A Survey of the Quadratic Assignment Problem

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ABSTRACT
The quadratic assignment problem (QAP) is very challengeable and interesting problem that can model many real-life problems. In this paper, we will simply discuss the meaning of quadratic assignment problem, solving techniques and we will give a survey of some developments and researches.

Keywords: Quadratic Assignment Problem, formulation, Exact Algorithm, NP-complete, Bound, Heuristic.

1. INTRODUCTION
Quadratic assignment problem (QAP) was firstly introduced in 1957 by **Koopmans and Beckmann [26]** as a mathematical model related to economic activities. Then QAP was used to model many applications in different areas such as operations research, parallel and distributed computing, and combinatorial data analysis, for example: locating machines, departments or offices within a plant , arranging indicators and controls in a control room to minimize eye fatigue, locating hospital departments , forest management, assignment of runners in a relay team ……. etc.

2. DESCRIPTION OF THE PROBLEM

We will discuss generally a common problem, **the linear assignment problem (LAP)**, then, we will define the quadratic assignment problem (QAP)

2.1 The Linear Assignment Problem (LAP)
The linear assignment problem (LAP) as used by **Hanan and Kurtzberg [27]**, involves the assignment of n agent to n tasks. For each task assignment, there is a related cost c_{ij} of assigning agent i to task j. The assignment of each agent to one and only one task in a manner that minimizes the total cost is the objective. Mathematically, the formulation of this problem as follows:

$$\min \sum_{i=1}^{n} c_{i\pi(\mathrm{i})} \quad \text{,over all permutations } \pi \in S_n$$

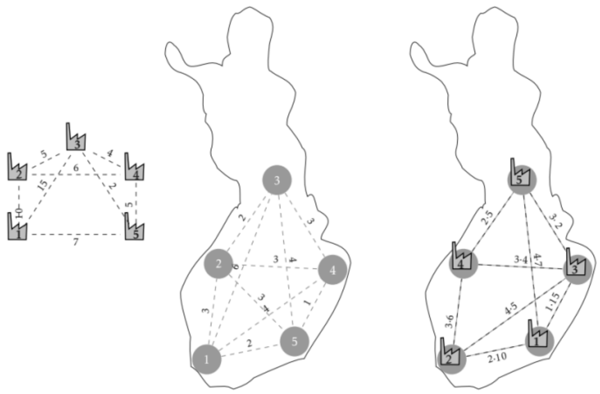
where S_n is the set of permutations of {1, 2,…, n}, and j= π(i) is the task assignment of agent i. Notice that there are n permutations from which to choose the optimal assignment, i.e., there are n! distinct ways in which n task can be assigned to n agent. Also notice that for large values of n, examination of all possible permutations is infeasible, For example, assigning n= 10 agents to 10 tasks as above, then there are 10! to be examined , or approximately 3.63 million different permutations. Obviously, we need more efficient algorithms for solving nontrivial forms of the LAP.

2.2 The Quadratic Assignment Problem
Simply, QAP can be illustrated as:
n facilities have to be assigned to n different locations, assuming that a flow of value f= [f_{ij}] has to go from facility I to facility j and that the distance between the locations k and l is D= [d_{kl}], the objective is to assign the facilities to the locations that minimizes the total cost which is the sum of the products flow × distance. example: let the matrices A and Bare the distance and flow matrices of a facility location problem.

$$A=\begin{bmatrix} 0\,3\,6\,4\,2 \\ 3\,0\,2\,3\,3 \\ 6\,2\,0\,3\,4 \\ 4\,3\,3\,0\,1 \\ 2\,3\,4\,1\,0 \end{bmatrix} \qquad B=\begin{bmatrix} 0\;10\;15\;0\;7 \\ 10\;0\;5\;\;6\;\;0 \\ 15\;5\;0\;\;4\;\;2 \\ 0\;\;6\;\;4\;\;0\;\;5 \\ 7\;\;0\;\;2\;\;5\;\;0 \end{bmatrix}$$

As illustrated in **Fig.1.** optimal solution p=[2,4,5,3,1] **.**The factories to be allocated to specific locations are shown to the left. The picture to the right shows the optimal allocation of the factories.

Fig.1. A facility location problem



As with the LAP, there are n! permutations from which to choose the optimal assignment. However, there is a difference between two problems that makes QAP considered more difficult.
The difference between LAP & QAP is that the LAP assignment of task j to agent i was made independently of the assignments of the other agents. With QAP the assignments are not independent, so when assign agent i to task j we must consider the assignments of all other agents who have nonzero affinity for agent i which represents the amount of face-to-face communication between agents.

