

Comparative performance study between the Time-varying LMS (TVLMS) algorithm, LMS algorithm and RLS algorithm.

Kapil Belpatre

Dept. of Electronics & Telecommunication,
G.H.Raisoni Institute of Technology,
Pune University.

Mrs. Bachute .M.R

Dept. of Electronics & Telecommunication,
G.H.Raisoni Institute of Technology,
Pune University.

ABSTRACT

This paper presents a comparative performance study between the recently proposed time-varying LMS (TVLMS) algorithm and other two main adaptive approaches: the least-mean square (LMS) algorithm and the recursive least squares (RLS) algorithm. Implementational aspects of these algorithms and their computational complexity are examined. Using computer simulations, the successive trade-off between the computational complexity and system noise cancellation ability, as one proceeds from the Wiener estimate to the LMS with fixed step size, becomes apparent. Three performance criteria are utilized in this study: the algorithm execution time, the minimum mean squared error (MSE), and the required filter order. The study showed that the selection of the filter order is based on a trade-off between the MSE performance and algorithm executive time. Results also showed that the execution time of the RLS algorithm increases more rapidly with the filter order than other algorithms. Recently adaptive filtering was presented, have a nice tradeoff between complexity and the convergence speed. This paper also compares a new approach for noise cancellation in speech enhancement using the two new adaptive filtering algorithms named fast affine projection algorithm and fast Euclidean direction search algorithms for attenuating noise in speech signals. The simulation results demonstrate the good performance of the two new algorithms in attenuating the noise.

Keywords

TV-LMS, LMS, NLMS

1. INTRODUCTION

Adaptive techniques use algorithms that enable the adaptive filter to adjust its parameters to produce an output that matches the output of the unknown system. Recently, considerable effort has been directed towards the realization of adaptive digital filters using the efficient block digital filtering technique where the system signals are processed in blocks [2]. Several block FIR adaptive algorithms have been introduced, including the Block Least Mean-Squares (BLMS) algorithm in [3]

and [4], the Optimum Block Adaptive (OBA) [5], and the optimum block adaptive shifting (OBAS) algorithms [6]. The optimum block algorithms (OBA and OBAS) employ a time-varying convergence factor that is the same for the adaptive filter coefficients, but is updated at each block iteration. Recently an OBA algorithm with individual adaptation of parameters (OBAl) has been proposed. This algorithm employs an individual convergence factor that is updated for each adaptive filter coefficient, at each iteration [7]. This algorithm has been shown to be an estimate of the Wiener solution at each iteration [8]. Processing the system signals in blocks leads to computational advantages and efficient implementation using the fast Fourier Transform (FFT). Sequential algorithms that process signals sequentially, i.e. the block length equals one, have been proposed. The individual (IA) and the homogeneous (HA) algorithms [9],[10] belong to a family of optimum adaptive sequential algorithms. These algorithms employ time-varying convergence factors which minimize the mean square error (MSE) resulting in vast improvements in adaptation accuracy and faster convergence at the expense of a relatively modest increase in the number of computations per data sample. The IA algorithm uses a convergence factor that is updated for each adaptive filter coefficient, at each iteration. In case of the HA algorithm, the convergence factors are the same for all the filter coefficients, but are updated at each iteration.

Table I shows the computational requirements of the OBAl, OBA, OBAS, BLMS, HA and LMS algorithms in terms of the number of real multiplications and divisions per iteration (MADPI). L is the length of the data block and N is the number of independent coefficients in the adaptive filter. For the LMS and HA algorithms the block length $L = 1$. While the OBA and the BLMS algorithms perform one iteration per L input samples, the other algorithm can be implemented to perform one iteration per each input sample. It is seen that the HA, OBA, OBAS and the OBAl algorithms require more computations than the LMS and the BLMS algorithms.

