Modeling Methods of Three Phase Induction Motor

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ABSTRACT
This paper presents the *qd* and voltage-behind reactance model of three phase induction motor. The accurate behavioral modeling of induction motor helps in designing controller for the machine and is also useful in detection of faults in machines. The rotor subsystem is expressed in *qd* coordinates and the stator subsystem is expressed in *abc* phase coordinates. Computer studies of an induction motor for Voltage-behind-reactance model demonstrate the improvement in computational efficiency as compared with the *qd* or PD model. In this paper Voltage-behind-reactance model is developed in stationary reference frame using MATLAB-SIMULINK platform.

Index terms
Coupled-circuit(CC) model, *qd* model, voltage-behind-reactance (VBR) model ,phase-domain model, induction machine, dynamic simulation.

NOMENCLATURE

v_{as}, v_{bs}, v_{cs}	- Input voltage for phase a, b, c respectively in Volts.
v_{qs}, v_{ds}	- Stator q and d axis voltages in Stationary reference frame respectively in Volts.
v_{qr}, v_{dr}	- Rotor q and d axis Voltages in Stationary reference frame respectively in Volts.
i_{qs}, i_{ds}	- Stator q and d axis current in Stationary reference frame respectively in Volts.
i_{qr}, i_{dr}	- Rotor q and d axis current in Stationary referece frame respectively in Volts.
$R_{S,}$	- Stator Resistence/phase in Ω.
L_S	- Stator Self Inductance/phase in H.
L_M	- Mutual Inductance in H.
R_r	- Rotor Resistance/phase in Ω.
L_r	- Rotor self inductance in H.
ω_r	- rotor speed in rad/sec.
J	- Inertia of Motor in Kg- m^2 .
T_e	- Electrical Torque in N-m.
$T_{L.}$	- Load Torque in N-m.
p	- Number of Poles.
$\lambda_{mq}, \lambda_{md}$	- q and d axis magnetizing flux linkage

1. INTRODUCTION

The induction motor (IM) is largely used in many industrial applications due to low cost, good torque density and robustness. Analytical model are commonly used and are appreciated for their speed. The modeling approach for this machine may be roughly divided into three categories: finite element method; equivalent magnetic circuit approach; and coupled electric approach [1].The most popular representation for ac machines for transient simulation is the so-called *qd* model based on a series of mathematical transformations. The direct and quadrature axis model based on the space phasor theory is widely used to study the dynamic behavior of three-phase inductor motor. Rotating reference frame, e.g. stationary, rotor or synchronous are used to transform physical (*abc*) variables of the machine into fictitious (*qd*) variable [1][5] . By having the voltage and current quantities in *qd* frame, it is possible to control the speed of the machine by controlling the flux and torque independently. It is also a method of sensor less measurement.

The advantages of the *qd* induction machine models: 1) the time –varying inductances between stator and rotor winding are eliminated; 2) the flux linkage equations are decoupled; 3) zero sequence quantities disappear for balanced operation; 4) the average-value modeling of machine converter system is simplified when expressing the machine in terms of *qd* variables [1].

Model based on finite element (FE) computations allow a higher accuracy of the induction motor performance prediction, taking into account the iron saturation and the current density distribution within the rotor slots [2]. FE approach requires a higher computation time.

The voltage-behind-reactance (VBR) machine model has been recently proposed for the electro-magnetic transient programs (EMTP)-type simulation programs. The VBR model provide many interface with the external network with greatly improved numerical accuracy and simulation efficiency [1][4]. The EMTP and its derivative programs are extensively used by industry and academia as powerful and standard simulation tools [5].

The two new models are nodal reduced current-flux (NR-CF) and nodal reduced current-current (NR-CC) model of the induction machine, both of them having an overall superior performance than the VBR [3]. The discrete NR-CC model is the discrete PD model. The NR-CF model is a PD model with stator voltage equations with a structure similar to that of the VBR model, but expressed in terms of current.

The main thrust of this paper is on the of VBR model for three phase induction motor with a direct interface to the external power system.