3. PROBLEM FORMULATIONS
3.1 Koopmans-Beckman QAP
As we discussed above let F= [f_{ij}] is a matrix of flows, D= [d_{kl}] is a matrix of distances and consider n is the set of positive integers {1, 2, …, n}, and S_n be the set of permutations of {1, 2, …, n}. Then the Quadratic Assignment

Problem can be defined as: $\min \sum_{i=1}^{n}\sum_{j=1}^{n} f_{ij} d_{\pi(i)\pi(j)}$, over all permutations $\pi \in S_n$

The objective of the quadratic assignment problem with flow matrix F and distance matrix D is to find the permutation $\pi_0 \in S_n$ that minimizes the double summation over all i, j. notice that the notation $d_{\pi(i)\pi(j)}$ as above, refers to permuting the rows and columns of the matrix D by some permutation π. That is, $D^{\pi} = [\, d^{\pi}_{ij} \,] = d_{\pi(i)\pi(j)}$, for $1 \le i, j \le n$.

3.1.1 The Generalized Koopamans-Beckmann QAP

A slightly different problem also addressed as a QAP, Besides the two matrices" flow" and "distance" we are given a third matrix "the cost of placing facility i at location j", this problem is called **the generalized Koopamans-Beckmann QAP**. In the case that the cost of placing facility i at location j = 0, for all $1 \le i, j \le n$, we get the problems formulated in (3.1).

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n} f_{ij} d_{\pi(i)\pi(j)} + \sum_{i=1}^{n} c_{i\pi(\mathrm{i})}$$

, over all permutations $\pi \in S_n$

3.2 A Quadratic 0-1 Formulation

There is another formulation of QAP problem called Quadratic 0-1 Formulation. Originally, this formulation was used by **Koopmans-Beckman [26]**. It is formulated using an nxn matrix is a permutation matrix $X = [x_{ij}]$ that represents permutations $\pi \in S_n$ by 0-1 form .To be considered a permutation matrix , it should satisfy the three following conditions:

$$\sum_{i=1}^{n} X_{ij} \qquad , j=1,\ldots, n$$

$$\sum_{j=1}^{n} X_{ij} \qquad , i=1,\ldots, n$$

$$X_{ij} \in \{0,1\} \qquad , i,j=1,\ldots, n$$

If the three conditions are satisfied, then QAP can be formulated as:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ij} d_{kl} X_{ik} X_{jl}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} X_{ij} \qquad , j=1,\ldots, n$$

$$\sum_{j=1}^{n} X_{ij} \qquad , i=1,\ldots, n$$

$$X_{ij} \in \{0,1\} \qquad , i,j=1,\ldots, n$$

3.3 Trace Formulation

The following formulation is based on the traces of the matrices F and D. The trace of a square matrix is defined as the sum of its diagonal elements. Given a square matrix T, then:

$$trace(T) = \sum_{i=1}^{n} T_{ii}$$

As previously defined given flow, distance, and permutation matrices then the formulation is:

$$\min trace(\mathrm{FXD}^t X^t),$$

$$\text{s.t.} \quad X \in \pi_X$$

where π_x represents the set of permutation matrices, and $\cdot^{\,t}$ is the transpose of the given matrix.

4. COMPUTATIONAL COMPLEXITY

However, QAP is applied in many practical applications, only very small instances of QAP have been solved to optimality in practice. As discussed before there is enormous number of all n! feasible solutions and we should search for the optimal solution, so QAP belongs to a class of computationally hard problems known as non deterministic polynomial- complete "**NP-complete" (Sahni and Gonzalez in 1976) [28]**.

5. LOWER BOUNDS

Lower bounds are not only used to evaluate the goodness of solutions produced by heuristics, but also they are an essential component of branch and bound procedures **(which will be discussed in 6.1)**. For QAP, there are three main categories of lower bounds: **Gilmore-Lawler bound [29, 30]**, **eigenvalue related bounds [15, 32]**, and **bounds based on reformulations [36]**. Briefly, they will be simply discussed.

5.1 Main Lower Bounds

5.1.1 Gilmore-Lawler Bound (GLB)

The GLB is one of the first lower bounds and is the most widely used lower bound for QAP. GLB involves solving LAP with cost matrix. It requires only $O(n^3)$ computation time for a Koopmans-Beckman QAP. So, it is simple and quick to compute. The drawback of The GLB is that when the size of QAP increases the GLB become slowly.

5.1.2 Eigenvalue Related Bounds (EVB)

The EVB based on eigenvalues of the flow and distance matrices F and D**.** It based on the trace formulation of QAP. The EVB are computed using some iterations, each iteration requiring $O(n^3)$ computing time . The drawback of The EVB is high computation time.