TABLE I: COMPUTATIONAL REQUIREMENTS (MADPI) OF THE OBAI, OBA, OBAS, BLMS, HA, AND LMS ALGORITHMS

ALGORITHMS	MADPI(L,N)
OBAI	$7N^2 - 5N + 9$
OBA	$3LN + 2N + L + 1$
OBAS	$L^2 + LN + 3N + 4L$
BLMS	$2LN + N$
HA	$2N + 1$
LMS	$N + 1$

The most popular adaptive algorithms are the least mean square (LMS) algorithm and the recursive least square (RLS) algorithm. The performance of these adaptive algorithms is highly dependent on their filter order and signal condition and also compared with the help of computer simulations. Furthermore, the performance of the LMS algorithm also depends on the selected convergence parameter. As for the RLS algorithm, it is also dependent on (commonly known as “forgetting factor” or “exponential weighting factor” [12]). In this work we will specifically use the version of RLS named as the “growing window” RLS algorithm (with $\lambda = 1$) [12]. Recently, a new version of the LMS algorithm with time varying convergence parameter has been proposed [14]. The time-varying LMS (TV-LMS) algorithm has shown better performance than the conventional LMS algorithm in terms of faster convergence and less MSE.

The TV-LMS algorithm is based on utilizing a time-varying convergence parameter with a general power decaying law for the LMS algorithm. The basic idea of this TV-LMS algorithm is to utilize the fact that the LMS algorithms need a larger convergence parameter value to speed up the convergence of the filter coefficients to their optimal values. After the coefficients converge to their optimal values, the convergence parameter should be small for better estimation accuracy [12]. In other words, we set the convergence parameter to a large value in the initial state in order to speed up the algorithm convergence. As time passes, the parameter will be adjusted to a smaller value so that the adaptive filter will have a smaller mean-squared error. In this paper we conduct a comparative study of the conventional LMS algorithm, the TV-LMS algorithm, and the RLS algorithm in terms of their execution time, filter order, and MSE performance.

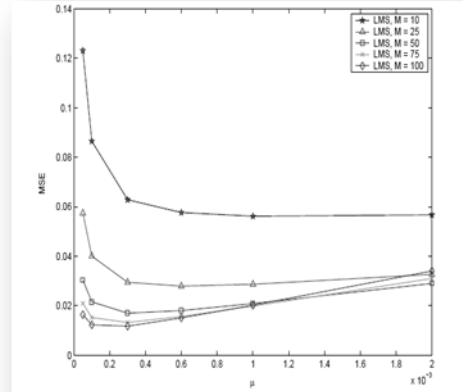


Fig. 1. Mean-squared error (MSE) performance of the LMS algorithm with different μ (SNR = 2 dB, single-tone at $f_o = 100$ Hz).

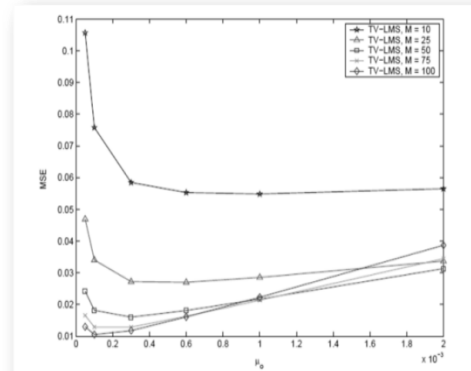


Fig. 2. Mean-squared error (MSE) performance for the time-varying LMS (TV-LMS) algorithm with different μ_o (SNR = 2 dB, single-tone at $f_o = 100$ Hz, $C = 2$, $a = 0.01$, $b = 0.7$).

Fig. 1 and Fig. 2 show that both algorithms provide similar performance results. Hence, for both TV-LMS and conventional LMS algorithms, we should use a higher filter order to provide a better MSE performance for the system. Fig. 2 also shows that the overall performance of the TV-LMS algorithm is better than that of the conventional LMS algorithm. .

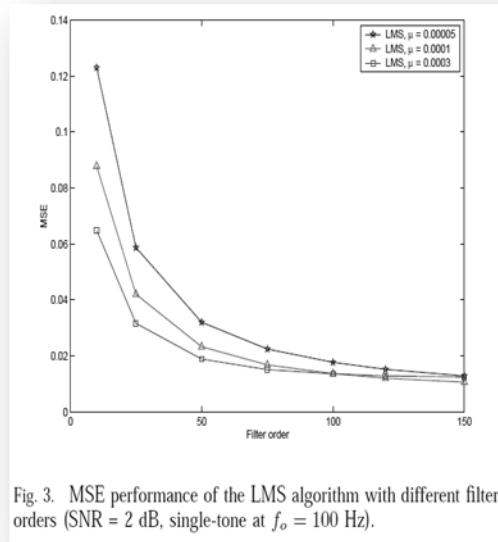


Fig. 3. MSE performance of the LMS algorithm with different filter orders (SNR = 2 dB, single-tone at $f_o = 100$ Hz).