2. COUPLED -CIRCUIT MACHINE MODEL

To better understand the proposed advanced models, and for consisting purposes, coupled circuit (CC) model is reviewed here. Cross sectional view of induction machine is shown in

Fig. 1.The winding arrangement for a 2-pole, 3-phase, wye-connected, symmetrical induction machine is shown in Fig. 2.
The stator windings are identical, sinusoidally distributed windings displaced by 120 degree, with N_s equivalent turns and resistance r_s. The rotor windings will also be considerd as three identical sinusoidally distributed winding, displaced 120 degree, with N_r equivalent turns and resistance r_r The corresponding voltage equation may be expressed in matrix form as

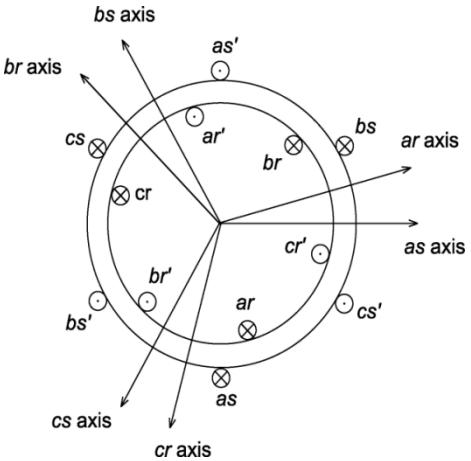

Fig 1: Basic structure of induction machine model

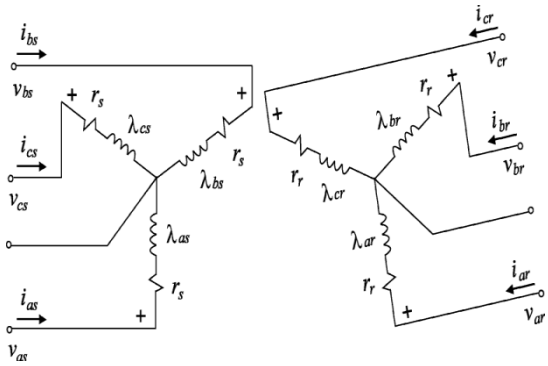

Fig 2: CC model of induction machine

$$\begin{bmatrix} V_{abcs} \\ V_{abcr} \end{bmatrix} = \begin{bmatrix} r_s & \\ & r_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i_{abcr} \end{bmatrix} + p \begin{bmatrix} \lambda_{abcs} \\ \lambda_{abcr} \end{bmatrix} \quad (1)$$

where the stator and rotor diagonal resistance matrices are 3×3 and denoted by r_s, r_r, respectively. The operator *p* denotes*d/dt*. The corresponding flux linkage equation is,

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda_{abcr} \end{bmatrix} = \begin{bmatrix} L_s & L_{sr} \\ L_{sr}^T & L_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i_{abcr} \end{bmatrix} \quad (2)$$

where te stator and rotor self-inductance matrices are L_s *and* L_r ,respectively, and are constant due to machine symmetry The expression for **L***s* and **L***r*..The mutual inductances as,

$$L_s = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls} + L_{ms} \end{bmatrix}$$

$$L_r = \begin{bmatrix} L_{lr} + L_{mr} & -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & L_{lr} + L_{mr} & -\frac{1}{2}L_{mr} \\ -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} & L_{lr} + L_{mr} \end{bmatrix}$$

$$L_{sr} = L_{ms} \begin{bmatrix} \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) & \cos\theta_r \end{bmatrix} \quad (3)$$

The developed electromagnetic torque is expressed as

$$T_e = \left(\frac{P}{2}\right)(i_{abcs})^T \frac{\partial}{\partial\theta_r}[L_{sr}]i_{abcr} \quad (4)$$

3. QD MACHINE MODEL

The CC induction machine models is often transformed into the *qd* arbitrary reference frame (ARF) [1], where the flux linkages become decoupled. For convenient derivation of the VBR models, the *qd* model is included in decoupled form. In particular, the voltage equations in the ARF are given as,

$$v_{qs} = r_s i_{qs} + \omega\lambda_{ds} + p\lambda_{qs} \quad (5)$$

$$v_{ds} = r_s i_{ds} + \omega\lambda_{qs} + p\lambda_{ds} \quad (6)$$

$$v_{0s} = r_s i_{0s} + p\lambda_{0s} \quad (7)$$

$$v_{qr} = r_s i_{qs} + (\omega - \omega_r\lambda_{dr} + p\lambda_{qr} \quad (8)$$

