On Developing a Decision-Making Tool for General Applications to Computer Vision

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ABSTRACT

The general applications to computer vision are full of problems expressed in terms of mathematical energy optimization. In this context developing a reliable optimal design process for the non-uniform rational b-spline (NURBS) curves and surfaces which in fact has a wide and foundational application in image processing, computer aided geometry design (CAGD), computer aided design (CAD) and computer animation, is the focus of this work. Yet the optimal design and parameter tuning of the NURBS is a highly non-linear and complicated multiobjective optimization (MOO) problem. The complexity of the problem is even increased when the criteria of product beauty is included to the design process. In this article for an optimal configuration, the operating design parameters are tuned within the proposed interactive multicriteria decision making (MCDM) environment where the decision maker (DM) is included into the process. Along with presenting the NURBS's optimal design problem the drawbacks to the former approaches are reviewed, and the applicability of the proposed decision-making tool in the general applications to computer vision is described.

Keywords:

Energy optimization, computer vision, interactive multicriteria decision making, computer vision, reactive search optimization, multiobjective optimization

1. INTRODUCTION

The general applications to computer vision are full of problems expressed in terms of mathematical energy optimization [1]. Problems as such are often complicated, highly non-linear and multiobjective in nature. In this context the optimal design of the NURBS curves and surfaces [2,60] is considered as an interesting case study as it has a wide application in computer vision [48,51], as well as other fileds of industry [7,9,12,13,16]. The applications include a wide range of problems from medical image processing [18,76], CAGD [57] and CAD [73] to computer animation [62] in which could also be further seen in [10,38,44,49,52,63,64,66]. Yet the optimal design and parameters tuning of the NURBS is a highly non-linear and complicated MOO problem [2,29,30,56]. In fact the mathematical modeling of the NURBS optimal design problem results in a MOO problem which cannot be handled as such by traditional single objective optimization algorithms [2]. Furthermore the complexity of the problem is even increased when the criteria of product beauty is included to the design process. In this article the optimization process of NURBS including four conflicting and highly non-liner design objectives is of the particular interest.

Applied optimization over the past few years have dramatically advanced, particularly with the availability of efficient MOO algorithms e.g. [34,40] which facilitates a DM to consider more than one conflicting goals at the time. In a MCDM problem [19,24] for the reason of decision-making on the optimality and further selecting the preferred solution with the aid of the MOO algorithms many conflicting objectives are traded off simultaneously. To doing so numerous biology-inspired metaphors e.g. evolutionary algorithms (EA)s with in fact a very limited learning capabilities, have been widely utilized so far [2,6,7,27,28]. Yet in this article for an optimal configuration, alternatively the operating design parameters are tuned in an interactive MCDM environment [14,15,37,39,40,42,54], which in fact is inpired by [24], where the DM is included into the process. By involving the DM interactively in the loop intelligent expertise is loaded to the algorithm leading to increasing the learning capabilities. Here it is assumed that integration of machine learning techniques into the search heuristics along with utilizing the advanced visualization tools would automate the algorithem selection, adaptation and integration for approaching a robust solution [15,34,53].

1.1 Statement of the Problem

A tensor product NURBS is defined as; $S(u,v) = \sum_{i=0}^{n-1}\sum_{j=0}^{m-1} \mathbf{P}_{\mathbf{i,j}}\, R_{\boldsymbol{i,j}}(s,t)$, where $\boldsymbol{P_{i,j}}$ are control points of the surface with the orders and the numbers of n and m. $R_{\boldsymbol{i,j}}(s,t)$ are the NURBS basis function, depended on the design variables including weights, $\boldsymbol{w}$, the knot vectors, $\boldsymbol{u}$ & $\boldsymbol{v}$, the d_u & d_v orders of the surface and the parameterization, s & t. Handling the parameterization, knot vectors, interpolation and NURBS weights is further described in [2,5,57,60,68]. Tuning NURBS weights and knot vector all together dramatically increases the number of DOF which is proportional to $n * m$.

