ABSTRACT
This paper introduces a new subclass of univalent analytic functions and derived various properties like coefficient inequality, distortion theorem, radius of starlikeness and convexity, Hadamard product, extreme points, closure theorems for functions belonging to this class with the help of fractional differential operator.

Keywords
Univalent functions, fractional derivative operator, Hadamard product.

1. INTRODUCTION
Let $S_k$ denote the subclass of functions $f(z)$ of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k,$$

which are analytic and univalent in the unit disc $U = \{z: |z| < 1\}$ of $f(z)$ and $g(z)$ of class $S$.

Let $g(z) = z - \sum_{k=2}^{\infty} b_k z^k \in S_k$, $b_k \geq 1$ (1.2)

which also analytic and univalent in $U$.

The following definitions which are used for working in the classes of analytic functions

**Definition (i):** A function $f(z) \in S_k$ is said to be starlike of order $\alpha$, $0 \leq \alpha < 1$ if

$$Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha, \quad z \in U$$

(1.3)

**Definition (ii):** A function $f(z) \in S_k$ is said to be convex of order $\alpha$, $0 \leq \alpha < 1$ if

$$Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, \quad z \in U$$

(1.4)

**Definition (iii):** If $f(z), g(z) \in S_k$ then their Hadamard product is $f(z) * g(z) = z - \sum_{k=2}^{\infty} a_k b_k z^k \in S_k$ (1.5)

for $f(z) = z - \sum_{k=2}^{\infty} a_k z^k$, $g(z) = z - \sum_{k=2}^{\infty} b_k z^k$.

**Definitions (iv):** (Fractional integral operator)

The fractional integral of order $\alpha$, $0 < \alpha \leq 1$ is defined for a function $f(z)$ by

$$D^\alpha_z f(z) = \frac{1}{\Gamma(\alpha)} \int_0^z \frac{f(\psi)}{(z-\psi)^{1-\alpha}} d\psi,$$

where $0 < \alpha \leq 1$.

**Definitions (v):** (Fractional derivative operator)

The fractional derivatives of order $\alpha$, is defined for a function $f(z)$ by

$$D^\alpha_z f(z) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{f(\psi)}{(z-\psi)^{\alpha}} d\psi,$$

where $0 < 1 - \alpha < \delta < 1$ (1.7)

and the multiplicity of $(z-\psi)^{-\alpha}$ is removed as in definition (iv).

**Definitions (vi):** (Extended fractional derivative operator)

Under the hypothesis of definition (v) The fractional derivative of order $n + \delta$ is defined for a function $f(z)$, by

$$D^{n+\delta}_z f(z) = \frac{d^n}{dz^n} D^\delta_z f(z)$$

where $0 < \delta < 1 ; n \in \mathbb{N}$

Using the above definition Srivastava and owa [2] introduced the operator

$$\Omega^\delta f(z) = \frac{\Gamma(2-\lambda)}{\Gamma(1-\lambda)} f(z)$$

(1.8)

where

$$\Phi(k, \lambda) = \frac{\Gamma(k+1)\Gamma(2-\lambda)}{\Gamma(k+1-\lambda)}$$

Using the definition ,S.M.Khairnar and Meena More [1] that a function $f(z)$ is in the class $S(\alpha, \beta, \gamma, \mu)$ if and only if,

$$\left|\frac{z^{m+1} f(z)}{D^\alpha f(z)} - 1\right| < \beta$$

(1.9)

for $|z| < 1$. Where

$$0 < \beta \leq 1, \frac{1}{2} \leq \gamma \leq 1, \quad 0 \leq \alpha \leq \frac{1}{2}, \frac{1}{2} < \mu \leq 1.$$

In this paper all the investigated results are motivated by S.M.Khairnar and S.M.Rajas[1],G.Murugusundaramoory And R. Themangani[2], M.Darus[3].

