Speed Control of BLDC Motor using a Tuned LQR Controller

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ABSTRACT
Brushless Direct Current motors are one of the motor types rapidly gaining popularity. The major problem in BLDC drive system is that some disturbances are originated in the drive which will results in errors and reduces the stability of the system. Conventional controller is used to control the speed of the motor, but the response of the system is affected by steady state error and represents a poor transient response. To regulate the speed of the motor at desired speed is an important application in automotive industries. So we use a Linear Quadratic Regulator controller and Linear Quadratic Gaussian to regulate the speed of the motor. The state variables and control variables of the BLDC drive system are synthesized in this paper. The main objective of this paper is to formulate the control law which results in minimum performance index. This paper spotlights both the design and simulation of optimal & Robust control systems for BLDC motor drive system.

General Terms
Your general terms must be any term which can be used for general classification of the submitted material such as Pattern Recognition, Security, Algorithms et. al.

Keywords
Brushless DC (BLDC) motor drives, Linear Quadratic Regulator (LQR),State Variables, Performance Index, Control Variables

1. INTRODUCTION
Brushless Direct Current (BLDC) are becoming prominent as the demand for efficiency, precise speed and torque control, reliability and ruggedness increases. BLDC provide high efficiency and exemplary precision of control when compared to conventional motors. The most important among them are the lower maintenance due to the elimination of the mechanical commutator and brushes [1], [2]. They are more efficient and have lower rotor losses due to the absence of field windings. This drive can be used for variable speed applications like Electrical Vehicles, Robotics etc.

Modeling and simulation of BLDC motor drive are described in [1], [2]. A mathematical model of PMSM is given in [3].

A fuzzy PID controller of BLDC motor drive is implemented Using digital signal processor in [12]. A phase locked observer is proposed to extract the speed and position of the motor [5]. Tae-won chun [7] proposed a hysteresis comparator to compensate the phase delay of the back emf constant. A novel digital control technique for Brushless DC motor drives is given in [8], [9]. Anand Sathyan et.al [4] presented an FPGA-based novel digital control scheme for BLDC motor drives. A transfer function for the BLDC drive is derived in this paper. They have not investigated an optimal controller for the BLDC system.In this paper, a LQR regulator system is designed for the digitally PWM controlled BLDC motor drive system. The structure of this paper is as follows. Section 2 describes about the digital model of BLDC drives.. Section 3 spotlights the design of LQR. Section 4 discuses about the tuning of LQR. Simulation results is given in section 6.conclusion discuss in section 7 and 8 respectively.

2. MODELLING OF BLDC MOTOR
The speed of a BLDC motor can be controlled by changing the applied voltage across the motor phases. This can be achieved by pulse amplitude modulation, PWM or hysteresis control. Another method of speed control involves sensor less techniques. An FGPA-based novel digital PWM control scheme for BLDC motor drives have been presented in [4]. Fig.1 shows the block diagram for digital PWM control for a BLDC motor drive system. A controller has been designed in this paper.

The torque equation is given by,

$$T_{em} = J\frac{d\omega}{dt} + B\omega + T_L \quad (1)$$

Where Tem , ω(t), B, J and TL denote electromagnetic torque, rotor angular velocity, viscous friction constant, rotor moment of inertia and load torque respectively.

$T_{em}\ \alpha I$ (2)

$$T_{em} = k_t I \quad (3)$$

$$k_t I = J\frac{d\omega}{dt} + B\omega + T_L \quad (4)$$

where Kt= torque constant and I=average current

For the purpose of analysis, the digital controller was considered equivalent to a proportional controller with high gain and saturation.