According to the input points, $\boldsymbol{Q}_{i,j}$, and the design variables, the control points, $\boldsymbol{P_{i,j}}$, via utilizing the linear least squares fitting, are calculated and the surface is created [8].

Let $\boldsymbol{M}$ be the collocation matrix used for surface fitting; $\boldsymbol{Q}_x$, $\boldsymbol{Q}_y$, $\boldsymbol{Q}_z$ are the coordinates of Q, the data to be fitted; $diag(x)$ a diagonal matrix whose entries are the vector x.

$$t = M * w,$$

$$X = diag(Q_x) \quad Y = diag(Q_y) \quad Z = diag(Q_z)$$

$$v_x = X * t \quad v_y = Y * t \quad v_z = Z * t$$

The position of the surface's control points $\boldsymbol{p}_x, \boldsymbol{p}_y, \boldsymbol{p}_z$ are given by least solution of the following equations: $d_x = M * v_x \quad d_y = M * v_y \quad d_z = M * v_z$

1.2 Optimization Objectives

The goal of the optimization process is to produce a set of NURBS surfaces which approximates a set of input points, $\boldsymbol{Q} = \boldsymbol{Q}_{0,0}, \dots, \boldsymbol{Q}_{N-1,M-1} \in \mathbb{R}^d$, $d = 2, 3$, and are optimal with respect to the specified design objectives. Once the surface is created the quality of it could be considered by evaluating a set of specified design objectives, i.e. $O_1(S(s,t))$, . . ., $O_k(S(s,t)$. The optimization process includes four conflicting and highly non-liner design objectives described in the following.

Approximation Error, O_1, the distance between the surface and the points $\boldsymbol{Q}$ measured at the parametrization points $\boldsymbol{s_i}, \boldsymbol{t_i}$, is often subjected to minimization;

$$O_1 = \min(\sum_{i=0}^{n-1} \sum_{j=0}^{m-1} \left\| \boldsymbol{S}(s_i, t_i) - \boldsymbol{Q}_{i,j} \right\|^2),$$

under L_2 norm,

$O_1 = \max(\| \boldsymbol{S}(s_i, t_i) - \boldsymbol{Q}_{i,j} \|)$, $i = 0, \dots, n-1; j = 0, \dots, m-1$, under L_∞ norm.

Surface Area, O_2, in conflict with approximation error, controls artifacts due to over-fitting; $O_2 = \int_0^1 \int_0^1 \left\| \frac{dS}{ds} \times \frac{dS}{dt} \right\| dsdt$.

Surface Elastic Energy, O_3, as an other conflicting objective is a highly non-linear term;

$O_3 = \int_0^1 \int_0^1 \| k_{min}^2 + k_{max}^2 \| dA$, where A is the surface area.

