Output Feedback Controller for a Single Input Single Output Smart Structure

Ragini Singh
Instrumentation & Control Department, Galgotias College of Engineering & Technology
Greater Noida, U.P., India

Jayalekshmi Nair
Instrumentation Department,
V.E.S Institute of Technology
Chembur, Mumbai

ABSTRACT

A smart structure cantilever beam is a distributed parameter system that employs sensors and actuators at different finite element locations on the beam and makes use of controllers that respond to inputs obtained from the sensors.

The paper uses the mathematical modeling of the smart structure using finite element method and Euler Bernoulli beam assumptions. A state space model of the beam as a Single Input Single Output (SISO) system with two vibratory modes is obtained and the Eigenstructure assignment for linear system with output feedback is studied based on which a controller is designed for two vibratory modes. The effectiveness of the proposed controller is established by the simulation of closed-loop system in MATLAB and the results show that the controller stabilizes SISO system with a remarkable reduction of settling time of the impulse response.

Keywords: Smart Structure, eigenstructure assignment, finite element methodology, output feedback controller.

1. INTRODUCTION

A. Smart Structures

Modeling and control of smart structures has received lot of attention in the past decade [1], [2]. This is due to the fact that the beam is a fundamental element of many engineering structures and its characteristics are well understood [3], [4]. A Smart Structure is a distributed parameter flexible cantilever aluminium beam with piezoelectric patches symmetrically bonded on both sides to provide structural bending. Pair of piezoelectric patches is used for both sensing and actuating purposes. Outputs obtained from the sensors are processed by the controller designed to generate actuating signals to the actuators. This approach ensures that the smart structure takes a corrective action to disturbances and structural deformations due to wind, stress and other forces that may act on the system.

A diagrammatic representation of a controlled smart structure is depicted in fig. 1.

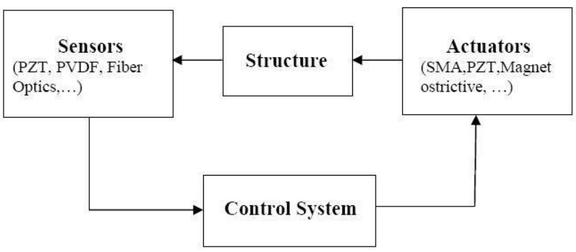

Fig. 1. Block diagram of a smart structure

B. Finite Element Model of the Smart Structure

Basically, the Finite Element method consists of piecewise application of classical variational methods to smaller and simpler sub domains called finite elements, connected to each other in a finite number of points called nodes [5] as shown in fig. 2. The sensor-actuator dynamics is also included in this context while obtaining the model. These values are subsequently used to realize the state space model of the beam, which is used for the controller design [4], [5]. This results in a large dimensional system i.e. a higher order system, typically 4th or higher, depending upon the vibratory modes.

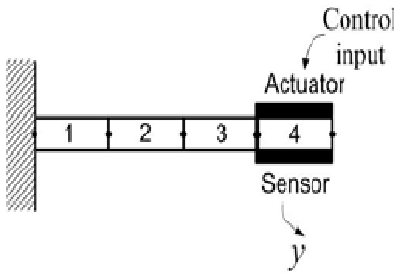

Fig. 2. Smart structure divided into 4 finite elements

2. REVIEW OF EIGEN STRUCTURE ASSIGNMENT TECHNIQUE

Eigenstructure assignment is a design technique which may be used to assign the entire eigenstructure (eigenvalues and eigenvector) of a closed loop linear system via a constant gain full state feedback or output feedback control law. It is a useful tool that allows the designer to satisfy damping, settling time, and mode decoupling specifications directly by choosing eigenvalues and eigenvectors. For a linear time-invariant system the governing equations using the standard notations are given by,

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$

$$y(t) = Cx(t)$$

where $x \in \Re^n$ the state vector , $u \in \Re^m$ the control input vector, $y \in \Re^r$ the output vector, and $A \in \Re^{n.x.n}$ and $B \in \Re^{n.x.m}$ and $C \in \Re^{r.x.n}$ the system matrices respectively. Each of the eigenvalues/vectors of matrix A satisfy the identity,

$$Av_i = \lambda_i v_i \qquad (2)$$

where λ_i is the i^{th} eigenvalue and v_i is the corresponding eigenvector. The free transient response of the system to a non-zero initial condition x0 is given by the equation [6],

$$x(t) = e^{A.t} x_0 \qquad (3)$$

Assuming the eigenvalues of A to be distinct, a non-singular modal matrix Φ consisting of eigenvector can be found, where

$$\phi = [v_1 \quad v_2 \quad v_3 \quad . \quad . \quad v_n] \tag{4}$$

and

$$A = \phi \Lambda \phi^{-1}$$

where Λ is the diagonal matrix of eigenvalues. The equation (2.3.4) can now be written as,

$$x(t) = \phi . e^{\Lambda . t} \phi^{-1} x_0 \tag{5}$$

By defining,

$$\phi^{-1} = [w_1 \quad w_2 \quad w_3 \qquad w_n]^T$$

and

$$\theta_k = \sum_{j=1}^{n} w_{kj} x_{oj}$$

Substituting the above equations in Equation (4), we get,

$$x_i(t) = v_{1i}\theta_1 e^{\lambda_1 t} + v_{2i}\theta_2 e^{\lambda_2 t} + v_{3i}\theta_3 e^{\lambda_3 t} + ...v_{ni}\theta_n e^{\lambda_n t}$$
(6)

