Fuzzy Robustic Technique for Color Image De-noising

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ABSTRACT
De-noising in color images needs to spin straw into the noisy images for eliminating outlier pixels that degrades quality. In this approach a rule based fuzzy logic and a three channel robust estimation are used for noise detection and estimation to reduce Gaussian-impulse noise mixture. Gaussian smoothing is performed over a fuzzy peer group by a weighted averaging (its membership coefficients), which is computed through its membership coefficient. Visual analysis and experimental results using PSNR and MSE values proves improved performance over the existing methods.

General Terms
Fuzzy Similarity, Membership Coefficient, Robust Estimator.

Keywords
Gaussian Impulse Noise, Robust Estimator, Noise Suppression

1. INTRODUCTION
It is a fact that images may have unavoidable additional features due to fault or error in the acquisition and transmission process. Eliminating these odd pixels from the images is known as de-noising. For decades, new algorithms and methods have emerged as a result of active research.

In images, mixture of Gaussian-impulse noise may cause negative impact on imaging applications. The impulse noise corrupts pixels to very high or very low intensity values gives salt and pepper appearance [1] and the additive Gaussian noise are zero-mean Gaussian distribution [2]

Numerous techniques are derived to remove impulse noise from corrupted images, such as adaptive median filters center weighted median filter [3]

Decision or switching based median filters [4] [5] identifies possible noisy pixels and replace them by using the median value or its variants leaving all other pixels unchanged. These filters are good for noise detection. The main drawback is the noisy pixels are replaced by some median value in their vicinity without considering local features.

Fuzzy bilateral filters and fuzzy noise detection methods detect edges and details by means of local statistics and smooth them to preserve their sharpness [6, 7] but these methods skip noise pixels as edges yields low performance. In case of Gaussian noise all the pixels are corrupted.

In this paper an efficient algorithm to remove Gaussian-Impulse noise mixture based on fuzzy similarity and robust estimation is proposed.

2. IMPULSE - GAUSSIAN MIXTURE
Let F denote the color image under consideration and F₁, the pixel value in the image. The occurrence of Gaussian-impulse noise mixture can be modeled with equation (1)

\[ A(i,j) = \begin{cases} F(i) \text{ with probability } 1-pr \end{cases} \]

Where pr is the probability that a pixel is corrupted with noise, and A is the corrupted image.

3. FUZZY PEER GROUP
The peer group of an image pixel is a pixel similarity based concept which has been successfully used to devise image de-noising methods. Since it is inefficient to define the pixel similarity in a crisp way, it is represented in fuzzy expressions. A fuzzy peer group is defined as a fuzzy set that takes a peer group as support set and where the membership degree of each peer group member will be given by its fuzzy similarity with respect to the pixel under processing. The fuzzy peer group of each image pixel will be determined by means fuzzy logic-based procedure. The noise detection and estimation use the same fuzzy peer group, which leads to computational savings.

4. ROBUST STATISTICS
The field of robust statistics is concerned with estimation problems in which the data contains outliers. Robust estimation algorithms can be classified into three large types of estimators: M-estimator, L-estimator, and R-estimator. An M-estimator is a maximum likelihood-type estimator, and it is obtained by solving a minimization problem.

The M-estimators were initially proposed by Huber (1964) [8] as a generalization of the maximum likelihood estimator. The M estimator addresses the problem of finding best fit to the model \( d = \{d_0, d_1, \ldots, d_n\} \) to another model. \( e = \{e_0, e_1, \ldots, e_n\} \) in cases where the data differs statistically from the model assumptions. It finds the value that minimizes the size of the residual errors between \( d \) and \( e \). This minimization can be written using the equation (2).

\[ \min_{\sigma \in S} \sum_{i \in S} \rho((e_i - d_i), \sigma) \]

where \( \sigma \) scale parameter that controls the outlier rejection point, and \( p \) is M-estimator.

Reducing \( \rho \) will cause the estimator to reject more measurements as outliers. \( S \) is the set of all chosen values. \( \sigma \),
is the input model and \( c_i \) is the best fit model. To minimize above, it is necessary to solve the equation (3) & (4)

\[
\sum \Psi((c_i - d_i), \sigma) = 0
\]  

(3)

Where the influence function given by the equation (4),

\[
\Psi(x, \sigma) = \frac{\partial \rho(x, \sigma)}{\partial x}
\]

(4)

Generally, robustness is measured using two parameters: influence function and breakdown point. The influence function gives the change in an estimate caused by insertion of outlying data as a function of the distance of the data from the (uncorrupted) estimate. Breakdown point is the largest percentage of outlier data points that will not cause a deviation in the solution.

To increase robustness, re-descending estimators are considered for which the influence of outliers tends to zero with increasing distance [9]. Lorentzian estimator [10] an Influence function which tends to zero for increasing estimation distance and maximum breakdown value.