$$v_{dr} = r_{\ r} i_{dr} + \omega\lambda_{qr} + p\lambda_{dr} \quad (9)$$

$$v_{0s} = r_r i_{0r} + p\lambda_{0r} \quad (10)$$

The flux linkage equations are expressed as,

$$\lambda_{qs} = L_{ls} i_{qs} + \lambda_{mq} \quad (11)$$

$$\lambda_{ds} = L_{ls} i_{ds} + \lambda_{md} \quad (12)$$

$$\lambda_{qr} = L_{lr} i_{qr} + \lambda_{mq} \quad (13)$$

$$\lambda_{dr} = L_{lr} i_{qr} + \lambda_{md} \quad (14)$$

where magnetizing fluxes are defined as

$$\lambda_{mq} = L_m\,(i_{qs} + i_{qr}) \quad (15)$$

$$\lambda_{md} = L_m\,(i_{ds} + i_{dr}) \quad (16)$$

$$L_m = \frac{3}{2}\ L_{ms} \quad (17)$$

The developed electromagnetic torque in terms of transformed *qd* variables is given as

$$T_e = \frac{3P}{4}\ (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \quad (18)$$

4. VOLTAGE-BEHIND-REACTANCE MODEL-I

Derivation of the first model is performed by first solving (13) and (14) for currents and substituting the result into (15) and (16).The magnetizing fluxes are then expressed as

$$\lambda_{mq} = L_m^{"} (i_{qs} + \frac{\lambda_{qr}}{L_{lr}}) \quad (19)$$

$$\lambda_{md} = L_m^{"} (i_{ds} + \frac{\lambda_{dr}}{L_{lr}}) \quad (20)$$

where

$$L_m^{"} = (\frac{1}{L_m} + \frac{1}{L_{lr}})^{-1} \quad (21)$$

Substituting (21) and (22) into (11) and (12), respectively, the stator flux linkage equations as,

$$\lambda_{qs} = L" \, i_{qs} + \lambda_q^{"} \quad (22)$$

$$\lambda_{ds} = L" \, i_{ds} + \lambda_d^{"} \quad (23)$$

where the sub transient inductance is defined as,

$$L^{"} = L_{ls} + L_m^{"} \quad (24)$$

The sub transient flux linkages are defined as,

$$\lambda_q^{"} = L_m^{"} \frac{\lambda_{qr}}{L_{lr}} \quad (25)$$

$$\lambda_d^{"} = L_m^{"} \frac{\lambda_{dr}}{L_{lr}} \quad (26)$$

Substituting (22) and (23) into (5) and (6), respectively, the stator voltage equation as

$$v_{qs} = r_s \, i_{qs} + \omega L^{"} i_{ds} + pL^{"} \, i_{qs} + \omega \lambda_d^{"} p \lambda_q^{"} \quad (27)$$

$$v_{ds} = r_s \, i_{ds} + \omega L^{"} i_{qs} + pL^{"} \, i_{ds} + \omega \lambda_q^{"} + p \lambda_d^{"} \quad (28)$$

The rotor currents are derived from (13) and (14) and are given by

$$i_{qr} = \frac{1}{L_{lr}} (\lambda_{qr} - \lambda_{mq}) \quad (29)$$

$$i_{dr} = \frac{1}{L_{lr}} (\lambda_{dr} - \lambda_{md}) \quad (30)$$

From (8), (9) and (29), (30), the rotor voltage equations maybe rewritten in state-space as,

$$p\lambda_{qr} = -\frac{r_r}{L_{lr}} (\lambda_{qr} - \lambda_{mq}) - (\omega - \omega_r)\lambda_{dr} + v_{qr} \quad (31)$$

$$p\lambda_{dr} = -\frac{r_r}{L_{lr}} (\lambda_{dr} - \lambda_{md}) - (\omega - \omega_r)\lambda_{qr} + v_{dr} \quad (32)$$