The transfer function for a BLDC motor is given by [4],

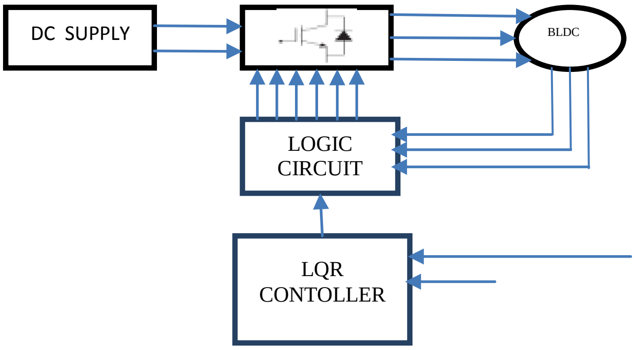

Fig: 1Block Diagram for Digital PWM Control for a BLDC Motor Drive System

$$\frac{\omega(s)}{V(s)} = \frac{Kt/JLa}{s^2 + \left(\frac{JRa + BLa}{JLa}\right)s + \left(\frac{BRa + KtKe}{JLa}\right)} \quad (5)$$

The state variable equation for this BLDC drive is given by,

$$\dot{x}_1 = x_2 \quad (6)$$

$$\dot{x}_2 = -\frac{(BRa + k_t k_e)}{JLa} x_1 - \frac{(JRa + BLa)}{JLa} x_2 + \frac{k_t}{JLa} u \quad (7)$$

Arranging in matrix form we get,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{(BRa + k_t k_e)}{JLa} & -\frac{(JRa + BLa)}{JLa} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_t}{JLa} \end{bmatrix} u \quad (8)$$

where $A = \begin{bmatrix} 0 & 1 \\ -\frac{(BRa + k_t k_e)}{JLa} & -\frac{(JRa + BLa)}{JLa} \end{bmatrix};$

$$B = \begin{bmatrix} 0 \\ \frac{k_t}{JLa} \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The output equation is given by,

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (9)$$

This is in the form,

$$\dot{x} = Ax + Bu \quad (10)$$

$$y = Cx \quad (11)$$

The BLDC drive system parameters are shown in Table

SL. NO.	Parameter	Symbol	Unit	Value
1.	Stator Winding Resistance	Ra	Ω	1.4
2.	Stator Winding Inductance	La	H	0.0066
3.	Rotor inertia	J	Kg-m^2	0.00176
4.	Motor Viscous Friction Coefficient	B	Nm/rad/sec	0.0003888
5.	Torque Constant	K_t	Nm/Amp	0.03
6.	Velocity Constant	K_e	Volts/rad	0.0000181

TABLE 1 BLDC Drive Parameters

3. STATE VARIABLE FEEDBACK

The design of a state feedback BLDC motor control system is based on a suitable selection of a feedback system structure. The stability of BLDC motor drive system is a major concern. If the state variables are known, then they can be utilized to design a feedback controller so that the input becomes U=KX. It is necessary to measure and utilize the state variables of the system in order to control the speed of the BLDC motor. This design approach of state variable feedback control gives sufficient information about the stability of the BLDC drive system. The design of a feedback control system for BLDC drive using state variables are discussed in this section [10], [11].

The vector differential equation of BLDC drive system is given in equation (8).

We will choose a feedback control system so that,

$$u(t) = -k_1 x_1 - k_2 x_2 \quad (12)$$

Then the equation (6) and (7) becomes,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{(BRa + k_t k_e + k_1 k_t)}{JLa} x_1 - \frac{(JRa + BLa + k_2 k_t)}{JLa} \quad (13)$$

ω_A

Arranging in matrix form, we get,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{(BRa + k_t k_e + k_1 k_t)}{JLa} & -\frac{(JRa + BLa + k_2 k_t)}{JLa} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

which is in the form

$$\dot{x} = Ax - kx = (A-k)x = Hx \quad (14)$$

where,

$$H = \begin{bmatrix} 0 & 1 \\ -\frac{\left(BRa + k_t k_e + k_1 k_t\right)}{JLa} & -\frac{\left(JRa + BLa + k_2 k_t\right)}{JLa} \end{bmatrix} \quad (15)$$

Let $k_1 = 1$ and determine a suitable value for k_2 so that the performance index is minimized.