The Lorentzian estimator \( \rho_{LOR}(\sigma) \) is defined by the equation (5)

\[
\rho_{LOR}(x) = \log(1 + \frac{x^2}{2\sigma^2})
\]

(5)

and it is described by the influence function \( \Psi_{LOR}(\sigma) \) given by the equation (6)

\[
\Psi_{LOR}(x) = \rho'_{LOR}(x) - \frac{2x}{2\sigma^2 + x^2}
\]

(6)

Where \( x \) is the Lorentzian estimation distance and \( \sigma \) is the breakdown point.

5. PROPOSED ALGORITHM

This method constructs fuzzy peer group for each pixel which forms the base for mixed noise detection and estimation. Impulse noise is replaced with robust Estimation and Gaussian noise is smoothed by weighted averaging operation over the fuzzy peer group members separately for each color components.

The technique has three phases,

(i) Impulse noise detection phase

(ii). Impulse Noise Estimation phase

(iii). Gaussian Noise Smoothing phase

5.1 Impulse noise detection phase

A Fuzzy Similarity function [11] which is used to find the similarity between the central pixel under consideration and its 3X3 neighborhood as in equation (7)

\[
\text{Sim}(F_i,F_j) = \frac{e^{-||F_i-F_j||}}{F_o}
\]

(7)

Where \( ||F_i-F_j|| \) is the Euclidean distance between Fi &Fj

\( F_o \) value is given in the Table I \( F_o \) is a fuzzy parameter that depends on the density of noise in images.

### Table I

<table>
<thead>
<tr>
<th>Suggested Values for ( F_o ) Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard deviation ( \sigma ) of</strong></td>
</tr>
<tr>
<td><strong>Gaussian noise</strong></td>
</tr>
<tr>
<td>Fσ</td>
</tr>
<tr>
<td>Very Low (in [0 10])</td>
</tr>
<tr>
<td>Low (in [10 20])</td>
</tr>
<tr>
<td>Medium (in [20 30])</td>
</tr>
<tr>
<td>High (in [30 40])</td>
</tr>
<tr>
<td>Very High(in [40 50])</td>
</tr>
</tbody>
</table>

A color image pixel can be represented as a 3-component vector constituting the three color components. Each image pixel’s similarity with its neighbors in the window \( W \) is computed using a fuzzy similarity function and sorted in their decreasing order. i.e. \( W=\{F_0,F_1,F_2,..F_n\} \) such that \( \text{Sim}(F_1,F_0)\geq \text{Sim}(F_2,F_0)\geq \text{Sim}(F_3,F_0)\text{...}\geq \text{Sim}(F_n,F_0) \).

To construct a Fuzzy Peer Group two fuzzy sets \( L_{F0}^F \), \( C_{F0}^F \) ,two fuzzy Rules CFR1, CFR2 and accumulated similarity \( A_{F0}^F \) are defined. \( A_{F0}^F \) for the pixels \( F_i \) in \( W \) is defined using equation (8).

\[
(F_i) = \sum_{k=1}^{9} \text{Sim}(F_0,F_k), \text{i.e} \{0,1,..9\}
\]

(8)

And,

\[
L_{F0}^F(F_i) = \#(A_{F0}^F(F_i)) = \frac{1}{(m^2-1)}(A_{F0}^F(F_i)-1)(A_{F0}^F(F_i)-2m^2+1)
\]

(9)

Where, \( i=0,1,..,n^2-1 \)

CFR1 to find the best no of pixels to be added in the FPG and is computed as

\[
\text{CFR1}(m) = C_{F0}^F(F_m), L_{F0}^F(F_m)
\]

(10)

CFR1 is computed for each \( m \in N_n \) where \( N_n = \{1,2,..n^2-1\} \)

\( n=3 \) and \( m \) the best no of elements in FPG is found using equation (11)

\[
m_i = \max_{m \in N_n}
\]

(11)

The fuzzy rule CFR2 is used for impulse noise detection and it is computed using equation (12)

\[
\text{CFR2}(m) = C_{F0}^F(F_m), L_{F0}^F(F_m)
\]

(12)

i.e CFR2(F0)=CFR1(m)
5.2 Impulse Noise Estimation phase

Lorentzian estimator is used for estimation. The estimation phase has the following steps [12]:

1. Select the pixels in the fuzzy peer group.

2. Find \( x \), the difference of each selected pixel with the median value \( S_{med} \) and use the function \( f(x) \) given in the equation (13)

\[
f(x) = 2x/(2\sigma + x)
\]

(13)

Where \( \sigma \) is the outlier rejection point, is given by the equation (14),

\[
\sigma = \frac{\tau_s}{\sqrt{2}}
\]

(14)

Where \( \tau_s \) is the maximum expected outlier and is given by equation (15),

\[
\tau_s = \zeta \sigma_N
\]

(15)

Where \( \sigma_N \) is the local estimate of the image standard deviation and \( \zeta \) is a smoothening factor. Here \( \zeta = 0.3 \) is taken for medium smoothening.

3. Pixel is estimated using the equations (16) and (17)

\[
S_1 = \sum_{x\in L} \frac{\text{pixel}(I) \times f(x)}{x}
\]

(16)

\[
S_2 = \sum_{x\in L} \frac{f(x)}{x}
\]

(17)

Where \( L \) is the number of selected pixels in the fuzzy peer group.