2. COEFFICIENT ESTIMATES

**Theorem 1:** Let $f(z) = z - \sum_{k=2}^{\infty} a_k z^k \in S_k$. Then $f(z) \in S(\alpha, \beta, \gamma, \mu)$ if and only if

$$\sum_{k=2}^{\infty} \left|1 - k + 2\beta(1 - \beta) - \beta(1 - \mu)\right| a_k^{\beta(1 - \beta)} \leq 2\beta(1 - \alpha)$$

(2.1)

where $0 \leq \alpha < \frac{1}{2}, 0 < \beta \leq \frac{1}{2}, \frac{1}{2} \leq \gamma \leq 1$. 

\[ \Phi(k, \lambda) = \frac{f'(k + 1) f'(2 - \lambda)}{f'(k + 1 - \lambda)} \]

The result is sharp for the function
\[ f(z) = z - \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} z^k \]

**Proof:** Assume that the inequality (1.9) holds true and \( |z| = 1 \). Then obtain
\[ |z\Omega^2f(z) - \Omega^2 f(z)| \]
\[ -\beta \left| 2\gamma \left[ z\Omega^2 f(z) - a\Omega^2 f(z) \right] - \mu \left[ z\Omega^2 f(z) - \Omega^2 f(z) \right] \right| \]
\[ \leq \sum_{k=2}^{\infty} |(1 - k)\Phi(k, \lambda) a_k z^k | \]
\[ = \frac{\sum_{k=2}^{\infty} |(1 - k)\Phi(k, \lambda) a_k z^k |}{\sum_{k=1}^{\infty} |(1 - k)\Phi(k, \lambda) a_k z^k |} \]
\[ \leq \sum_{k=2}^{\infty} (1 - k)\Phi(k, \lambda) a_k \]
\[ \leq \frac{2\beta\gamma(1 - \alpha)}{1 + \frac{2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} z^k \]

by hypothesis. Hence by maximum modulus principle, \( f(z) \in S(\alpha, \beta, \gamma, \mu) \)

Conversely, let \( f(z) \in S(\alpha, \beta, \gamma, \mu) \).

Then
\[ \left| \frac{z^{\Omega^2 f(z)} - 1}{2\gamma \left( z^{\Omega^2 f(z)} - a\Omega^2 f(z) \right)} \right| < \beta \]

for all \( z \in U \), i.e.
\[ \left| \frac{\sum_{k=2}^{\infty} (1 - k)\Phi(k, \lambda) a_k z^k}{\sum_{k=1}^{\infty} (1 - k)\Phi(k, \lambda) a_k z^k} \right| < \beta \]

(2.2)

Since \( |\text{Re}(z)| \leq |f(z)| \) for all \( z \), we have
\[ \left| \frac{\sum_{k=2}^{\infty} (1 - k)\Phi(k, \lambda) a_k z^k}{\sum_{k=1}^{\infty} (1 - k)\Phi(k, \lambda) a_k z^k} \right| \]
\[ \leq \beta \]

(2.3)

Since \( z^{\Omega^2 f(z)} \) is real and upon clearing the denominator of the above expression, we choose the value of \( z \) on real axis and allowing \( z \to 1 \) through real values.
\[ \left| \frac{\sum_{k=2}^{\infty} (1 - k)\Phi(k, \lambda) a_k z^k}{\sum_{k=1}^{\infty} (1 - k)\Phi(k, \lambda) a_k z^k} \right| \]
\[ \leq \beta \]

(2.4)

which obviously is required assertion (2.1)

Finally, sharpness follows if we take
\[ f(z) = z - \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} z^k \]

, \( k = 2, 3, 4, \ldots \) (2.5)

**Corollary:** If \( f(z) \in S(\alpha, \beta, \gamma, \mu) \) then
\[ a_k \leq \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} z^k \]

, \( k = 2, 3, 4, \ldots \) (2.6)

3. GROWTH AND DISTORTION THEOREM

**Theorem 2:** If the function \( f(z) \in S(\alpha, \beta, \gamma, \mu) \) then
\[ \left| \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} \right| \]
\[ \leq \left| f(z) \right| \]
\[ \leq \left| \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} \right| \]
\[ \leq \left| f(z) \right| \]
\[ \leq \left| \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} \right| \]

The result is sharp for
\[ f(z) = z - \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} z^k \]