To minimize the performance index J, consider the following two equations, (16) & (17)

$$J = \int_0^\infty X^T X dt = X^T(0) P X(0) \quad (16)$$

$$H^T P + PH = -I \quad (17)$$

$$\begin{bmatrix} 0 & -\frac{\left(BRa + k_t k_e + k_t\right)}{JLa} \\ 1 & -\frac{\left(JRa + BLa + k_2 k_t\right)}{JLa} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -\frac{\left(BRa + k_t k_e + k_t\right)}{JLa} & -\frac{\left(JRa + BLa + k_2 k_t\right)}{JLa} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad (18)$$

Completing matrix multiplication, addition and solving, we obtain,

$$P_{11} = \frac{\left(JRa + BRa + k_t k_e + k_t\right)}{2\left(JRa + BLa + k_2 k_t\right)} + \frac{\left(JRa + BLa + k_2 k_t\right)}{2\left(BRa + k_t k_e + k_t\right)} \quad (19)$$

$$P_{12} = \frac{JRa}{2\left(JRa + k_t k_e + k_t\right)} \quad (20)$$

$$P_{22} = \frac{JLa\left(JLa + BRa + k_t k_e + k_t\right)}{2\left(JRa + BLa + k_2 k_e + k_t\right)\left(BRa + k_t k_e + k_t\right)} \quad (21)$$

$$J = P_{11} + 2P_{12} + P_{22} \quad (22)$$

To minimize as a function of k_2,

$$\text{Set } \frac{\partial J}{\partial k_2} = 0 \quad (23)$$

Therefore

$$k_2 = \frac{-JRaBLa}{k_t} \pm \frac{\sqrt{\left(JRaBLa\right)^2 - \left(JLa + BRa + k_t k_e + k_t\right)\left(BRa + k_t k_e + k_t + JLa\right) + J^2 Ra^2 + 2JRaBLa + B^2 La^2}}{k_t} \quad (24)$$

$$k_2 = 1.01499 \quad (25)$$

$$J_{\min} = 1.47 \quad (26)$$

The system matrix H obtained for the compensated system is,

$$H = \begin{bmatrix} 0 & 1 \\ -2629.476 & -2833.7 \end{bmatrix} \quad (27)$$

The feedback control signal is obtained as,

$$u = -x_1 - 1.015 x_2 \quad (28)$$

This compensated system is considered to an optimal system which results in a minimum value for the performance index. The simulation of this compensated system is listed below and shown in figure 5 & figure 6.

4. LINEAR QUADTRATIC REGULATOR (LQR)

This section deals with the design of a stable control system for BLDC drive based on quadratic performance indexes. The main advantage of using the quadratic optimal control scheme is that the system designed will be stable, except in the case where the system is not controllable. The matrix 'P' is determined from the solution of the matrix Riccatti equation. This optimal control is called the Linear Quadratic Regulator (LQR) [10], [11].

The optimal feedback gain matrix k can be obtained by solving the following Riccatti equation for a positive-definite matrix 'P'.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (29)$$

$$\text{Let } Q = \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix} (\mu \geq 0) \quad (30)$$

Put the value of A,B,Q in equation(29)

$$\begin{bmatrix} 0 & -\frac{(\mathbf{B} R_a + k_t k_e)}{J L_a} \\ 1 & -\frac{(J R_a + B L_a)}{J L_a} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} +$$

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -\frac{(\mathbf{B} R_a + k_t k_e)}{J L_a} & -\frac{(J R_a + B L_a)}{J L_a} \end{bmatrix} -$$

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{k_t}{JLa} \end{bmatrix} [1] \begin{bmatrix} 0 & \frac{k_t}{JLa} \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix} = 0$$

Solving we obtain the following three equations,

$$\frac{P12^2 k_t^2}{J^2 L_a^2} + \frac{2(B R_a + k_t k_e)}{J L_a} - 1 = 0 \qquad (31)$$

$$P_{11} - \frac{P_{12}(J R_a + B L_a)}{J L_a} - \frac{P22(B R_a + k_t k_e)}{J L_a} - \frac{P_{12} P_{22} k_t^2}{J^2 L_a^2} = 0 \qquad (32)$$

$$2 P_{11} - \frac{2 P_{22}(J R_a + B L_a)}{J L_a} - \frac{P22^2 k_t^2}{J^2 L_a^2} + \mu = 0 \qquad (33)$$