4. Ratio of \( S_1 \) and \( S_2 \) gives the estimated pixel value. Replace impulse pixel with an estimated one.

5.3. Gaussian Noise Smoothing phase

A weighted averaging operation among color vectors in the FPG with the weighting coefficient as its membership degree to the FPG is used to smooth the pixel which as given in equation (18).

\[
F_{out} = \frac{\sum_{m'=1}^{m'} F_{m'} F_{m'}^F (F_{m'})}{\sum_{m'=1}^{m'} F_{m'} F_{m'}^F (F_{m'})}
\]

(18)

Where \( F_{out} \) is the estimated pixel and \( m' \) is the number of pixels in FPG.

\( F_{m'}^F \) is the FPG of \( F_0 \).

6. PERFORMANCE EVALUATION

The effectiveness of the proposed algorithm is evaluated by adding different percentages of impulse noise. Gaussian noise and Gaussian-impulse noise mixture with equal probabilities to three standard color images Flower.png, Lena.png, and Motorbikes.png.

The quantitative values using PSNR computation are listed in Table III, IV, V for gaussian impulse noise mixture, impulse noise, and Gaussian noise.

6.1 Peak Signal to Noise Ratio

The PSNR block computes the peak signal-to-noise ratio between two images. The higher the PSNR, the better the quality of the reconstructed image. PSNR in decibels (dB) is computed by using the equation (19)

\[
PSNR = 10 \log_{10} \left( \frac{255^2}{MSE} \right)
\]

(19)

Where MSE is given by equation (20)

\[
MSE = \frac{1}{MN} \sum_{y=1}^{M} \sum_{x=1}^{N} \left( I(x,y) - I'(x,y) \right)^2
\]

(20)

Where \( I(x,y) \) is the original image, \( I'(x,y) \) is the reconstructed image and \( M, N \) are the dimensions of the images.

The quantitative results of the three standard images Flower.png, Lena.png, and Motorbikes.png are shown in Table III, IV, V respectively and shown graphically in Fig. 1, 2 and 3. This shows that with increase in noise density the PSNR value decreases. But from the consistent results on different images it is proved that the method holds well for different images with different characteristics. The visual results are presented in Fig. 4 to 7(b) that the proposed algorithm is good in noise suppression and image detail preservation.

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>Impulse Noise</th>
<th>PSNR VALUES(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10%</td>
<td>29.33 28.94 28.65</td>
</tr>
<tr>
<td>20</td>
<td>20%</td>
<td>28.72 28.65 28.38</td>
</tr>
<tr>
<td>30</td>
<td>30%</td>
<td>28.09 28.12 28.09</td>
</tr>
<tr>
<td>40</td>
<td>40%</td>
<td>27.76 27.74 27.72</td>
</tr>
<tr>
<td>50</td>
<td>50%</td>
<td>27.55 27.49 27.44</td>
</tr>
</tbody>
</table>
Fig. 1 PSNR plot for de-noising Gaussian and impulse noise mixture on different images.

Table IV. PSNR FOR DE-NOISING IMPULSE NOISE

<table>
<thead>
<tr>
<th>Noise Density</th>
<th>PSNR VALUES (db)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>35.32</td>
</tr>
<tr>
<td>20</td>
<td>32.86</td>
</tr>
<tr>
<td>30</td>
<td>31.39</td>
</tr>
<tr>
<td>40</td>
<td>30.31</td>
</tr>
<tr>
<td>50</td>
<td>29.48</td>
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</tbody>
</table>

Fig. 2 PSNR plot for different impulse noise densities.

Table V. PSNR FOR DE-NOISING GAUSSIAN NOISE FOR DIFFERENT COLOR IMAGES

<table>
<thead>
<tr>
<th>σ</th>
<th>PSNR VALUES (db)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>37.47</td>
</tr>
<tr>
<td>20</td>
<td>33.21</td>
</tr>
<tr>
<td>30</td>
<td>31.09</td>
</tr>
<tr>
<td>40</td>
<td>29.93</td>
</tr>
<tr>
<td>50</td>
<td>29.19</td>
</tr>
</tbody>
</table>

Fig. 3 PSNR plot for de-noising Gaussian noise on different images.

Fig. 4 Flower.png

Fig. 5 (a) Flower.png corrupted with 70% impulse noise (b) De-noised image

Fig. 6 (a) Flower.png corrupted with σ = 30 Gaussian noise (b) De-noised image

Fig. 7 (a) Flower.png corrupted with σ = 30 Gaussian and 30% impulse noise (b) De-noised image

7. CONCLUSION

The fuzzy relationship between the pixels effectively isolates the noisy pixel from its neighbor by fuzzy peer group construction. Fuzzy peer group performs noise detection and estimation over 3X3 window which reduces computational savings which is a basic requirement in color image processing. The robust estimation for impulse noise estimation can successfully handle intensity discontinuities.
and performs well in predicting the accurate estimated value. weighted averaging operation using the membership degree of the pixel in the fuzzy peer group holds well in smoothing Gaussian noise. Experimental results show that the method yields good performance and visual analysis prove that its effectiveness in preserving image details.

8. REFERENCES