From the above equation it can be interpreted that every solution representing a free response of system in equation (6), depends on three quantities:- a) Eigenvalues, which determine the decay/growth rate of response, b) Eigenvectors, which determines the shape of the response, c) Initial condition, which determines the degree to which each mode will participate in the free response.

More recently, eigenvalue assignment via state feedback has been studied more deeply by many researchers. Full state feedback requires that all of the state variables are measurable, which is often not possible. For some system in which states are not measurable, full state feedback is not practical and in such cases the output feedback is used [7], [8].

Output feedback based control algorithms are more practical compared to state feedback based algorithms

[9], [10]. Thus, Eigenstructure assignment by output feedback has been a focal point in multivariable system, as discussed by many researchers [9], [12], [13]. Davison showed that if the system is controllable and if rank [C] = r, then a linear feedback control law of the form *u(t) = F y(t)* , can always be found so that “r” eigenvalues of the closed loop system matrix A+BFC, are arbitrarily close (but not necessarily equal) to the “r” pre-assigned values.

3. EIGENSTURCTURE ASSIGNEMENT BASED OUTPUT FEEDBACK CONTROLLER DESIGN

Controller design consists, essentially, of the following steps: a) Choose a set (or sets) of possible closed-loop eigenvalues (or poles). b) Compute the associated so called allowable eigenvector subspace, which describe the freedom available for closed-loop eigenvector assignment. c) Select specific eigenvectors from the allowable eigenvector subspaces according to some design strategies. d) Calculate a control law, appropriate to the chosen eigenstructure.

In many practical situations, complete specification of vid is neither required or it is unknown, but rather the designer is interested only in certain elements of the eigenvector. Thus, assumes that it has the following structure,

$$v_i^d = [v_{1i} \quad x \quad x \quad x \quad x \quad v_{ij} \quad x \quad x \quad v_{ni}]^T$$
(7)

where v_{ij} are designer specified components and x is an unspecified components. Define, as shown by Andry et al. [4], a reordering operation is done so that,

$$v_i^d = \begin{bmatrix} l_i \\ d_i \end{bmatrix} \tag{8}$$

where l_i is a vector of specified components of v_i^d and d_i ia a vector of unspecified components of v_i^d . The rows of the matrix (λi I - A)-1B are also reordered to conform with the reordered components of v_i^d . Thus,

$$(\lambda_i I - A)^{-1} B = \begin{bmatrix} \tilde{L}_i \\ D_i \end{bmatrix} \tag{9}$$

Then as shown by Andry et al. [14], the achievable eigenvector v_i^d is given by

$$v_i^a = (\lambda_i I - A)^{-1} B \tilde{L}_i^* l_i \tag{10}$$

where (.)* denotes the appropriate pseudoinverse of (.). The output feedback gain matrix using eigenstructure assignment is described by,

$$F = (Z - A_1 V)(CV)^{-1} \tag{11}$$

where A1 is the first m rows of the matrix A in equation (1), V is the matrix whose columns are the r achievable eigenvectors, Z is a matrix whose columns are $\lambda_i z_i$ where the i^th eigenvector vi is partitioned as,

$$v_i = \begin{bmatrix} z_i \\ w_i \end{bmatrix}$$

, with z_i an *(m x 1)* vector and *C* is the output matrix in equation (1)

4. ANALYSIS OF AN SISO SYSTEM WITH TWO VIBRATORY MODES AND RESULTS

The SISO Smart Flexible Cantilever beam is divided into 4 finite elements (FE) and the sensor actuator pair is bonded to the master structure as a collocated pair at one position only, says the fixed end (see Fig. 2) [3]. The state space model of this smart cantilever beam with sensor actuator at FE position 4 for *two vibratory modes* as shown in Fig.2 is given by [3],

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2.5e4 & 5.1e-8 & -2.53 & 5.1e-12 \\ 8.2e-11 & -3.92e2 & 8.2e-15 & -3.9e-2 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0.2 \\ -0.0172 \end{bmatrix};$$

$$C = [0 \quad 0 \quad 4.9e-4 \quad -4.11e-5];$$

and

$$D = [0]$$

For this system the open loop eigenvalues are

λ_1= -1.27+j158, λ_2= -1.27-j158,

λ_3= -0.195e-1+j19.8, λ_4= -0.195e-1-j19.8

Here *rank* [C] = 1, i.e. we will be able to modify one closed loop eigenvalue. The chosen desired closed loop eigenvector is,

$$\begin{bmatrix} x \\ x \\ x \\ 1 \end{bmatrix}$$

where x defines unspecified condition. The feedback gain of the controller is computed for different desired eigenvalues as shown in Table.1. The open loop and closed loop impulse response for the above desired eigenvalues is shown below in Fig.3, Fig.4, Fig.5 and Fig.6.