1.3 Revision

As mentioned above the mathematical modeling of the NURBS curves and surfaces design problem results in a MOO problem which cannot be effectively handled as such by traditional single objective optimization algorithms. Considering the problem with conjugate gradient and Newton-based approaches, the optimization process is divided into several phases and each functional is optimized separately [3,4,5]. In the approaches as such the MOO problem is solved via a single objective optimization algorithm. However the results obtained reported to be not promising [2,7]. The detailed description of the problem, applications and previous approaches are available in [2,16,38], where the use of MOO algorithms enhances the design process by enabling optimization of several design objectives at once. The general form of a MOO problem [14,24,27], is stated as; minimize $\mathbf{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), \dots, f_m(\boldsymbol{x})\}$, subjected to $\boldsymbol{x} \in \Omega$, where $\mathbf{x} \in \mathbb{R}^n$ is a vector of n decision variables; $\mathbf{x} \subset \mathbb{R}^n$ is the feasible region and is specified as a set of constraints on the decision variables; $\mathbf{f} : \Omega \to \mathbb{R}^m$ is made of m objective functions subjected to be minimization. Objective vectors are images of decision vectors written as $\mathbf{z} = \mathbf{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), \dots, f_m(\boldsymbol{x})\}$. Further an objective vector is considered optimal if none of its components can be improved without worsening at least one of the others. An objective vector $\mathbf{z}$ is said to dominate $\mathbf{z}'$, denoted as $\mathbf{z} \prec \mathbf{z}'$, if $z_k \leq z_k'$ for all k and there exist at least one h that $z_h \leq z_h'$. A point $\hat{\boldsymbol{x}}$ is Pareto optimal if there is no other $\boldsymbol{x} \in \Omega$ such that $\mathbf{f}(\boldsymbol{x})$ dominates $\mathbf{f}(\hat{\boldsymbol{x}})$. The set of Pareto optimal points is called Pareto set (PS). And the corresponding set of Pareto optimal objective vectors is called Pareto front (PF). Considering solving MOO problems EAs are among the most popular *a posteriori* methods for generating Pareto optimal points of a MOO problem. The EAs of MOO for solving MCDM problems have been around for up to two decades now. EAs are natural choices for MOO since at each step the algorithm keeps a population, which is a set of solutions instead of a single, optimal, solution. Because of the robustness and efficient handling of highly non-linear objective functions and constrains the use of EAs in geometrical problem has proved to be a powerful technique [6,7]. In fact evolutionary multiobjective optimization algorithms (EMOA) [6,27] are well suited to search for a set of PS to be forwarded to the DM while aiming at building a set of points near the PF. Afterward, a single preferred solution is chosen from the obtained set by using a MCDM procedure [28]. In this way EMOA application helps a DM to analyze different trade-offs before choosing the final one. However the DM has to go through analyzing many different solutions to be able to confidently make the final decision. This is done by considering too many possible solutions within the multiobjective and multicriteria trade-offs [2]. Although the EMOA may employ plenty of complications in usage, efficiency, robustness, and decision-making on the final solution when the number of objectives increases. In fact in a number of case studies e.g. [11,16,38,56,58], by increasing the number of objectives, EMOAs have been reported ineffective. The problem of MOO of curves and surfaces [2,56] would be indeed a good example for such ineffective attempt and increasing complexity. Previously an evolutionary MOO algorithm [2,16] was used to handle this case. In this approach due to the robustness and efficiency of the evolutionary algorithms the problem was well modeled. Nevertheless the approaches to solving MOO of the NURBS curves and surfaces problems whether *a priori* or *a posteriori*, would involve plenty of various complications. The reason is that the proportion of PF in a set grows very rapidly with the dimension $\boldsymbol{m}$.

Yet for an ideal and seamless approach to solving the MOO problems a reliable multicriteria decision making environment builds its bases on software tools used for a large number of applications in computer vision from modeling activities, optimization and decision-making tasks, to performance's simulation and beauty evaluations. Furthermore the addition of new tools is intended to extend the support to the creative part of the design process and also the capability to deal with big data. This support allows the DM to improve the performance of their concepts, allowing computers to take part on the generation of variants, and on the judgment, by true modeling of these variants. Integration of data mining, modeling, learning, and interactive decision-making are all parts of a reliable software tool that can nurture the knowledge of designers to generate new solutions, based on many separate ideas leading to the new design concepts.

The task of MCDM in the proposed decision-making environment unlike the former MOO approaches [26,27], where the workflow is divided into two different parts of optimization and decision-making, is seen as a single task. Although both processes of optimization, to discover conflicting design trad-offs, and decision-making, to choose a single preferred solution among them, are considered as two joint tasks, yet they have been previously treated as a couple of independent activities. For instance EMOA [6] have mostly concentrated on the optimization aspects, developing efficient methodologies of finding a PS. However finding a set of trade-off optimal solutions is just half the process of optimal design in the multicriteria decision making environments. This has been the reason why EMOA researchers were looking to find ways to efficiently integrate both optimization and decision making tasks in a convenient way. The efficient MOO algorithms facilitate the DMs to consider multiple and conflicting goals of a MCDM problem simultaneously. Some examples of such algorithms and potential applications could be found in [9,10,11,12,13]. Within the known approaches to solving complicated MCDM problems there are different ideologies and considerations in which any decision-making task would find a fine balance among them.