Solving these three equations we get,

$$P = \begin{bmatrix} -1.09\times 10^{-3} + \sqrt{1+132.98\mu} & 3.8\times 10^{-4} \\ 3.8\times 10^{-4} & -3.18\times 10^{-5} + 3.35\times 10^{-5}\sqrt{1+132.98\mu} \end{bmatrix} \qquad (34)$$

The optimal feedback gain matrix is obtained as,

$$k = R^{-1} B^T P \qquad (35)$$

$$k = \left[0.981 \quad -0.08 + 0.0865\sqrt{1+132.98\mu}\right] \qquad (36)$$

$$u = -kx = -0.981 x_1 - \left(-0.08 + 0.0865\sqrt{1+132.98\mu}\right) x_2 \qquad (37)$$

Let assume μ =1, the control law u = $0.981x_1 - 0.921x_2$ This control signal yields an optimal result for any initial state under the given performance index. Figure (2) shows the block diagram for optimal control of the BLDC drive system.

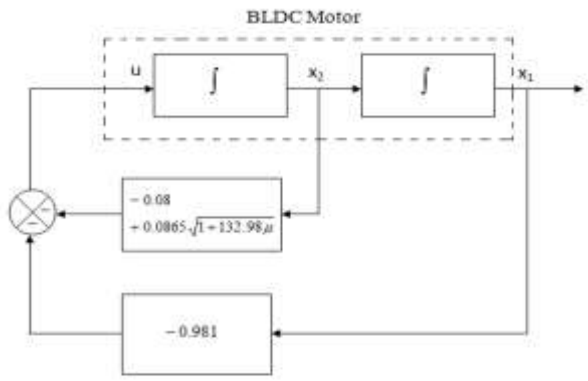

Fig.2 Optimal Control of the BLDC Drive System

5. TUNING OF Q & R MATRIX IN LQR

In LQR the cost function which is to minimized is
$J = \int_0^\infty (X^T Q X + U^T R U)\, dt$(38)

The two matrices Q and R are selected by the design engineer by trial and error method. Generally speaking, selecting a large value for Q requires the value of J to be small. On the other hand, selecting a large value for R, the control input *u* must be smaller to keep value of *J* smallOne should select value of Q to be positive semidefiniteand R to be positive definite. This means that the scalar quantity $X^T Q X$is always positive or zero at each time *t*. The Q & R matrix is tuned by trial & error method. The trial & method is done by MATLAB coding's. The best value of the Q & R matrix is calculated by checking the step response of the system.The best value of

Q=$\begin{vmatrix} 0.41 & 0 \\ 0 & 0.0001 \end{vmatrix}$&R=[1]

By tuning Q & R matrix the value of K_1 = 0.225 &K_2 = 0.0039,the control law

6. SIMULATION RESULTS

MATLAB software package is used to determine the *response* of the system. Tuning of the Q & R matrix is done by separate coding. The regulation of speed and rate of change of speed is determined with and without tuning of Q & R matrix and the tracking of the motor is also determined.

