A GA based Approach to Find Minimal Vertex Cover

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ABSTRACT

Genetic Algorithms are a class of Optimization Techniques which has been developed under inspiration of the Darwinian Theory of Survival of the Fittest. This technique has been successfully used to solve many optimization problems which otherwise pose huge challenges for computation. This paper presents a GA based approach to solve the Minimal Vertex Cover problem of Graph Theory.

Keywords

Minimal Vertex Cover, Genetic-Algorithms, Chromosomes, Mutation, Generations.

INTRODUCTION

Optimization problems form an important class of problems that are encountered in all forms of scientific research.Ideally, the number of solutions available to this class of problems is more than one and each have a different level of accuracy and therefore the acceptance of the solutions vary.The acceptance of a given solution is based on certain pre-determined criteria, which usually are presented in the form of real valued objective functions. Classical methods of solving optimization problems are hindered by their inability to solve functions which are not continuous differentiable [1]. Real life problems however, pose issues which are more likely to be having objective functions which are not continuous and therefore the applications of classical optimization techniques to these problems are limited at best. Another issue bothering the acceptance of the classical techniques is their tendency of settling down at a local minima or maxima instead of a globally optimal solution. The computational efforts required to solve such problems using the classical methods are also extremely high and therefore the quest for alternatives continue. Intelligent search strategies like hill climbing,local beam search, simulated annealing etc., have returned considerably good solutions to such problems. However, hill climbing suffers from the serious problem of settling to sub-optimal solutions remaining in the search space as local optimal points [2]. Genetic Algorithms (GAs) provide a way around in some of these situations. GAs try to mimic the process of natural evolution through natural selection based on the Darwinian principle of survival of the fittest.

In graph theory, the term vertex cover of a graph refers to a set of vertices such that each edge of the graph is incident to at least one vertex of the set. Formally, it can be stated as: For a graph G = (V, E), S ⊆ V is a vertex cover if ∀ {u, v} ∈ E: u ∈ S ∨ v ∈S . Minimum vertex cover for G would be a vertex cover S that minimizes |S|.

The problem of finding a minimum vertex cover is a classical optimization problem in computer science. A large body of work has been devoted in finding efficient approximation algorithms for the minimum vertex cover problem[3].

Papadimitriou and Yannakakis [4] showed that the problem is NP-hard and is a typical example of an NP-hard optimization problem that has an approximation algorithm,which transpires to the fact that it belongs to the class of problems which are at least as hard as the hardest problems in NP. Its decision version, the vertex cover problem, was one of Karp's 21 NP-complete problems and is therefore a classical NP-complete problem in computational complexity theory.

The classical approach to solve this problem would yield an algorithm as shown APPROX_VERTEX_COVER(G:Graph). However, the cost of the algorithm is too high and the complexity levels already discussed.

Procedure Vertex Cover(G: Graph)

1. C ← { }
2. E' ← E[G]
3. While E' is not empty do
4. Let (u, v) be an arbitrary edge of E'
5. C ← C U {u, v}
6. Remove from E' every edge incident on either u or v
7. Return C

Efforts have been put in to find cheaper solutions to such problems with the help of Evolutionary Algorithms, which are randomized search heuristics widely used for solving combinatorial optimization problems [5]. Khuri and Back [6] proved through experimental results that a Genetic Algorithm (GA) performs very well on instances of sizes n=100 and n=202 of the Papadimitriou- Stieglitz (PS) graph [6] since it finds the optimal cover on average 6 times out of 10 in a runtime of cn^2. In other cases, though they settle in a local maximum, which may be the case with many optimization techniques, the chances presented by GA are better. In this paper, a Genetic Algorithm based approach is proposed to find the minimum vertex cover of an undirected graph.

1. GENETIC ALGORITHM

Computationally, GA is a maximization process[1]. The problems which are solved using Genetic Algorithms usually have a very large search space with probable multiple local maxima inside it. The GA process ensures that it finds the global maximum point, or gives the globally optimumwithout being trapped to the local optima, which in case of the GA would be the local maxima.