The terms $p\lambda_q^{"}$ and $p\lambda_d^{"}$ in respective stator voltage equation (27) and (28) may be eliminated by taking the derivative of (25) and (26) and substituting (31) and (32) into the resulting equation, the stator voltage equation may be written as,

$$v_{qs} = r^{"} \, i_{qs} + \omega L^{"} i_{ds} + pL^{"} i_{qs} + e_q^{"} \quad (33)$$

$$v_{qs} = r^{"} \, i_{qs} + \omega L^{"} i_{ds} + pL^{"} i_{qs} + e_q^{"} \quad (34)$$

where

$$r^{"} = r_s + \frac{L_m^{"2}}{L_{lr}^2} r_r \quad (35)$$

and

$$e_q^{"} = \omega_r \lambda_d^{"} + \frac{L_m^{"} r_r}{L_{lr}^2} (\lambda_q^{"} - \lambda_{qr}) + \frac{L_m^{"}}{L_{lr}} v_{qr} \quad (36)$$

$$e_d^{"} = \omega_r \lambda_q^{"} + \frac{L_m^{"} r_r}{L_{lr}^2} (\lambda_d^{"} - \lambda_{dr}) + \frac{L_m^{"}}{L_{lr}} v_{dr} \quad (37)$$

The stator voltage equation (7), (33), and (34) may now be transformed back into the *abc* phase coordinates by applying inverse arbitrary reference transformation (**K***s*)−1 . This final step gives the voltage equation in ARF form as,

$$v_{abcs} = r_{abcs}^{"} \, i_{abcs} + L_{abcs}^{"} p \, i_{abcs} + e_{abcs}^{"} \quad (38)$$

where

$$e_{abcs}^{"} = [K_s]^{-1} \begin{bmatrix} e_q^{"} & e_d^{"} & 0 \end{bmatrix}^T \quad (39)$$

Here, the resistance matrix is given by

$$r_{abcs}^{"} = \begin{bmatrix} r_S & r_M & r_M \\ r_M & r_S & r_M \\ r_M & r_M & r_S \end{bmatrix} \quad (40)$$

where

$$r_S = r_s + r_a \quad (41)$$

$$r_M = -\frac{r_a}{2} \quad (42)$$

$$r_a = \frac{2}{3} \frac{L_m^{"}}{L_{lr}^2} r_r \quad (43)$$

$$L_{abcs}^{"} = \begin{bmatrix} L_S & L_M & L_M \\ L_M & L_S & L_M \\ L_M & L_M & L_S \end{bmatrix} \quad (44)$$

$$L_S = L_{ls} + L_a \quad (45)$$

$$L_M = -\frac{L_a}{2} \quad (46)$$

$$L_a = \frac{2}{3} L_m^{"} \quad (47)$$

Note that in (38), sub-transient resistance matrix (40) and inductance matrix (44) are constant due to machine symmetry, and are independent of any reference frame. These are veryDesirable properties that make the VBR model more efficient than the CC model. Thus equation (10), (31),(32),(36),(37) and (38) define the VBR formulation. the developed electromagnetic torque can be expressed using the magnetizing fluxes as .

$$T_e = \frac{3P}{4} (\lambda_{md} i_{qs} - \lambda_{mq} i_{ds}) \quad (48)$$

5. VOLTAGE-BEHIND-REACTANCE MODEL-II

The VBR-I model is difficult to implement in simulation because using a full resistance matrix. So the model may be simplified by using the diagonal stator matrix r_s . This formulation constitutes the second VBR model (VBR-II). Which is expressed as?

$$v_{abcs} = r_s i_{abcs} + L''_{abcs}\, p i_{abcs} + v''_{abcs} \qquad (49)$$

Here, other equations are the same as in VBR-I, except the back emf v''_{abcs} is define as

$$v''_{abcs} = \left[K_s\right]^{-1}\left[v''_q \quad v''_d \quad 0\right]^T \qquad (50)$$

6. VOLTAGE-BEHIND-REACTANCE MODEL-III

In previous formulations, the stator inductive branches are coupled. A further simplification may be achieved if the stator inductance matrix (44) and resistance matrix (40) (in VBR-I are made diagonal.

$$i_{as} + i_{bs} + i_{cs} = 3i_{0s} \qquad (51)$$

$$pi_{as} + pi_{bs} + pi_{cs} = 3pi_{0s} \qquad (52)$$

Here, we use (49) and (50) to reconsider the VBR-I formulation. After algebraic manipulation, (38) can be rewritten as

$$v_{abcs} = \begin{bmatrix} r_D & 0 & 0 \\ 0 & r_D & 0 \\ 0 & 0 & r_D \end{bmatrix} i_{abcs} + \begin{bmatrix} L_D & 0 & 0 \\ 0 & L_D & 0 \\ 0 & 0 & L_D \end{bmatrix} pi_{abcs}$$
$$+ e''_{abcs} + \left(3r_M \begin{bmatrix} i_{0s} \\ i_{0s} \\ i_{0s} \end{bmatrix} + 3L_M \begin{bmatrix} pi_{0s} \\ pi_{0s} \\ pi_{0s} \end{bmatrix} \right)$$