In MCDM algorithms [19,31,47,55,61,70] the single optimal solution is chosen by collecting the DM's preferences where MOO and decision making tasks are combined for obtaining a point by point search approach. In addition in MOO and decision-making, the final obtained solution must be as close to the true optimal solution as possible and the solution must satisfy the preference information. Towards such a task, an interactive tool to consider decision preferences is essential. This fact has motivated novel researches to properly figure out the important task of integration between MOO and MCDM [6,27,28]. Naturally in MOO, interactions with the DM can come either during the optimization process, such as in the interactive EAs optimization [21,75], or during the decision-making process [22].

Alternatively a MCDM procedure could be integrated with an EMOAs to find the preferred PF where the search is concentrated on the most important region of the PF. This would let the optimization task to evaluate the preferences of the DM interactively. The approaches to interactive evolutionary algorithms are numerous. The researches in various problem domains in which an EA is carried out by the involvement of the DM reviewed by Takagi [23]. Additionaly a summary can be found in the text by Miettinen [19]. Some of the popular approaches are interactive surrogate worth trade-off method [20], the reference point method [20] and the NIMBUS approach [24]. All procedures require a DM to provide the preferences. A search workflow is then used to find the optimum of the objective task. This procedure is repeated many times until the DM is satisfied with the obtained final solution. For instance in [27] an EMOA procedure is applied to a complicated design problem and then a interactive methodology is employed to choose a single solution. In [28], EMOA is combined with MCDM procedures, and an interactive procedure is suggested where the EMOA methodologies are combined with a certain and efficient MCDM technique. The work later was extended by involving more MCDM tools and integrations with further software packages such as MATLAB, for providing better working on more real-life study case.

2. DRAWBACKS to SOLVING the MOO PROBLEMS UTILIZING EAs

Applied optimization over the years have dramaticaly advanced, particularly with the availability of efficient MOO algorithms which facilitate a DM to consider multiple conflicting and nonlinear goal at the time. Optimization methods for computer vision, based on evolutionary design, currently have been used to obtain optimal geometric solutions [2,7]. They are evolving to configurations that minimize the cost of trial and error and perform far beyond the abilities of the most skilled DMs. Although in developing a MCDM environment relying only on evolutionary design components, in today's ever-increasing complexity when often numerous design objevtives involved, is not sufficient where in fact most studies in the past concentrated in finding the optimum corresponding to a single goal. In a reliable decision-making environment the procedure searches through possible feasible solutions and at the end identifies the best solution. In fact the reality of applied DM has to consider plenty of priorities and drawbacks to both interactive and non-interactive approaches.

Although the mathematical representative set of the DM model is often created however presenting a human DM with numerous representative solutions on a multi-dimensional PF is way complicated. This is because the typical DM cannot deal with more than a very limited number of information items at a time [43]. Therefore an improved decision procedures should be developed according to human memory and his data processing capabilities. In this context utilizing decision-support tools have been reported effective for the reason of reducing the design space in some cases [17,35,36,46,50,67,59,69,71,47,77]. Neverthelss often DMs cannot formulate their objectives and preferences at the beginning. Instead they would rather learn on the job. This is already recognized in the MOO formulation, where a combination of the individual objectives into a single preference function is not executed. Considering the problem in [2] the DM is not clear about the preference function. This uncertainty is even more increased when the objectives such as beauty involved. This fact would employ lots of uncertainty and inconsistency.

Consequently interactive approaches try to overcome some of these difficulties by keeping the user in the loop of the optimization process and progressively focusing on the most relevant areas of the PF directed by DM. This is done when the fitness function is replaced by a human user. However most DMs are typically more confident in judging and comparing than in explaining. They would rather answer simple questions and qualitative judgments to quantitative evaluations. In fact the identified number of questions that has to be asked from the DM a crucial performance indicator of interactive methods. This would demand for selecting appropriate questions, for building approximated models which could reduce bothering the DM.

The above facts, as also mentioned in [34], and later in [39] demand a shift from building a set of PF, to the interactive construction of a sequence of solutions, so called brain-computer optimization [40], where the DM is the learning component in the optimization loop, a component characterized by limited rationality and advanced question-answering capabilities. This has been the reason for the systematic use of machine learning techniques for online learning schemes in optimization processes available in the novel optimization software architectures [42].