5. RESULTS: IMPULSE RESPONSE

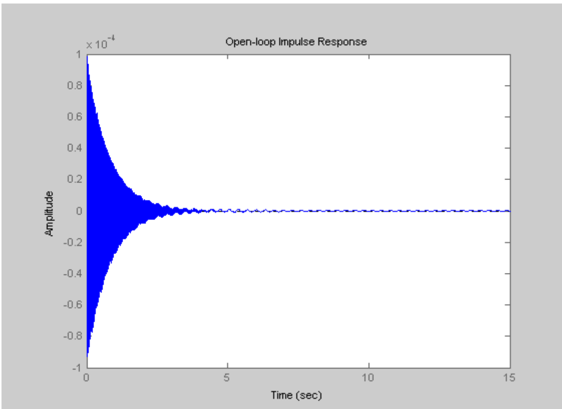

Fig. 3. Open-loop response of SISO system

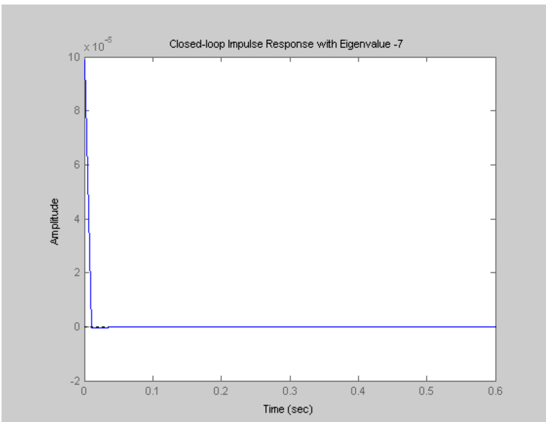

Fig. 4. Closed-loop Impulse response with -7 desired eigenvalues

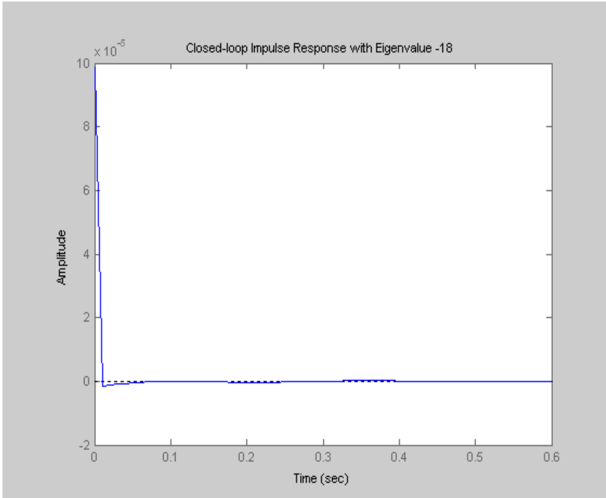

Fig. 5. Closed-loop Impulse response with -18 as desired eigenvalue

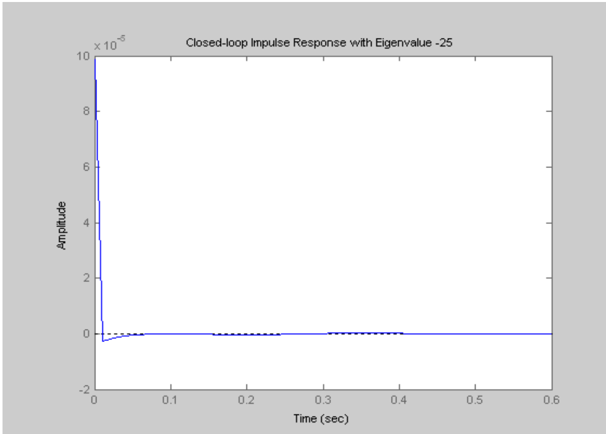

Fig. 6. Closed-loop Impulse response with -25 as desired Eigenvalue

TABLE I

ANALYSIS WITH DIFFERENT DESIRED EIGENVALUES FOR A SISO SMART SYSTEM WITH TWO VIBRATORY MODES

Desired Eigen Values	Controller Gain (F)	Closed-loop eigenvalues				Settling time (Impulse Response) in sec.
		λ_1	λ_2	λ_3	λ_4	
-7	-2.5884e+007	-2550	-7	-1.40+j23.4	-1.40-j23.4	0.04
-18	-1.1419e+007	-1110	-18	-2.24+j22.1	-2.24-j22.1	0.06
-25	-8.8308e+006	-845	-25	-2.22+j21.4	-2.22-j21.4	0.07
-35	-6.8237e+006	-637	-35	-2+j20.9	-2-j20.9	0.07

Analysis of the above results show that the system clearly exhibits moderate settling time but with low impulse response settling time of 0.04s for the system with desired eigenvalue of (-7). Correlating this with the gain of the controller, it is observed that the better results are obtained with increased gain of the controller.