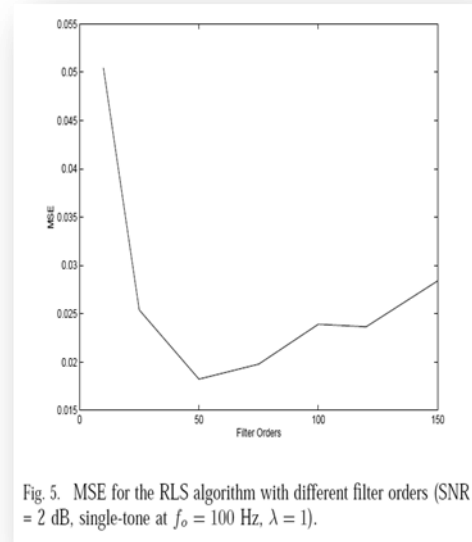


Fig. 5. MSE for the RLS algorithm with different filter orders (SNR = 2 dB, single-tone at $f_o = 100$ Hz, $\lambda = 1$).

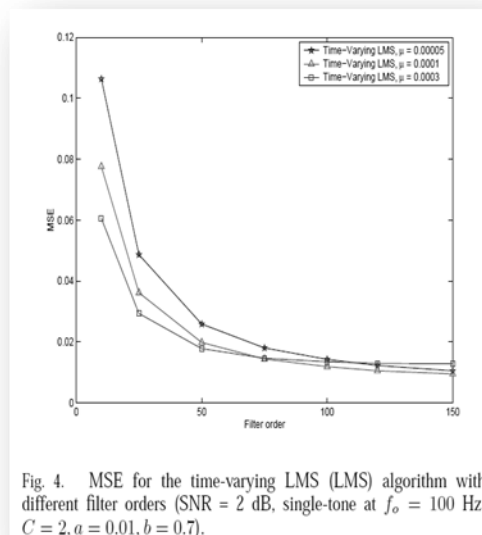


Fig. 4. MSE for the time-varying LMS (LMS) algorithm with different filter orders (SNR = 2 dB, single-tone at $f_o = 100$ Hz, $C = 2$, $a = 0.01$, $b = 0.7$).

Fig. 3 and Fig. 4 show the MSE performance of the TV-LMS algorithm and conventional LMS algorithm with different filter orders. Again, both LMS algorithms provide similar performance results. The TV-LMS algorithm performs much better than the conventional LMS algorithm in the low filter order region. Fig. 3 and Fig. 4 also show that both algorithms provide better MSE performance when filter order increases.

Fig. 5 shows the MSE performance for the RLS algorithm with different filter orders. As the RLS is highly sensitive to numerical instability [11], [12], [13], the filter order will severely affect the performance of the algorithm. Fig. 5 shows that the RLS performance does not improve when the filter order increases. Its optimal filter order in this case is around 50. Hence, a careful selection of the filter order is needed for optimal performance.

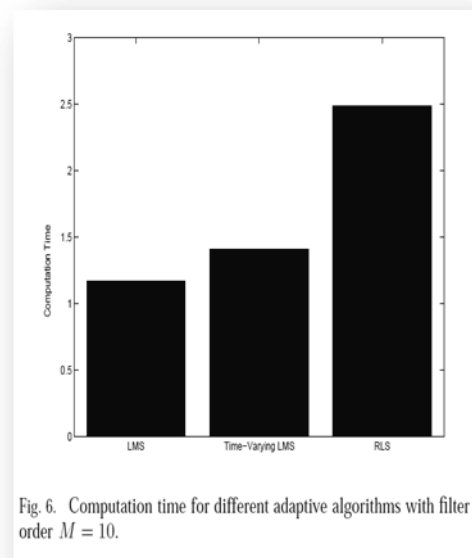


Fig. 6. Computation time for different adaptive algorithms with filter order $M = 10$.