**Proof:**
\[ f(z) = z - \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} z^k \]

(3.1)

Similarly,
\[ \left| \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} \right| \]
\[ \leq \left| f(z) \right| \]
\[ \leq \left| \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} \right| \]

(3.2)

Combining (3.1) and (3.2), we get
\[ \left| \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} \right| \]
\[ \leq \left| f(z) \right| \]
\[ \leq \left| \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} \right| \]

Theorem 3: If the function \( f(z) \in S(\alpha, \beta, \gamma, \mu) \) then
\[ \left| \frac{4\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} \right| \]
\[ \leq \left| f(z) \right| \]
\[ \leq \left| \frac{4\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} \right| \]

The result is sharp for
\[ f(z) = z - \frac{2\beta\gamma(1 - \alpha)}{[1 - k + 2\beta\gamma(k - \alpha) - \beta\mu(k - 1)]\Phi(k, \lambda)} z^k \]

4. RADIUS OF STARLIKENESS AND CONVEXITY

**Theorem 3:** If the function \( f(z) \in S(\alpha, \beta, \gamma, \mu) \) then \( f(z) \) is a starlike of order \( \alpha, 0 \leq \alpha < 1 \) in \( |z| < R \) where
\[ f(z) = z - \frac{2\beta y(1 - \alpha)}{1 - k + 2\beta y(k - \alpha) - \beta \mu(k - 1)} \Phi(k, \lambda) z^k \]

**Proof:** Let \( f(z) \in \mathbb{S}(\alpha, \beta, \gamma, \mu) \) is convex of order \( \alpha, 0 \leq \alpha < 1 \) if

\[ \text{Re} \left[ 1 + \frac{2f''(z)}{f'(z)} \right] > \alpha \]

That is if

\[ \frac{|zf''(z)|}{f'(z)} < 1 \] (4.5)

which simplifies to

\[ \sum_{k=2}^{\infty} \frac{k(k-\alpha)\alpha_k|z|^{k-1}}{(1-\alpha)} \leq 1 \] (4.6)

By equation (2.6) we have

\[ \alpha_k \leq \frac{2\beta y(1 - \alpha)}{1 - k + 2\beta y(k - \alpha) - \beta \mu(k - 1)} \Phi(k, \lambda) \] (4.7)

Using equation (4.6) and (4.7),

\[ |z|^{k-1} \leq \frac{(1-\alpha)(1 - k + 2\beta y(k - \alpha) - \beta \mu(k - 1)) \Phi(k, \lambda)}{2\beta y(1 - \alpha) k(k - \alpha)} \] (4.8)

Thus \( |z| < R \)

\[ \inf_k \left( \frac{(1-\alpha)(1 - k + 2\beta y(k - \alpha) - \beta \mu(k - 1)) \Phi(k, \lambda)}{2\beta y(1 - \alpha) k(k - \alpha)} \right)^{1/k} \]

5. EXTREME POINTS

**Theorem 5:** Let \( f_k(z) = z - \frac{2\beta y(1 - \alpha)}{1 - k + 2\beta y(k - \alpha) - \beta \mu(k - 1)} \Phi(k, \lambda) z^k \), then

\( f(z) \in \mathbb{S}(\alpha, \beta, \gamma, \mu) \) iff it can be expressed in the form of \( f(z) \in \mathbb{S}(\alpha, \beta, \gamma, \mu) \)

**Proof:** Suppose

\[ f(z) = \sum_{k=2}^{\infty} \lambda_k f_k(z) \]

\[ \sum_{k=2}^{\infty} \lambda_k \left( z - \frac{2\beta y(1 - \alpha)}{1 - k + 2\beta y(k - \alpha) - \beta \mu(k - 1)} \Phi(k, \lambda) z^k \right) \]

(5.1)