3. BRAIN-COMPUTER OPTIMIZATION (BCO) APPROACH TO STOCHASTIC LOCAL SEARCH

As Battiti et al. [15,39] also clearly state, the aim of stochastic local search is to find the minimum of the combinatorial optimization function f, on a set of discrete possible input values X. To effectively and interactively doing so the focus in [60] devoted to a local search, hinting at reactive search optimization (RSO) with internal self-tuning mechanisms, and BCO i.e. a DM in the interactive problem-solving loop. Accordingly in this context the basic problem-solving strategy would start from an initial tentative solution modifying the optimization function. According to [15,41,42] the local search starts from an acceptable configuration $X(0)$ and builds a search trajectory $X^{(0)}, \ldots, X^{(t+1)}$. Where X is the search space and $X^{(t)}$ is the current solution at iteration t, time. $N(X^{(t)})$ then would be the neighborhood of point $X^{(t)}$,obtained by applying a set of basic moves $\mu_0, \mu_1, \ldots, \mu_M$ to the configuration of $N(X^{(t)}) = \{X \in x$ Such that $X = \mu_i(X^{(t)}), i = 0, \ldots, M\}$. If the search space is given by binary strings with a given length $L: X = \{0,1\}^L$, the moves can be those changing the individual bits, and therefore L is equal to the string length M. The accuracy of the achieved point is a point in the neighborhood with a lower value of f to be minimized. The search then would stop if the configuration is a local minimizer [14].

Y $\leftarrow$ IMPROVING-NEIGHBOR ($N(X^{(t)})$)

$$X^{(t+1)} = \begin{cases} Y & \text{if } f(Y) < X^{(t)} \\ X^{(t)} & \text{otherwise (search stops)} \end{cases} \quad [15]$$

Here the local search works very effectively. As the improving-neighbor returns an improving element in the neighborhood. This is manily because most combinatorial optimization problems have a very rich internal structure relating the configuration X and the f value [5]. In the neighborhood the vector containing the partial derivatives is the gradient, and the change of f after a small displacement is approximated by the scalar product between the gradient and the displacement.

3.1 Learning Component; DM in the Loop

In problem-solving methods of stochastic local search, proposed in [40], where the free parameters are tuned through a feedback loop, the user is considered as a crucial learning component in which different options are developed and tested until acceptable results are obtained. As explained in [14,15] by inserting the machine learning the human intervention is decreased by transferring intelligent expertise into the algorithm itself. Yet in order to optimize the outcome setting the parameters and observing the outcome, a simple loop is performed where the parameters in an intelligent manner changed until a suitable solution is identified. Additionaly to operate efficiently, RSO uses memory and intelligence, to recognize ways to improve solutions in a directed and focused manner.

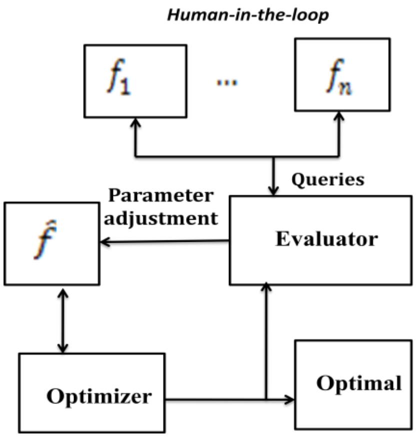

Fig 1: Schematic flowchart of the proposed optimization process; human-in-the-loop [34,39,40].

In the RSO approach of problem solving the brain-computer interaction is simplified. This is done via learning-optimizing process which is basically the insertion of the machine learning component into the solution algorithm. In fact the strengths of RSO are associated to the brain charactristics i.e. learning from the past experience, learning on the job, rapid analysis of alternatives, ability to cope with incomplete information, quick adaptation to new situations and events [14,15]. Moreover the term of intelligent optimization in RSO refers to the online and offline schemes based on the use of memory, adaptation, incremental development of models, experimental algorithmics applied to optimization, intelligent tuning and design of heuristics. In this context with the aid of advanced visualization tools implemented within the software architecture packages [42] the integration of visualization and automated problem solving and optimization would be the centere of attention.

