2.536</td>
<td>2.836</td>
</tr>
<tr>
<td>0.450</td>
<td>0.559</td>
<td>0.520</td>
</tr>
</tbody>
</table>

3.3 Result and Discussion

As we have seen from results that for selection of reliable windows operating system problem the appropriate methods of fuzzy are AHP and TOPSIS. In this paper results of fuzzy TOPSIS i.e. second alternative is preferred compared to other alternatives. So we can say that Windows 8 is the most reliable version of windows operating system. fuzzy TOPSIS outcome is preferred because on comparing fuzzy AHP and fuzzy TOPSIS in terms of amount of computations fuzzy TOPSIS requires less computation than fuzzy AHP and works
well for one-tier decision tree unlike fuzzy AHP which is preferable for widespread hierarchies. In fuzzy AHP increase in number of criteria and alternative increases risk and pair wise comparison. Also fuzzy TOPSIS provides meaningful information by measuring alternative distances to negative ideal solution and positive ideal solution of each alternative and then gives ranking as compared to fuzzy AHP where pair wise comparison and priority weights of alternative is done by decision makers using extent analysis method. Hence we get Windows 8 as the most reliable windows operating system compared with latest version Windows 8.1 and previous version Windows 7.

4. CONCLUSION

Component based software is recently use in the field of software engineering emphasizes on design and development of component based software system. The reliable windows operating system is determined in this project by the use of fuzzy multi-criteria decision making approaches i.e. fuzzy AHP and fuzzy TOPSIS. Reliability factors of windows operating system is determined which form selection criteria for evaluating most reliable windows operating system. Criteria’s are Interoperability, Ease of use, Security, System configuration, File management, Memory Management, Backup and Restore, System restore and On/Off transition experience. Preference order of alternatives is determined by using fuzzy AHP and fuzzy TOPSIS and from result discussion Windows 8 is found as the most reliable windows operating system.

5. FUTURE SCOPE

There is further scope of improvement in reliability of component based software system by finding sub criteria which may improve result of selection of reliable windows operating system. The method proposed in this project can be applied to other problems required decision from multiple criteria like selecting reliable android version so that people can choose best version Smartphone out of different version available, personnel selection, material selection for companies. Also other multi-criteria methods like fuzzy PROMETHEE and ELECTRE can also be used to find out the reliable windows operating system.

6. REFERENCES

[17] WANG Dong HUANG Ning YE Ming .2008.Reliability Analysis of Component-Based Software Based on Relationships of Components. IEEE.
[22] Deepak Panwar , Pradeep Tomar.2011.New method to find the maximum number of faults by analyzing reliability and reusability in Component-Based Software.3rd International Conference on Trendz in Information Sciences & Computing