(53)

where

$$r_D = r_S - r_M \qquad (54)$$

and

$$L_D = L_S - L_M \qquad (55)$$

The voltage equation for the stator zero sequence is

$$v_{0s} = r_S i_{0s} + L_{ls}\, pi_{0s} \qquad (56)$$

which gives the following state equation

$$pi_{0s} = \frac{1}{L_{ls}}\left(v_{0s} - r_s i_{0s}\right) \qquad (57)$$

Substituting (55) into (51), the VBR model formulation II has the following form

$$v_{abcs} = \begin{bmatrix} r_D & 0 & 0 \\ 0 & r_D & 0 \\ 0 & 0 & r_D \end{bmatrix} i_{abcs} + \begin{bmatrix} L_D & 0 & 0 \\ 0 & L_D & 0 \\ 0 & 0 & L_D \end{bmatrix} pi_{abcs}$$
$$+ e''_{abcs} + \left(3r_M - \frac{3L_M r_S}{L_{ls}} \right) \begin{bmatrix} i_{0s} \\ i_{0s} \\ i_{0s} \end{bmatrix} + \frac{3L_M}{L_{ls}} \begin{bmatrix} v_{0s} \\ v_{0s} \\ v_{0s} \end{bmatrix}$$

(58)

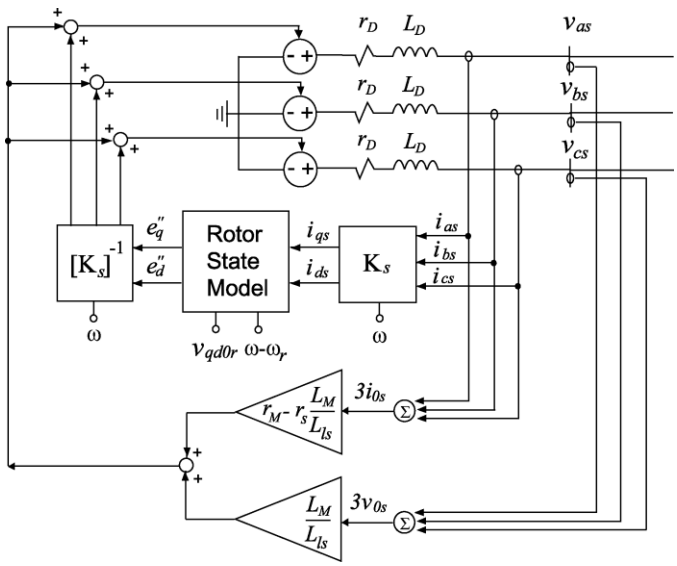

Fig 3: Proposed VBR model

7. PHASE-DOMAIN MODEL

The general form of Phase-domain (PD) model is the coupled-circuit model expressed in physical variable coordinates. The voltage equation can be expressed as

$$\begin{bmatrix} v_{abcs} \\ v_{qdr} \end{bmatrix} = R \begin{bmatrix} i_{abcs} \\ i_{dqr} \end{bmatrix} + p \begin{bmatrix} \lambda_{abcs} \\ \lambda_{qdr} \end{bmatrix} \qquad (59)$$

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda_{qdr} \end{bmatrix} = L(\theta_r) \begin{bmatrix} i_{abcs} \\ i_{dqr} \end{bmatrix} \qquad (60)$$

With the inductance matrix now depending on the position of the rotor

$$L(\theta_r) = \begin{bmatrix} L_s(\theta_r) & L_{sr}(\theta_r) \\ L_{rs}(\theta_r) & L_r \end{bmatrix} \qquad (61)$$

The electromagnetic torque as

$$T_e = \frac{1}{2}\begin{bmatrix} i_{abcs} \\ i_{dqr} \end{bmatrix}^T \frac{\partial}{\partial \theta_r} L(\theta_r) \begin{bmatrix} i_{abcs} \\ i_{dqr} \end{bmatrix} \qquad (62)$$

The main advantage of this model for the EMTP solution is that the stator circuit is directly integrated with the electrical network, thereby avoiding the interfacing and stability problems common in the *qd* model.