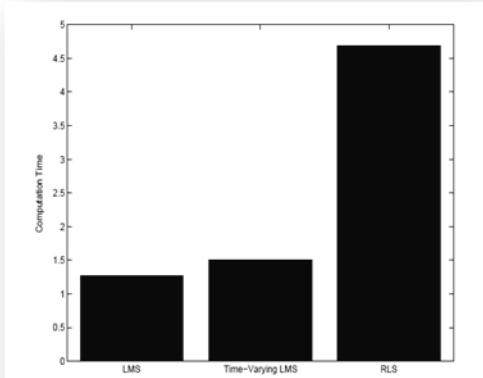


Fig. 7. Computation time for different adaptive algorithms with filter order $M = 50$.

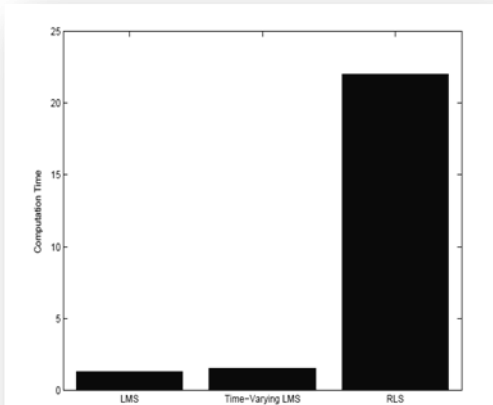


Fig. 8. Computation time for different adaptive algorithms with filter order $M = 100$.

Fig. 6, Fig. 7, and Fig. 8 show the computational time for different algorithms with different filter orders. The computation time for the conventional LMS algorithm and the TVLMS algorithm is relatively similar and much less than that of the RLS algorithm. The above figures also show that the RLS computation time is increasing rapidly and non-linearly with the filter order.

It is well known that two of most frequently applied algorithms for noise cancellation [1] are normalized least mean squares (NLMS) [15]-[18] and recursive least squares (RLS) [19]-[23] algorithms. Considering these two algorithms, it is obvious that NLMS algorithm has the advantage of low computational complexity. On the contrary, the high computational complexity is the weakest point of RLS algorithm but it provides a fast adaptation rate. Thus, it is clear that the choice of the adaptive algorithm to be applied is always a

tradeoff between computational complexity and fast convergence. The convergence property of the FAP and FEDS algorithms is superior to that of the usual LMS, NLMS, and affine projection (AP) algorithms and comparable to that of the RLS algorithm [11]-[14]. In these algorithms, one of the filter coefficients is updated one or more at each time instant, in order to fulfill a suitable tradeoff between convergences rate and computational complexity [15].

Figs. 10-15 show the filter coefficients evolutions of the, LMS, NLMS, AP, FEDS, FAP and RLS algorithms. Again, the results show that the performance of the FEDS and FAP is better than the LMS, NLMS and AP algorithms and comparable with the RLS algorithm.

TABLE II. SNR IMPROVEMENT IN DB

Algorithm	SNRI(db)
LMS	13.5905
NLMS	16.8679
APA	20.0307
FEDS	22.2623
FAPA	24.9078
RLS	29.7355

1. Conclusion

In this paper we presented a comprehensive comparative study between the above time-varying LMS (TV-LMS) algorithm and other two well-known algorithms: the conventional LMS and the recursive least-squared (RLS) algorithm. In a stationary white Gaussian noise environment with filter order M is set at a larger value (e.g., $M = 100$), simulations showed that the TV-LMS algorithm provides the best MSE than the conventional LMS and the RLS algorithms. However, when we choose to use a smaller filter order (e.g., $M = 10$), the RLS algorithm provides the best MSE performance as compared to TV-LMS and the conventional LMS algorithms.

When the computational time is vital to the application, the TV-LMS and the conventional LMS algorithms will be a better choice than the RLS algorithm. Both the TV-LMS and the conventional LMS algorithms provide less computational time than the RLS. In narrow-band applications, like single-tone noise

reduction scheme, the RLS does not provide any further benefit when increasing the filter order, but on the contrary, it requires more computational time in this case.

According to our simulations, the best scenario is to use the TV-LMS algorithm with a larger filter order; this provides an optimal MSE performance with a computation time close to that of the LMS algorithm. As a result, a careful selection of filter order is needed depending on the signal condition and the trade-off between the algorithm complexity and its computational time.