Now \( f(z) \in \mathbb{S}(\alpha, \beta, \gamma, \mu) \) since

\[ \sum_{k=2}^{\infty} \frac{[1 - k + 2\beta y(k - \alpha) - \beta \mu(k - 1)] \Phi(k, \lambda)}{2\beta y(1 - \alpha) k(k - \alpha)} \lambda_k \]

\[ = \sum_{k=2}^{\infty} \lambda_k = 1 - \lambda_1 \leq 1 \]
Conversely, suppose that \( f(z) \in S(\alpha, \beta, \gamma, \mu) \) then by theorem 1,
\[
a_k \leq \frac{2\beta \gamma (1-\alpha)}{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}
\]
Setting
\[
\lambda_k = \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)}
\]
\( k = 2, 3, \ldots \)
And \( \lambda_1 = 1 - \sum_{k=2}^{\infty} \lambda_k \) we notice that \( f(z) = \sum_{k=1}^{\infty} \lambda_k f_k(z) \).
Hence the result.

6. \textbf{HADAMARD PRODUCT}

\textbf{Theorem 6.1:} Let \( f(z), g(z) \in S(\alpha, \beta, \gamma, \mu) \) then
\[
(fg)(z) = z - \sum_{k=2}^{\infty} a_k b_k z^k \in S(\alpha, \beta, \gamma, \mu)
\]
for \( f(z) = z - \sum_{k=2}^{\infty} a_k z^k \), \( g(z) = z - \sum_{k=2}^{\infty} b_k z^k \)
where
\[
\psi = \frac{2\beta \gamma (1-\alpha)(1-\beta)}{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda) + 2\beta \gamma (1-\alpha)(k-1 - 2\gamma (k-\alpha))}
\]
\textbf{Proof:} Let \( f(z), g(z) \in S(\alpha, \beta, \gamma, \mu) \) then
\[
\sum_{k=2}^{\infty} \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)} a_k \leq 1 \quad (6.1)
\]
\[
\sum_{k=2}^{\infty} \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)} b_k \leq 1 \quad (6.2)
\]
To find smallest number \( \psi \) such that
\[
\sum_{k=2}^{\infty} \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)} a_k b_k \leq 1 \quad (6.3)
\]
By Cauchy Schwarz inequality,
\[
\sum_{k=2}^{\infty} \sqrt{\frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)}} a_k b_k \leq 1 \quad (6.4)
\]
Thus it is enough to show that
\[
\sum_{k=2}^{\infty} \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)} a_k b_k \leq \sum_{k=2}^{\infty} \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)} \sqrt{a_k b_k}
\]
That is
\[
\sqrt{a_k b_k} \leq \psi \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)]}{\beta [1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)]} \quad (6.5)
\]
From (6.4)
\[
\sqrt{a_k b_k} \leq \frac{2\beta \gamma (1-\alpha)}{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)} \quad (6.6)
\]
Therefore, in view of (6.5) and (6.6) it is enough to show that
\[
2\beta \gamma (1-\alpha) \leq \frac{\psi [1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)]}{\beta [1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)]} \Phi(k, \lambda)
\]
\[
\frac{\psi [1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)]}{\beta [1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)]} \Phi(k, \lambda) \leq \frac{\psi (1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1))}{\beta (1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1))} \Phi(k, \lambda)
\]
Which simplifies to
\[
\psi \leq \frac{2\beta \gamma (1-\alpha)(1-k)}{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda) + 2\beta \gamma (1-\alpha)(k-1 - 2\gamma (k-\alpha))}
\]
7. \textbf{CLOSURE THEOREMS}