5.1.3 Bounds Based On Reformulations

Like the eigenvalue based bounds, the reformulation bounds are computed using some iteration. n^2+1 LAP of size n must be solved for each iteration.The running time for the k^{th} iteration is $O(kn^5)$. The drawback is high computation time.

5.2 Other Lower Bounds

There are some contributions in the lower bounds like**:** the interior point bound by Resende et al. (1995), the level-1 RLT (or RLT1) dual-ascent bound by Hahn and Grant (1998), the dual-based bound by Karisch et al. (1999), the convex quadratic programming bound by Anstreicher and Brixius (2001), the level-2 RLT (or RLT2) interior point bound by Ramakrishnan et al. (2002), the bundle method bound by Rendl and Sotirov (2007), the lift-and-project SDP bound by Burer and Vandenbussche (2006), and the Hahn-Hightower level-2 RLT dual-ascent bound by Adams et al. (2007). (As shown in qaplib) [3].

6. EXACT ALGORITHMS

Finding QAP optimal solution is a great challenge. In order to achieve optimality for QAP there are three main methods: the **Branch and bound procedures [29, 37],** the **cutting plane techniques [38], and** the **Dynamic programming [22].**

6.1 The Branch and Bound Method

The most important and used method to achieve optimality for QAP is the Branch and bound as it is more efficient technique. The Branch and bound procedures was firstly introduced by **Gilmore [29] in 1962** who solved a QAP of size 8 = n. The Branch and bound method was named as it is applied. To apply The Branch and bound method; first, chose a heuristic procedure to get an initial feasible solution "suboptimal" and it is used as upper bound. Second, the problem is fragmented into subproblems with a lower bound, then formulate search tree by repeating the fragmentation and lower bounding of each subproblem .During iterations , an optimal permutation is being constructed.

6.2 The Cutting Plane Method

This method has two classes: traditional cutting plane methods and polyhedral cutting-plane or branch-and-cut methods. Traditional cutting plane algorithms for QAP first used by **Kaufman and Broekx in 1978** , these algorithms make use of mixed integer linear programming (MILP) formulations for QAP . Generally, the time needed for these methods is too large, and hence these methods may solve to optimality only very small QAPs. However, heuristics derived from cutting plane approaches produce good suboptimal solutions in early stages of the search. The polyhedral cutting planes or branch and cut algorithms make use of MILP formulations of QAP. The polyhedral cutting plane is not widely used in QAP because of the scarcity of knowledge about QAP polytypes.

6.3 The Dynamic Programming

The idea behind the dynamic programming method is quite simple. In general, to solve a given problem, different parts of the problem "subproblems" need to be solved, and then the solutions of the subproblems should be combined to reach an overall solution. **Christofides and Benavent [22]** used a dynamic programming approach to solve a special case of QAP. Dynamic programming solves a problem of size n by starting from subproblems of size 1,2,…,n-1. After solving subproblems of size k it upgrades the solutions to size k+1. Problems may arise in dynamic programming if the solution to a subproblem or the upgrade procedure cannot be performed in polynomial time.

There have been fewer applications For cutting plane and dynamic programming methods than the Branch and Bound method. As shown before, QAP is NP-hard due to the overwhelming complexity of QAP, most problems with large sizes remain nearly intractable by exact algorithms. Therefore, we use heuristics in order to get solutions with good quality for QAP in a reasonable computational time.

7. HEURISTICS

Because of obvious difficulties experienced in the development of exact solution procedures, a wide variety of heuristic approaches has been developed for QAP. Heuristic algorithms do not give a guarantee of optimality for the best solution obtained. In this context, we consider heuristic techniques as a procedure dedicated to the search of good quality solutions. These approaches can be classified into the following categories: **Construction methods [43]**, **Limited enumeration methods [25], Improvement methods [35], simulated annealing[15], Genetic algorithms[46], Greedy randomized adaptive search procedures [44 ,45], and Ant colonies [23].**(Improvement methods are frequently used in metaheuristics.)

7.1 Construction Methods (CM)

The construction method algorithm is considered one of the oldest heuristics in use **(Buffa, Armour and Vollmann (1964), Muller-Merbach (1970))**. These methods create suboptimal permutations by starting with a partial permutation that is initially empty. The permutation is expanded by repetitive assignments based on set selection criterion until the permutation is complete.