The simulation results demonstrate the good performance of the two new algorithms i.e. AFA & FEDS in attenuating the noise.

REFERENCES

- [1] W. Harrison, J. Lim, E. Singer, "A new application of adaptive noise cancellation," *IEEE Trans. Acoustic Speech Signal Processing*, vol.34, pp. 21-27, Jan 1986.
- [2] C. S. Burrus, "Block implementation of digital filters," *IEEE Trans. Circuit Theory*, vol. CT-18, pp. 697-701, Nov. 1971.
- [3] G. A. Clark, S. K. Mitra, and S. R. Parker, "Block implementation of adaptive digital filters," *IEEE Trans. Circuits Syst.*, vol. CAS-28, pp. 584-592, June 1981.
- [4] G.A.Clark, S. R. Parker, and S. K. Mitra "A unified approach to time- and frequency-domain realization of FIR adaptive digital filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-31, pp. 1077-1083, Oct. 1983.
- [5] W. B. Mikhael and F. H. Wu, "Fast gradient algorithms for block adaptive digital filters," in *Proc. IEEE Inf. Symposium on Circuits and Systems*, San Jose, CA, pp. 968-971, May 1986.
- [6] --, "Fast algorithms for block FIR adaptive digital filtering," *IEEE Trans. Circuits Syst.*, vol. CAS-34, pp. 1152-1160, Oct. 1987.
- [7] "A fast block FIR adaptive digital filtering algorithm with individual adaptation of parameters," *IEEE Trans. Circuits Syst.*, vol. CAS-36, pp. 1-10, Jan. 1989.
- [8] F.H.Wu, "Time-varying gradient algorithms for block implementation of adaptive digital filter," Ph. D. dissertation, West Virginia University, Morgantown, June 1987.
- [9] W. B. Mikhael, F.H.Wu, G. Kang, and L. Fransen, "Optimum adaptive algorithms with applications to noise cancellation," *IEEE Trans CircuitsSyst.*, vol. CAS-31, pp. 312-315, Mar. 1984.
- [10] W. B. Mikhael, F. H. Wu, L. G. Kazovsky, G. S. Kang, and L. L. Fransen, "Adaptive filters with individual adaption of parameters," *IEEE Tans. CircuitsSyst.* Vol.CAS-33, pp. 677-686, July 1986.
- [11] S.Haykin, *Adaptive Filter Theory*, Prentice Hall, 1986.
- [12] M. H. Hayes, *Statistical Digital Signal Processing and Modeling*, John Wiley & Sons, 1996.
- [13] H. Leung, and J. Lam, "Design of demodulator for the chaotic modulation communication system," *IEEE Transactions on Circuits and Systems-I: FUNDAMENTAL THEORY AND APPLICATIONS*, vol. 44, 1997.
- [14] YS. Lau, Z. M. Hussain, and R. Harris, "A time-varying convergence parameter for the LMS algorithm in the presence of white Gaussian noise," Submitted to the Australian Telecommunications, Networks and Applications Conference (ATNAC), Melbourne, 2003
- [15] B. Widrow, S. Stearn, *Adaptive Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1985.
- [16] G. Goodwin, k. Sin, *Adaptive Filtering Prediction and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1985.
- [17] J. R. Treichler, C. R. Johnson, M. G. Larimore, *Theory and Design of Adaptive Filters*, Wiley, 1987.
- [18] S. I. A. Sugiyama, "An adaptive noise canceller with low signal distortion for speech codes" *IEEE Trans. Signal Processing*, vol. 47, pp. 665-674, Mar 1999.
- [19] S. Haykin, *Adaptive Filter Theory*, 4th ed, Prentice Hall, 2002.
- [20] M. Honig, D. Messerschmitt, *Adaptive Filters: Structures, Algorithms and Applications*. Boston Kluwer Academic Publishers, 1984.
- [21] F. Broujeny, *Adaptive Filters: Theory and Applications*, Wiley, 2003.
- [22] A. H. Sayed, *Fundamentals of Adaptive Filtering*, Wiley, 2003.
- [23] P. S. R. Diniz, *Adaptive Filtering Algorithms and Practical Implementation*, 2 Editions, Kluwer, 2002.