3.2 RSO and Visualization Tools; an Effective Approach to Building a Reliable MCDM Environment for Applications to Computer Vision

Visualization is an effective approach in the operations research and mathematical programming applications to explore optimal solutions, and to summarize the results into an insight, instead of numbers [32,33]. Fortunately during past few years, it has been a huge development in combinatorial optimization, machine learning, intelligent optimization, and RSO [15], which have moved the advanced visualization methods even further. Previous work in the area of visualization for MCDM [39] allows the DM to better formulate the multiple objective functions for large optimization runs. Alternatively in our research utilizing RSO and visualization [37], which advocates learning for optimizing, the algorithm selection, adaptation and integration, are done in an automated way and the user is kept in the loop for subsequent refinements. Here one of the crucial issue in MCDM is to critically analyzing a mass of tentative solutions assusiated with bid data, which is visually mined to extract useful information [15,37]. In developing RSO in terms of learning capabilities there has been a progressive shift from the DM to the algorithm itself, through machine learning techniques [14].

Concerning solving the MCDM problems, utilizing RSO, the final user is not distracted by technical details, instead concentrates on using his expertise and informed choice among the large number of possibilities. Algorithms with self-tuning capabilities like RSO make life simpler for the final user. And to doing so the novel approach of RSO is to integrate the machine learning techniques, artificial intelligence, reinforcement learning and active learning into search heuristics. According to the original literature [15] during a solving process the alternative solutions are tested through an online feedback loop for the optimal parameters' tuning. Therefor the DM would deal with the diversity of the problems, stochasticity, and dynamicity more efficiently. Here are some study cases treated very promising by RSO [29,30,31,45].

3.3 More on Characteristics and Methodology of the Proposed Decision-Making Environment; Presenting the Results of the Case Study

For solving problems with a high level of complexity, modeling the true nature of the problem is of importance and essential. For this reason a considerable amount of efforts is made in modeling the MOO problems in Scilab which later will be integrated into optimizer package. Once the problem is modeled in scilab it is integrated to the optimizer via advanced interfaces to the RSO algorithm and its brain-computer EMO implementations and visualization. In this framework the application of learning and intelligent optimization and reactive business intelligence approaches in improving the process of such complex optimization problems is accomplished. Furthermore the problem could be further treated by reducing the dimensionality and the dataset size, multi-dimensional scaling, clustering and visualization tools [15]. Here in contrast to the EAs, the DM guides the optimization in the desirable search locations and the final desirable surface. In this case the computation cost is minimized and the preferences of the DM are effectively considered.

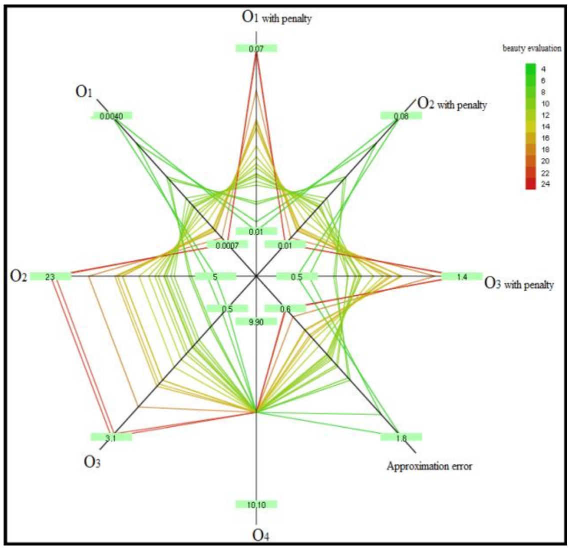

Fig 2: Considering four objectives of the study case in a multi-dimensional graph

During the process of solving the real-life problems exploring the search space, utilizing RSO, many alternative solutions are tested and as the result adequate patterns and regularities appear. While exploring, the human brain quickly learns and drives future decisions based on the previous observations and searching alternatives. For the reason of rapidly exploiting the most promising solutions the online machine learning techniques are inserted into the optimization engine of RSO. Furthermore with the aid of inserted machine learning a set of diverse, accurate and crucial alternatives are offered to the DM. In this context the feedbacks from the DM in the preliminary exploration phase can be incorporated so that a better tuning of the parameters takes the preferences into account. Further relevant characteristics of RSO, according to [15], could be summarized as; learning on the job, rapid generation, and analysis of many alternatives, flexible decision support, diversity of solutions and anytime solutions.