7.2 Limited Enumeration Methods (LEM)

(West (1983), Burkard and Bonniger (1983)) The limited enumeration methods are strongly related to exact methods like branch and bound and cutting planes. The idea behind these algorithms is that a good suboptimal solution may be produced early in an enumerative search. In addition, an optimal solution may be found earlier in the search while the rest of the time is spent on proving the optimality of this solution. There are many ways to limit enumeration of the search space**;** one approach is to impose a time limit. Enumeration stops when the algorithm reaches a time limit or no improvement has been made in a predetermined time interval. These pre-specified parameters can be problem specific. A second option is to decrease the requirement for optimality. For example, if no improvement has been made after a certain pre-specified time interval, then the upper bound is decreased by a certain percentage resulting in deeper cuts in the enumeration tree. Although the optional solution may be cut off, it differs from the obtained solution by the above percentage.

7.3 Improvement Methods (IM)

Improvement methods are the most researched class of heuristic. The most popular improvement methods are the local search and the tabu search [35]. Both methods work by starting with an initial basic feasible solution and then trying to improve it. **The local search** seeks a better solution in the neighborhood of the current solution, terminating when no better solution exists within that neighborhood. **The tabu search (TS) (SkorinKapov (1990), Taillard (1991))** works similarly to the local search. However, **the tabu search** is sometimes more favorable as it was designed to cope with the problem of a heuristic getting trapped at local optima.

7.4 Simulated Annealing (SA)

(Burkard and Rendl (1984) ,Wilhelm and Ward (1987)) This group of heuristics, which is also used for overcoming local optima, receives its name from the physical process that it imitates. This process, called annealing moves high energy particles to lower energy states with the lowering of the temperature, thus cooling a material to a steady state. Initially, in the initial state of the heuristic, the algorithm is lenient and capable of moving to a worse solution. However, with each iteration the algorithm becomes stricter requiring a better solution at each step.

7.5 Genetic Algorithms (GA)

(Fleurent and Ferland (1994), Tate and Smith (1985), Ahuja, Orlin, and Tewari(1998), Drezner (2001), Drezner and Zvi (2003), Yongzhong Wu and Ping Li(2007)) Genetic algorithms receive their name from an intuitive explanation of the manner in which they behave. This explanation based on Darwin's theory of natural selection. Genetic algorithms store a group of solutions and then work to replace these solutions with better ones based on some fitness criterion, usually the objective function value. Genetic algorithms are parallel and helpful when applied in such an environment.

7.6 Greedy Randomized Adaptive Search Procedures (GRASP)

GRASP is a relatively new heuristic used for solving combinatorial optimization problems. At each iteration, a solution is computed. The final solution is taken as the one that is the best after all GRASP iterations are performed. The GRASP was first applied to QAP by **Li, Pardalos, and Resende in 1994 [44]**. They applied the GRASP to 88 instances of QAP, finding the best known solution in almost every case, and improved solutions for a few instances.

7.7 Ant Colonies (AC)

This section gives you a glimpse of Swarm intelligence algorithms by discuss Ant Colonies as example. The ants based algorithms have been introduced with **Maniezzo, Colorni, and Dorigo [24].**They are based on the principle that using very simple communication mechanisms, an ant group is able to find the shortest path between any two points. During their trips**,** a chemical trail (pheromone) is left on the ground. The role of this trail is guiding the other ants towards the target point. For one ant, the path is selected according to the quantity of pheromone. Moreover, this chemical substance has a decreasing action over time, and the quantity left by one ant relies on the amount of food found and the number of ants using this trail. Applied on QAP, The AC is based on a hybridization of the ant system with a local search method, each ant being associated with an integer permutation. Modifications based on the pheromone trail are then applied to each permutation. The solutions (ants) found so far are then optimized using a local search method, update of the pheromone trail simulates the evaporation and takes into account the solutions produced in the search strategy.

7.8 Metaheuristics

Metaheuristic is more general techniques have appeared to adapt with the problem structure. Several of these techniques are based on some form of simulation of a natural behavior studied within another field of knowledge. With the adventages of metaheuristics, QAP research received new and increased interest which is reflected on QAP publications. **Fig.2.** shows the distribution of publications, categorized by solution techniques that were classified in this work as Heuristic Methods, Exact Methods and Metaheuristics.

Fig.3. shows the distribution of metaheuristic solution techniques used to QAP. In this figure, we have scatter search (SS), variable neighborhood search (VNS), greedy randomized adaptive search procedure (GRASP), genetic algorithm (GA), neural networks and others (NNO), tabu search (TS), ant colony(AC), simulated annealing (SA) and hybrid algorithms (HA). Hybrid procedures that result from different metaheuristic compositions are the most utilized solution procedure. However, when we look for a comparison among pure metaheuristics, the procedures based on more traditional simulated annealing and more recently defined ant colony are the most popular.