6. CONCLUSION

Controllers have been designed for the smart flexible cantilever beam using the eigenstructure assignment for linear system with output feedback control technique for SISO system to suppress the first two vibratory modes. The flexible cantilever beam was divided into 4 elements system and the sensor / actuator pairs were bonded to the structure at finite element 4(free end). The impulse responses are obtained for various desired eigenvalues. From the simulation results, it is observed that modeling a smart structure by including the sensor / actuator mass and stiffness and its location on the beam at the free end introduces a considerable change in the system's structural vibration characteristics. Thus, unlike static output feedback, the output feedback control technique always guarantees the stability of the closed loop control system. The results are verified from Figs. 3-6 and in section IV.

It has been observed that the closed-loop characteristics of a SISO system with two vibratory modes can be improved by using eigenstructure assignment technique. The open-loop behavior of SISO smart beam element is unstable and our controller design offers a stable behavior with a realistic settling time for selected band of desired eigenvalues. The open-loop and the closed-loop system responses with the controller are compared and it is seen that the vibrations die out very quickly with high controller gain. This study gives the application designer a confidence to build a closed-loop controller for higher order vibratory modes by a judicious choice of desired eigenvalues and controller gain for a good settling time behaviour.

7. REFERENCES

[1] Manjunath T.C. and Bandopadhyay B., “Smart control of cantilever structures using output feedback, A special issue on modeling, simulation and Decision support”, International Journal of Simulation Vol. 7 Nos. 4-5 pp. July 2006.

[2] Manjunath T.C., Bandyopadhyay B., Seshu P. and Umapathy M., “Active Vibration Control of Smart Structures using Fast Output Sampling Feedback Technique” Proceedings of ISSS 2005 International Conference on Smart Materials Structures and Systems July 28-30, 2005.

[3] Manjunath, T.C., and Bandyopadhyay, B., “Controller Design for Euler-Bernoulli Smart Structures Using Robust Decentralized POF via Reduced Order Modeling”, International Journal of Signal Processing, Vol. 3, No.3, pp. 205-221, 2006.

[4] Manjunath, T.C., and Bandyopadhyay, B., “Controller Design for Euler-Bernoulli Smart Structures Using Robust Decentralized FOS via Reduced Order Modeling,” International Journal of Intelligent Systems and Technologies, Vol. 1, No.2, pp. 111-130, June 2006.

[5] Bandyopadhyay B., Manjunath T.C and M.Umapathy, “Modeling, control and implementation of smart structures”. Springer, New York, 2007.

[6] Varadan, Vinoy, Gopalkrishanan, “Smart materials and structures”. John Wiley and sons, 2006.

[7] Liu, G.P. and Patton, R.J, “Eigenstructure assignment for control system”. John Wiley and sons, England 1998.

[8] Andry, A.N., Shapiro, Jr. E.Y and Sobel, K.M., “Eigenstructure Assignment” International Journal of control, Vol. 59 No.1, pp. 13-37, 1994.

[9] Kimura, H., “Pole assignment by Gain Output feedback”, IEEE Transaction on Automatic Control, August 1975.

[10] Srinathkumar S., “Eigenvalue/Eigenvector Assignment usingoutput feedback”, National Aeronautics and Space Administration (NASA) Technical Paper 1118, Hampton, Virginia, February 1978

[11] Moore, B.C., “On the flexibility offered by state feedback in Multivariable system beyond closed loop eigenvalue assignment”, IEEE Transaction on Automatic control, October 1976.

[12] Sobel K. M. and Shapiro E. Y., “Flight control examples of Robust eigenstructure assignment”, Proceedings of the 28thConference on Decision and Control, LA, CA December 1987

[13] Kautsky, J. Nichols, N.K, and Van Dooren, P., “Robust Pole Assignment in Linear State Feedback,” InternationalJournal of Control, Vol.41,No.5, pp.1129-1155, 1985.

[14] Andry, A.N., Shapiro, Jr. E.Y., and Chung, J.C, “Eigenstructure Assignment for Linear Systems” IEEE Transactions on Aerospace and Electronic Systems Vol. Aes-19, No. 5, pp.711-729, September 1983.