8. SIMULATION RESULT

Voltage behind reactance model (VBR-I) of induction motor is developed by using the equation from (19) to (48) in MATLAB/SIMULINK platform. The simulations are carried out using the motor data obtain from the No load test. 2.2KW motor used, motor line voltage is 415V and frequency is given 50 Hz. Variable-step type (stiff/Mod. Rrosenbrock) solver used for simulation work. The simulated waveforms are shown in Fig.4 to 9.

Table 1. Induction motor parameters

Parameter	Machine
Ls(H)	0.70608
Lr(H)	0.70608
Lm(H)	0.59181
Rs(Ω)	9.1091
Rr(Ω)	8.00634
J(Kg-m)	0.01
P	4

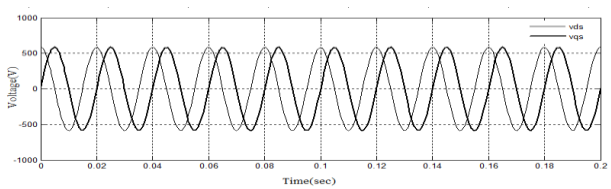

Fig 4: Stator voltage, in q and d axis

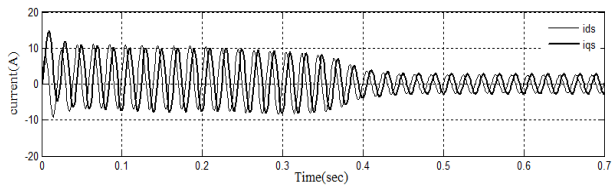

Fig 5: Stator current in q and d axis

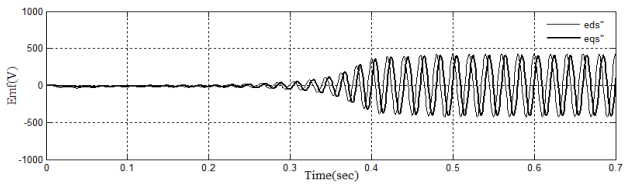

Fig 6: Stator emf in q and d axis

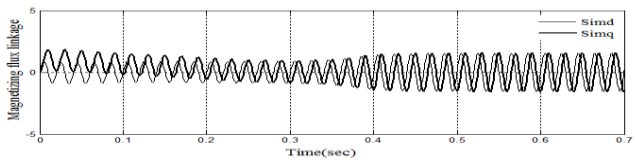

Fig 7: Magnetizing flux linkage in q and d axis

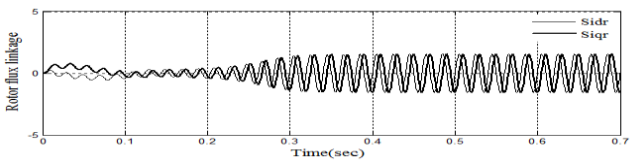

Fig 8: Rotor flux linkage in q and d axis

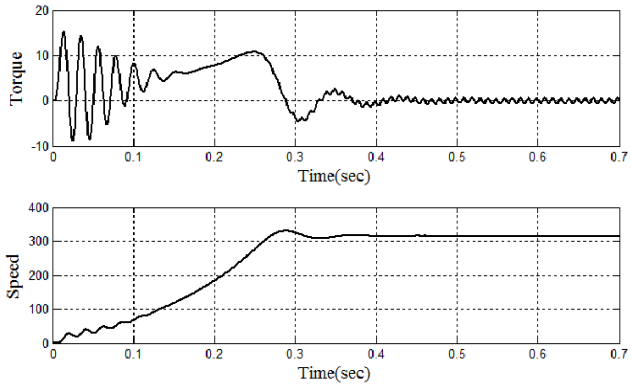

Fig 9: Torque and speed of motor

9. CONCLUSION

This paper has presented the models of 3-phase induction motor. The VBR modeling of the induction motor has been developed for simulation results. This model help to estimate the rotor flux linkage, magnetizing flux linkage, back emf, torque and speed in the machine. The recently proposed VBR model helps to achieve the required direct interface of the stator circuit with outside controllers also provides an improved numerical accuracy with reduced computational overhead.

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