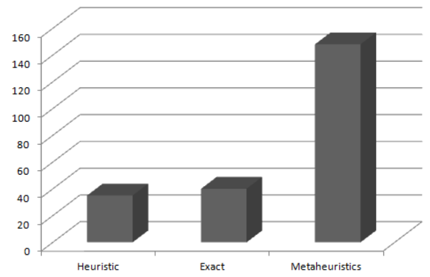


Fig. 2. Publications: solution techniques Metaheuristic

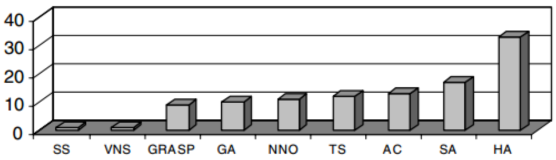


Fig. 3. Publications metaheuristics used to the QAP.

8. APPLICATION

There are enormous numbers of practical applications that can be modeled as QAPs. **Koopmans and Beckmann (1957)** first proposed QAP as a mathematical model regarding the economic activities. Since then , it has appeared in several practical applications: **Steinberg (1961)** used QAP to minimize the number of connections between components in a backboard wiring, **Heffley (1972, 1980)** applied it to economic problems, **Francis and White (1974)** developed a decision framework for assigning a new facility (police posts, supermarkets, schools) for serving a given set of clients, **Geoffrion and Graves (1976)** concentrated on scheduling problems, **Pollatschek et al. (1976)** mentioned QAP to define the best design for typewriter keyboards and control panels, **Krarup and Pruzan (1978)** applied it to archeology**. Hubert (1987)** applied it in statistical analysis, **Forsberg et al. (1994)**used it in the reaction chemistry analysis and **Brusco and Stahl (2000) used it** in numerical analysis. **Though**, the facilities layout problem is the most popular application for QAP: **Dickey and Hopkins (1972)** applied QAP to the assignment of buildings in a University campus, **Elshafei (1977)** in a hospital planning and **Bos (1993)** in a problem related to forest parks. **Benjaafar (2002)** introduced a formulation of the facility layout design problem for minimizing work-in-process (WIP). **Ben-David and Malah (2005)** looked into a special case of QAP called index assignment in order to minimize channel errors in vector-quantization. Vector-quantization is used when mapping images or speech to digital signals. A similar mapping problem is also found when configuring the layout of microarrays, which is a problem in bioinformatics presented as a QAP by **de Carvalho Jr. and Rahmann (2006)**. A more modern application of the same problem is the design of keyboards on touch screen devices **(Dell'Amico et al., 2009)**. The main difference in this approach is that on a touch screen only one finger is used, and the letters can be placed anywhere on the screen instead of in a rectangle as with normal keyboards.

9. SURVEY

Extensive research has been done on the quadratic assignment problem**.** QAP publications have the categories: applications, theory (i.e., formulations, complexity studies and lower bounding techniques) and algorithms. **Fig.4.** shows the distribution of QAP publications with respect to the categories. We observe that an explosion of interest in algorithm development.

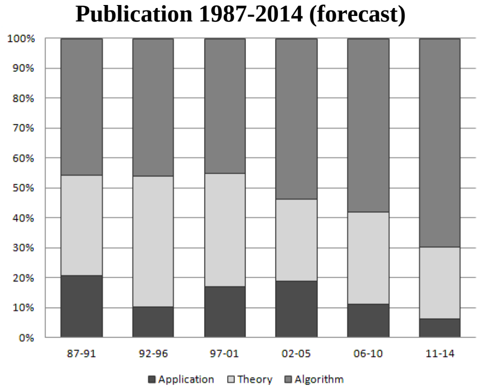


Fig.4. Publications classification according to their contents

In **Table 1.** We will give some PhD thesis, dissertations and articles about development on QAP:

Table 1. A Survey on Quadratic Assignment Problem

AUTHOR	TYPE	TITLE	UNIVERSITY	OBJECTIVE
T.A. Johnson(1992)	PhD thesis	New linear programming-based solution procedures for the quadratic assignment problem	Clemson University, Clemson, USA,	Introduce new solution procedures based on linear programming. The linear formulation derived in this thesis theoretically dominates alternate linear formulations for QAP.
T. Mautor(1992)	PhD thesis	Contribution to solving problems implanation: sequential and parallel algorithms for the quadratic assignment	Pierre et Marie Curie university, France	Focuse on parallel implementations and exploits the metric structure of the Nugent instances to reduce the branching tree considerably.
Y. Li(1992)	PhD thesis	Heuristic and exact algorithms for the quadratic assignment problem.	The Pennsylvania State University, USA	Introduce beside other ideas lower bounding techniques based on reductions, GRASP and a problem generator for QAP.
F.Malucelli(1993)	PhD thesis	Quadratic assignment problems: solution methods and applications.	University of Pisa, Pisa, Italy,	Propose a lower bounding technique for QAP based on a reformulation scheme and implemented it in a branch and bound code. Some new applications of QAP in the field of transportation were also presented.
E. Cela(1995)	PhD thesis	The quadratic assignment problem: special cases and relatives.	Graz University of Technology, Graz, Austria	Investigate the computational complexity of specially structured quadratic assignment problems, and consider a generalization of QAP, the so called biquadratic assignment problem.