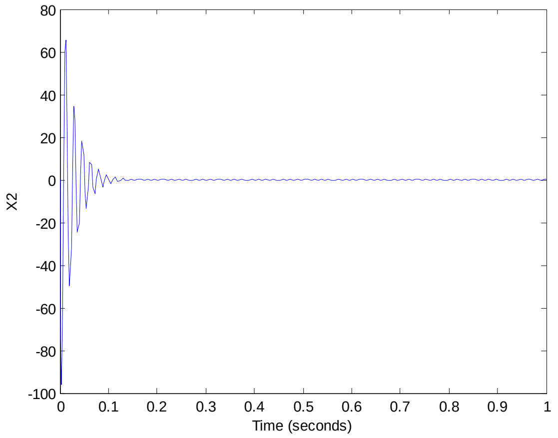

Fig.3 Regulation of rate of change of speed without tuning

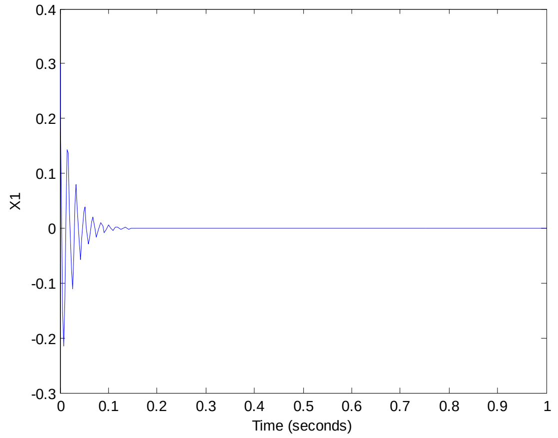

Fig.4 Regulation of speed without tuning

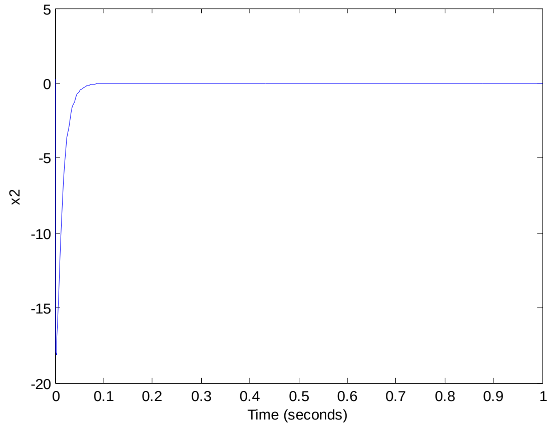

Fig.5 Regulation of rate of change of speed with tuning

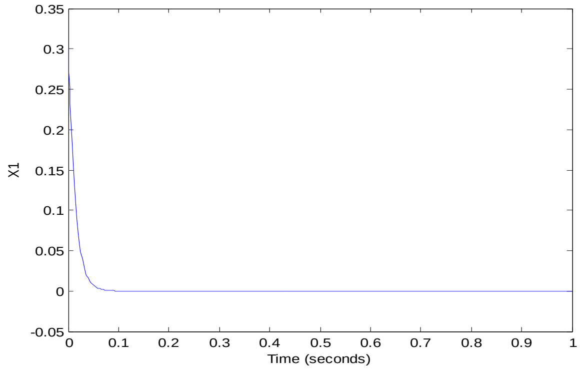

Fig.6 Regulation of speed with tuning

From figure 3 and figure 4 the system is regulated at 0.2 sec, but the system consists of large no of overshoot and undershoot. The system is not precisely regulated at this condition. From figure 5 and 6 the system is regulated at 0.1 sec without any oscillations. The system is completely controlled and the tracking of the motor at rated speed of 2200 rpm is shown in figure 7.The motor is tracked at rated speed at 5.5 sec.

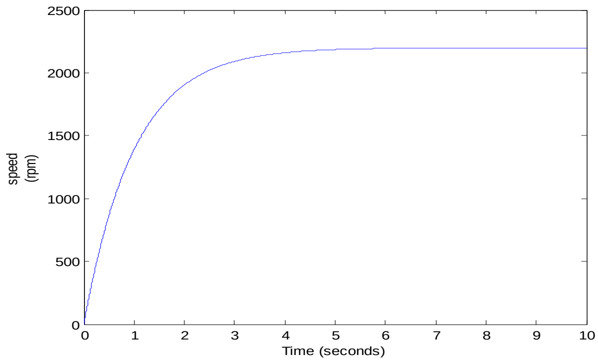

Fig.7 Tracking of BLDC motor at rated speed by LQR

7. CONCLUSION

In this paper a state variable feedback system was designed for BLDC drive system to achieve the desired system response. Also, an LQR system was designed for BLDC drive which results in a minimum value for the performance index.The LQR design provides an optimal state feedback control minimizes the quadratic state error and control effort This optimal controlled BLDC drive system results in a minimum value for the performance index. Also, the control law given by equation (40) yields optimal result for any initial state under the given performance index. Both the transient and steady state response of the system is improved with LQR controller. This design based on the quadratic performance index yields a stable control system for the BLDC drive system.

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