3.3.1 Communicating the results of the Case Study via Multi-Dimensional Graphs

For solving problems as such, with a high level of complexity, modeling the true nature of the problem is of importance and essential. Here, as an alternative to the previous approaches the robust and interactive MOO algorithm of RSO efficiently optimizes all the objectives at once including the criteria of beauty in which couldn't be completely considered in the previous attempts [2,3,4]. In this framework the quality of the surface, similar to the previous research workflows, is measured using a set of certain functions, then an optimization algorithm is applied in order to optimize the function to improve the quality of the surface.

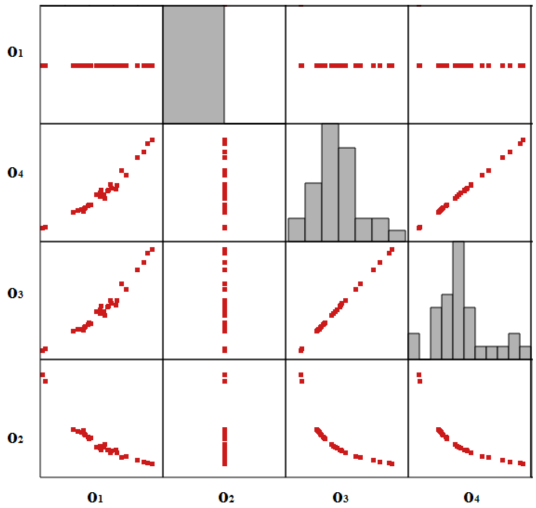

Fig 3: Considering four objectives of the case study in a multi-dimensional graph

The problem is modeled in scilab and the model is integrated to the optimizer via advanced interfaces to the RSO algorithm and its brain-computer evolutionary multiobjective optimization implementations and visualization [14]. In this framework the application of learning and intelligent optimization and reactive business intelligence approaches in improving the process of such complex optimization problems are described. Furthermore the problem is further reconsidered by reducing the dimensionality and the dataset size [37,34], multi-dimensional scaling, clustering and visualization tools [15]. Figure 2. And Figure 3. peresent the multi-dimensional graphs to the study case results.

6. CONCLUSIONS

In this paper along with presenting a highly nonlinear and multiobjective case study the aspects of data mining, modeling, and visualization the data related to computer vision, geometry and image processing are considered. A novel environment for optimization, analytics and decision support in general computer vision design problems is proposed. The new set of powerful integrated data mining, modeling, visualiztion and learning tools via a handy procedure stretches beyond a decision-making task and attempts to discover new optimal designs relating to decision variables and objectives, so that a deeper understanding of the underlying problem can be obtained. Here along with presenting the study case of NURBS optimal design, the interactive procedure is introduced which involves the DM in the optimization process helping to choose a single solution at the end. The method is well capable of handling the big data often associated with MCDM problems in computer vision and image processing.

The methodology implements a strong interface between a generic optimization algorithm and DM. While optimizing the systems produce different solutions, the DM is pursuing conflicting goals, and trade-off policies represented on the multi-dimensional graphs. Moreover the preliminary results of the proposed optimal design environment in the concrete context of optimal designing the NURBS have shown the effectiveness of the approach in rapidly reaching a design preferred by the DM via advanced visualization tools and the brain-computer novel interactions.

In addition the future research is set out to investigate the role that the proposed optimization strategy can play in the optimal skinning of circles and spheres [65] which is considered as an interesting subject in CAGD. Moreover customizing the proposed methodology for decision-making tasks in sustainable regional development [78], waste management [79], and materials selection [80], in the particular areas of construction and demolition would be a part of our future research.

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