\textbf{Theorem 7.1:} Let \( f_j \in S(\alpha, \beta, \gamma, \mu) \), \( j = 1, 2, \ldots, m \) then
\[
g(z) = \sum_{j=1}^{m} c_j f_j(z) \in S(\alpha, \beta, \gamma, \mu)
\]
Where \( \sum_{j=1}^{m} c_j = 1 \) and \( f_j(z) = z - \sum_{k=2}^{\infty} a_k z^k \).
\textbf{Proof:}
\[
g(z) = z - \sum_{j=1}^{m} c_j \sum_{k=2}^{\infty} a_k c_{j,k} z^k
\]
\[
= z - \sum_{k=2}^{\infty} \left( \sum_{j=1}^{m} c_j a_{j,k} \right) z^k
\]
\[
(7.1)
\]
\[
= z - \sum_{k=2}^{\infty} e_k z^k \quad (7.2)
\]
where \( e_k = \sum_{j=1}^{m} c_j a_{k,j} \)
Since \( f_j \in S(\alpha, \beta, \gamma, \mu) \) by Theorem 1,
\[
\sum_{k=2}^{\infty} \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)} a_{k,j} \leq 1 \quad (7.3)
\]
In view of (7.2), \( g(z) \in S(\alpha, \beta, \gamma, \mu) \)
\[
\sum_{k=2}^{\infty} \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)} e_n \leq 1
\]
Now
\[
\sum_{k=2}^{\infty} \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)} e_n \sum_{j=1}^{m} c_j a_{k,j}
\]
\[
= \sum_{j=1}^{m} c_j \sum_{k=2}^{\infty} \frac{[1 - k + 2\beta \gamma (k - \alpha) - \beta \mu (k-1)] \Phi(k, \lambda)}{2\beta \gamma (1-\alpha)} a_{k,j}
\]
In view of (8.3) and (8.4) it is enough to show that

\[ h(z) = 1. \]

Therefore \( g(z) = 1. \) using (7.3)

Theorem 8: Let \( f(z), g(z) \in S(\alpha, \beta, \gamma, \mu) \) then

\[ h(z) = z - \sum_{k=1}^{\infty} \left( a_k^2 + b_k^2 \right) z^k \]

is in \( S(\alpha, \phi, \gamma, \mu) \)

where

\[ \phi \geq \frac{4\beta^2\gamma(1-\alpha)(1-k)}{[1-k + 2\beta\gamma(k-\alpha) - \beta\mu(k-1)]\Phi(k,\lambda)} \]

Proof: Let \( f(z), g(z) \in S(\alpha, \beta, \gamma, \mu) \) and so

\[ \sum_{k=2}^{\infty} \left[ \frac{1-k + 2\beta\gamma(k-\alpha) - \beta\mu(k-1)}{2\beta\gamma(1-\alpha)} \Phi(k,\lambda) \right] a_k^2 \leq 1 \]

(8.1)

And

\[ \sum_{k=2}^{\infty} \left[ \frac{1-k + 2\beta\gamma(k-\alpha) - \beta\mu(k-1)}{2\beta\gamma(1-\alpha)} \Phi(k,\lambda) \right] b_k^2 \leq 1 \]

(8.2)

Adding (8.1) and (8.2)

\[ \sum_{k=2}^{\infty} \left[ \frac{1-k + 2\beta\gamma(k-\alpha) - \beta\mu(k-1)}{2\beta\gamma(1-\alpha)} \Phi(k,\lambda) \right] \left( a_k^2 + b_k^2 \right) \leq 1 \]

(8.3)

I must show that \( h(z) \in S(\alpha, \phi, \gamma, \mu) \), that is

\[ \sum_{k=2}^{\infty} \left[ \frac{1-k + 2\beta\gamma(k-\alpha) - \beta\mu(k-1)}{2\beta\gamma(1-\alpha)} \Phi(k,\lambda) \right] \left( a_k^2 + b_k^2 \right) \leq 1 \]

(8.4)

In view of (8.3) and (8.4) it is enough to show that

\[ \frac{1-k + 2\beta\gamma(k-\alpha) - \beta\mu(k-1)}{2\beta\gamma(1-\alpha)} \Phi(k,\lambda)^2 \]

which simplifies to

\[ \frac{4\beta^2\gamma(1-\alpha)(1-k)}{[1-k + 2\beta\gamma(k-\alpha) - \beta\mu(k-1)]\Phi(k,\lambda) + 4\beta^2\gamma(1-\alpha)[k-1-2\gamma(k-\alpha)]} \]

8. CONCLUSION

This paper derived the basic properties like coefficient inequality, distortion theorem, radius of starlikeness and convexity, Hadamard product, extreme points, closure theorems of univalent and analytic functions with negative coefficient belonging to the class \( S(\alpha, \beta, \gamma, \mu) \) with the help of fractional differential operator which are new result, can be used in engineering field.

9. REFERENCES