S.E. Karisch(1995)	PhD thesis	Nonlinear approaches for the quadratic assignment and graph partition problems	Graz University of Technology, Graz, Austria	Present nonlinear approaches for QAP. These provide the currently strongest lower bounds for problems instances whose distance matrix contains distances of a rectangular grid and for smaller sized general problems.
M. Rijal(1995)	PhD thesis	Scheduling, design and assignment problems with quadratic costs	New York University, New York, USA	Investigate structural properties of the QAP polytope. The starting point is the quadric Boolean polytope.
Q. Zhao(1996)	PhD thesis	Semidefinite programming for assignment and partitioning problems	University of Waterloo, Waterloo, Canada	Investigate semidefinite programming approaches for the QAP. Tight relaxations and bounds are obtained by exploiting the geometrical structure of the convex hull of permutation matrices.
A. Bouras(1996)	PhD Thesis	quadratic assignment problem of small rank: models, complexity, and applications	Joseph Fourier university, Grenoble, France	Consider special cases of QAP where the coefficient matrices have a low rank, especially rank one, and propose a heuristic based on matrix approximations by matrices with low rank.
V. Kaibel(1997)	PhD thesis	Polyhedral combinatorics of the quadratic assignment problem	University of Cologne, Cologne, Germany	Investigate the QAP polytope and derived the first large class of facet defining inequalities for these polytopes, the box inequalities..
Gunes Erdogan (2006)	PhD thesis	Quadratic assignment problem: linearizations and polynomial time solvable cases	BILKENT UNIVERSITY, Turkey	Focus on "flow-based" formulations, strengthen the formulations with valid inequalities, and report computational experience with a branch-and-cut algorithm.
Thomas Stützle(2006)	Article	Iterated local search for the quadratic assignment problem	Darmstadt, Germany	Present and analyze the application of ILS to the quadratic assignment problem (QAP).
Yi-Rong Zhu(2007)	dissertation	RECENT ADVANCES AND CHALLENGES IN QUADRATIC ASSIGNMENTAND RELATED PROBLEMS	the University of Pennsylvania	Contribute to the theoretical, algorithmic and applicable understanding of quadratic assignment and its related problems.
Tao Huang(2008)	dissertation	Continuous Optimization Methods for the Quadratic Assignment Problem	the University of North Carolina at Chapel Hill	Study continuous optimization techniques as they are applied in nonlinear 0-1 programming. Specically, the methods of relaxation with a penalty function have been carefully investigated.

Franklin Djeumou Fomeni(2011)	dissertation	New Solution Approaches for the Quadratic Assignment Problem	University of the Witwatersrand	Propose two new solution approaches to the QAP, namely, a Branch-and-Bound method and a discrete dynamic convexized method.
Francesco Puglierin(2012)	Master thesis	A Bandit-Inspired Memetic Algorithm for Quadratic Assignment Problems	University of Utrecht	Propose a new metaheuristic for combinatorial optimization,with focus on the Quadratic Assignment Problem as the hard-problem of choice, a choice that is refl ected in the name of the method, BIMA-QAP.
Wu et al. (2012)	Article	Global optimality conditions and optimization methods for quadratic assignment problems	School of Science, Information Technology and Engineering, University of Ballarat , Victoria, Australia Department of Mathematics, Shanghai University,China	Discuss some global optimality conditions for general quadratic {0, 1} programming problems with linear equality constraints, and then some global optimality conditions for quadratic assignment problems (QAP) are presented.
Duman et al. (2012)	Article	Migrating Birds Optimization: A new metaheuristic approach and its performance on quadratic assignment problem	Ozyegin University, Department of Industrial Engineering,Istanbul, Turkey Dogus University, Department of Computer Engineering, Istanbul, Turkey Marmara University, Department of Computer Engineering, Istanbul, Turkey	Propose a new nature inspired metaheuristic approach based on the V flight formation of the migrating birds which is proven to be an effective formation in energy saving. Its performance is tested on quadratic assignment problem instances arising from a real life problem and very good results are obtained.
Benlic et al.(2013)	Article	Breakout local search for the quadratic assignment problem	University of Angers,france	Present breakout local search (BLS) for solving QAP. BLS explores the search space by a joint use of local search and adaptive perturbation strategies.