To achieve this, GA works on a set of solutions (perhaps suboptimal) to the given problem instance, and evolves it through a number of generations. The evolution process stops when some predefined termination condition is satisfied. At each intermediate stage, the old generation is replaced by the new generation. The individuals of the population of a generation are processed with the help of a number of GA operators in such a way that the quality of the new generation, in general, is improved in comparison with old generation. In this way we obtain better and better solutions as the search proceeds until the end of the search when it is expected the best, or a near-best solution will be returned by the GA process [1] .The basic GA procedure is shown below:

Procedure Basic-GA

1. Initialize the population.

2. From initial population select using chosen operation the members to be sent to mating pool. Repeat 3 through 5 till termination condition is satisfied.

3. Crossover among pairs in mating pool to obtain new population. Use mutation at fixed or varied intervals to add variety to the population.

4. Replace the current population by the new population.

5. Repeat states 2 to 4 till terminating condition is satisfied.

6. Return the best solution of the current population.

1.1 Chromosomes

A chromosome is a stranded structure which stores the encoded traits of an individual. These are long strands ofgenes which genes are made up of two long thin strands of DNA in a double helix structure. The idea of chromosome as an encoded pattern of individual traits is used in Genetic Algorithms (GA), where each chromosome is considered to be an encoded solution to the problem.

The basic idea behind solving any problem using GA based techniques lies in this encoding. If a solution cannot be encoded as a DNA it cannot be solved using GA.

1.2 Natural Selection

Natural selection is Nature's way to support and perpetuate 'good' qualities in a species against the 'bad' qualities[1].Copying this type of phenomenon from nature, the GA can recreate more chromosomes which would themselves be encoded solutions to the problem under consideration. There are a number of techniques which can be employed to select the chromosomes. Some of the popular ones are RouletteStarting with a randomly generated initial selection, tournament selection etc. sometimes employed in selecting is Elitism. In this technique, only the fittest members of the pool are allowed to crossover. However, since there is no assurance that the offspring's of the fittest members would be fitter themselves, this technique is not popularly employed. This also limits the variations available in the pool by limiting the pairs to fitter members only.

1.3 Crossover

Crossover is a genetic operation, which results in exchange of genetic material between two chromosomes[1]. The chromosomes, represented as strings in a GA implementation can be operated upon by the crossover operator to realize new structure of the same type.

1.4 Mutation

In genetics, a gene mutation is a permanent change that may occur in the DNA sequence constituting a gene [1]. The same mutation operator is also used in the GA process to evolve newer and hopefully better solutions from the already existing gene pool.

2. EXPERIMENTAION AND RESULTS

It has been tested using the basic GA (procedure Basic-

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	0	1	2	3	4	5	16	17	18	19	20	0	0	0	0	0
2	1	0	0	0	0	0	7	8	0	0	0	0	0	0	0	0
3	2	0	0	0	0	0	0	9	10	0	0	0	0	0	0	0
4	3	0	0	0	0	0	0	0	11	12	0	0	0	0	0	0
5	4	0	0	0	0	0	0	0	0	13	14	0	0	0	0	0
6	5	0	0	0	0	0	0	0	0	0	15	0	0	0	0	0
7	16	7	0	0	0	6	0	0	0	0	0	0	21	0	0	0
8	17	8	9	0	0	0	0	0	0	0	0	0	0	22	0	0
9	18	0	10	11	0	0	0	0	0	0	0	0	0	0	23	0
10	19	0	0	12	13	0	0	0	0	0	0	0	0	0	0	24
11	20	0	0	0	14	15	0	0	0	0	0	25	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	25	0	0	0	0	0
13	0	0	0	0	0	0	21	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	22	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	23	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	24	0	0	0	0	0	0

GA)asin [1] on the problem shown therein. The adjacency matrix of the graph is as shown in Fig.1. The 16 nodes graph had a star like shape with node 1 being at the center. population of length 16 (for 16 nodes) and a population size of 128, interesting results came up with crossover probability of 0.65 and mutation probability of 0.04. The minimal vertex cover was returned by the GA in 18 generations. This shows slight improvement over previous work on the same problem [1], where a population size of 64 with same crossover and mutation probabilities yielded the result in 21 generations.
The results prove beyond doubt that the minimal vertex cover problem, which has been rated as NP-hard can be solved with comparative ease using the adaptive technique of GA.

3. CONCLUSIONS

The problem of vertex cover has been solved by the GA based technique. This work points at findings to prove that the higher population size has better chances of quicker solution to problems of this type. This however, cannot be generalized for all GA's, however, whether this particular problem consistently shows this trait has to be further investigated.

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