Klerk et al.(2014)	Article	Symmetry in RLT-type relaxations for the quadratic assignment and standard quadratic optimization problems	Department of Econometrics and OR, Tilburg University, The Netherlands Centrum Wiskunde & Informatica (CWI), Amsterdam, The Netherlands	Show that, in the presence of suitable algebraic symmetry in the original problem data, it is sometimes possible to compute level two RLT bounds with additional linear matrix inequality constraints.
Axel Nyberg (2014)	PhD Thesis	Some Reformulations for the Quadratic Assignment Problem	Department of Chemical Engineering Abo Akademi University , Finland	Reformulate the Quadratic Assignment Problem for optimization

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11. CONCLUSION

As discussed before, QAP can mimic enormous numbers of very important practical applications that can be modeled as QAPs. So, in this paper we gave fundamentals of QAP and some solving techniques, highlights some efforts on solving QAP. This paper can be directed towards help new researchers to improve QAP development results as well as modeling real world applications by getting and all over view about QAP and development s in this field of optimization.

12. REFERENCES

[1] E. M. Loiola, N. M.de Abreu, P.O. Boaventura-Netto, P.Hahn, T. Querido. A survey for the quadratic assignment problem. European journal of operational research, 27 December 2005.

[2] Quadratic Assignment Problem.(2012). retrieved May 2014 , from: http://www.neos-guide.org/content/quadratic-assignment-problem

[3] QAPLIB - A Quadratic Assignment Problem Library.(2012). Retrieved May 2014, from: http://www.seas.upenn.edu/qaplib

[4] P. Ji, Y. Wu, H. Liu. A Solution Method for the Quadratic Assignment Problem (QAP), Department of Industrial and Systems Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong ,China, August 2006.

[5] Quadratic assignment problem. (2013). Retrieved May 2014, from: http://en.wikipedia.org/wiki/Quadratic_assignment_problem

[6] I. Kudelska . Methods of using the quadratic assignment problem solution, LogForum 8, Issue 3, 2012.

[7] Assignment problem (2014). Retrieved May 2014, from: http://en.wikipedia.org/wiki/Assignment_problem

[8] T. Stützle. Iterated local search for the quadratic assignment problem, European Journal of Operational Research , Volume 174, Issue 3, 1 November 2006.

[9] Z. Wu, Y. Yang, F. Bai, J. Tian. Global optimality conditions and optimization methods for quadratic assignment problems , Volume 218, Issue 11, 5 February 2012.

[10] E. Duman, M. Uysal, A. F. Alkaya. Migrating Birds Optimization: A new metaheuristic approach and its performance on quadratic assignment problem, Information Sciences, Volume 217, 25 December 2012.

[11] U. Benlic, J.Hao. Breakout local search for the quadratic assignment problem, Applied Mathematics and Computation, Volume 219, Issue 9, 1 January 2013.

[12] E. .Klerk, M.Nagy, R. Sotirov, U. Truetsch. Symmetry in RLT-type relaxations for the quadratic assignment and standard quadratic optimization problems, European Journal of Operational Research, Volume 233, Issue 3, 16 March 2014.

[13] Y. Xia, and Y.Yuan. A new linearization method for quadratic assignment problems, Institute of Computational Mathematics and Scientific/Engineering Computing,The Academy of Mathematics and Systems Sciences,Chinese Academy of Sciences, Beijing, China.

[14] R. E. Burkard , E. Cela ,P. M. Pardalos , and L.S. Pitsoulis. The Quadratic Assignment Problem, Handbook of combinatorial optimization, 1998.

[15] C. W. Commander. A Survey of the Quadratic Assignment Problem, with Applications, Morehead Electronic Journal of Applicable Mathematics, Issue 4,1 March 2005.

[16] N.W. Brixius and K.M. Anstreicher. The Steinberg wiring problem. Working Paper, The University of Iowa, October 2001.

[17] A. N. Elshafei. Hospital layout as a quadratic assignment problem, Operational Research Quarterly, 1977.

[18] Juillet. comparison of iterative searches for the quadratic assignment problem, Montréal university, Canada,1994.

[19] E. M. Loiola, N. M. de Abreu, P. O. Boaventura-Netto, P. Hahn, and T. Querido. A survey for the quadratic

assignment problem, European Journal of Operational Research, 27 December 2005.

[20] F. D. Fomeni. New Solution Approaches for the Quadratic Assignment Problem, University of the Witwatersrand, Johannesburg, September 2011.

[21] R. K. Ahuja, K. C. Jha, J. B. Orlin and D. Sharma. Very large-scale neighborhood search for the quadratic assignment problem, Working Paper, MIT Sloan School of Management, July 2002.

[22] N. Christofides and E. Benavent. An exact algorithm for the quadratic assignment problem on a tree, Operation Research, 1989.

[23] E.-G. Talbi , O. Roux, C. Fonlupt, and D. Robillard. Parallel Ant Colonies for the quadratic assignment problem. Future Generation Computer Systems , Volume 17 ,France, 2001.

[24] M. Dorigo. Optimization, learning and natural algorithms, Ph.D. Thesis, Politecnico di Milano, Italy, 1992.

[25] A. kaveh. Algorithms and theoretical topics on selected combinatorial optimization problems. Simon fraser university,2006.

[26] T.C. Koopmans and M. Beckman. Assignment Problems and the Location of Economic Activities. Econometric ,1957.

[27] Hanan, M. and J.M. Kurtzberg . A Review of the Placement and Quadratic Assignment Problems, 1972.

[28] Sahni, S. and T. Gonzalez.P-Complete Aproximation Problems. Journal of the ACM ,1976.

[29] P.C. Gilmore . Optimal and Suboptimal Algorithms for the Quadratic Assignment Problem. SIAM Journal on Applied Mathematics , Volume 10, 1962.

[30] E.L. Lawler. The Quadratic Assignment Problem, Management Science, Volume 9, 1963.

[31] M. S. Hussin and T.Stutzle. Hierarchical Iterated Local Search for the Quadratic Assignment Problem, Institut de Recherches Interdisciplinaires, et de D´eveloppements en Intelligence Artificielle, Universite Libre de Bruxelles, Bruxelles, Belgium, June 2009.

[32] F. Rendl and H. Wolkowicz . Applications of Parametric Programming and Eigenvalue Maximization to the Quadratic Assignment Problem. Mathematical Programming , Volume 53, 1992.

[33] P.. M. Zvidrezner , and E. D. Taillard. Recent Advances for the Quadratic Assignment Problem with Special Emphasis on Instances that are Difficult for Meta-Heuristic Methods. Annals of Operations Research , Volume 139, 2005.

[34] P. Carraresi, and F. Malucelli. A New Lower Bound for the Quadratic Assignment Problem. Tech Report TR-34/92, Universita di Pisa, 1992.

[35] M. Dell'Amico, J.Carlos D´ıaz , M. Iori and R. Montanari . The single-finger keyboard layout problem. University of Modena and Reggio Emilia, Italy, November 2008.

[36] P.M. Pardalos, F. Rendl, and H. Wolkowicz. The Quadratic Assignment Problem: A Survey and Recent Developments, in Quadratic Assignment and Related Problems, DIMACS Series in Discrete Mathematics and Theoretical Computer Science, Volume 16, 1994.

[37] P. Hahn, T. Grant, and N. Hall. A Branch-and-Bound Algorithm for the Quadratic Assignment Problem Based on the Hungarian Method, European Journal of Operational Research, 1996.

[38] M. S. Bazaraa and H. D. Sherali. On the Use of Exact and Heuristic Cutting Plane Methods for the Quadratic Assignment Problem . The Journal of the Operational Research Society, Volume 33, Issue 11, Novmber 1982.

[39] G. Erdoğan. quadratic assignment problem:linearizations and polynomial time solvable cases. October, 2006.

[40] R. Bellman, An introduction to the theory of dynamic programming, 1953.

[41] Francesco. A Bandit-Inspired Memetic Algorithm for Quadratic Assignment Problems. University of Utrecht. August 2012.

[42] E. Cela. Assignment Problems .Technical University Graz, Institute of Mathematics,Steyrergasse 30, A-8010 Graz, Austria.

[43] G.C. Armour, and E.S. Buffa. Heuristic Algorithm and Simulation Approach to Relative Location of Facilities. Management Science, Volume 9, 1963.

[44] Y.Li, P. M. Pardalos, and M. G.C. Resende. A Greedy Randomized Adaptive Search Procedure for the Quadratic Assignment Problem. DIMACS Series in Discrete Mathematics and Theoretical Computer Science.

[45] B. Vahedinori, M. Kianpour and P. Fattahi. Using Greedy Randomize Adaptive Search Procedure for solve the Quadratic Assignment Problem, Volume 22, November 2011.

[46] H. M. Nehi, and S. Gelareh. A Survey of Meta-Heuristic Solution Methods for the Quadratic Assignment Problem. Applied Mathematical Sciences, Volume 1, Issue 46, 2007.