Synchronization of Two Tent Map Systems

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ABSTRACT
Synchronization is one of the most important requirements for designing a Digital Communication System. Synchronization here means feeding both the transmitter and the receiver with the same carrier signal for demodulating the modulated signal accurately. The more accurate is the synchronization, the more accurate is the demodulated signal. Unfortunately, synchronizing the transmitter system and the receiver system perfectly is very complex and difficult. Thus, in this paper, two Tent Map Systems have been synchronized. There is a master system which controls a slave system and synchronization is achieved quickly due to the feedback mechanisms and cascading connections made between both the systems.

General Terms
Cascading, Chaotic system, Digital communication, Feedback, Synchronization, Tent map.

Keywords
Feedback and Cascading Method (FCM).

1. INTRODUCTION
In the world of science and mathematics, new proofs, interesting phenomena, physical behavior, etc., are often discovered with little or more modifications for simplicity or betterment. This is what exactly happened to this paper. It turns out that synchronization of chaotic systems of the kind has been presented in this paper (identical synchronization) was discovered several times, often using different approaches. Below is the list in reverse historical order publications and other information that have been discovered after the initial work of this paper. The reverse order here is not the order in which they have been actually discovered.

- In 1989, Aranson and Rulkov published a paper analyzing synchronization zones in multidimensional dynamical systems. This paper studied the parameters and bifurcation diagrams of synchronized, driven microwave oscillators. This is an early study of synchronization in nonlinear multi-dimensional systems [1].
- In 1989, Volkovskii and Rul'kov studied the bifurcations which lead to stochastic locking in coupled, self oscillating systems with piecewise nonlinear vector fields. They studied both mutual and unidirectional coupling. As in so many Russian papers on this subject, the authors also built experiments to test the theory [2].
- Pikovskii published two papers that had synchronization as their theme in 1984 by coupling chaotic systems, which synchronized two chaotic attractors [3].
- Starting in 1983, Fujisaka and Yamada published a series of four papers on synchronization in chaotic systems in Progress in Theoretical Physics [4][5].

In this paper, a new method to synchronize two Tent map systems, has been proposed, by applying Feedback and Cascading methods. Two Tent map systems have been taken into consideration, one is considered as the master system and the other as the slave system. Both the systems are controlled by a control parameter μ.

2. CHAOTIC SYSTEM
Chaos theory describes the behaviour of certain nonlinear dynamic system that under specific conditions exhibit dynamics that are sensitive to initial conditions. The two basic properties of chaotic systems are the sensitivity to initial conditions and mixing property [6]. Chaotic systems can be very simple, but they produce signals of surprising complexities. One characteristic of a chaotic system is that the signals produced by a chaotic system do not synchronize with any other system. It, therefore, seems impossible for two chaotic systems to synchronize with each other, but if the two systems exchange information in just the right way, they can synchronize. The conditions for synchronization can be described mathematically, and extended to situations where entire arrays of chaotic oscillators are coupled to each other. When an array of synchronized oscillators becomes desynchronized through the changing of a parameter, the first differences that emerge between oscillators can occur on short or long spatial scales. Chaotic synchronization is very sensitive to noise added to the coupling signal, but some techniques for overcoming this sensitivity point to mechanisms that may already exist in systems of coupled neurons. Eventually, the theory of arrays of coupled chaotic oscillators led to developments in the theory of networks.

3. TENT MAP
The generation of random numbers is required in several applications, including measurement and testing of digital circuits and telecommunication systems (e.g., to perform their functional verification and evaluate their immunity to noise). The aim is to achieve a satisfactory tradeoff among the best merit factors. In RADAR or SONAR applications, linear chirps are the most typically used signals to achieve pulse compression.

In mathematics, the tent map with parameter μ is the real-valued function f_μ defined by:

$$f_\mu = \mu \min\{x, 1 - x\} \tag{1}$$

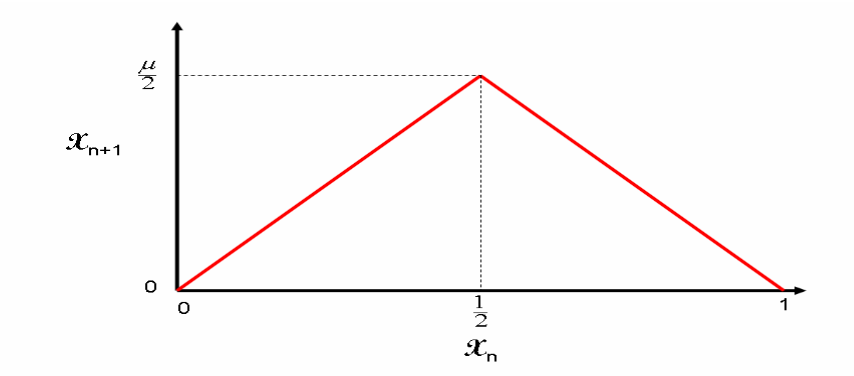

Fig 1: Graph of Tent Map Function.

(the name being due to the tent-like shape of the graph of fμ as shown in fig. 1).

For the values of the parameter μ within 0 and 2, f_μ maps the unit interval [0, 1] into itself, thus defining a discrete-time dynamical system on it (equivalently, a recurrence relation). In particular, iterating a point x_0 in [0, 1] gives rise to a sequence x_n:

$$x_{n+1} = f_\mu(x_n) = \begin{cases} \mu x_n & \text{for } x_n < \frac{1}{2} \\ \mu(1 - x_n) & \text{for } \frac{1}{2} \le x_n \end{cases} \qquad (2)$$

(where μ is a positive real constant).

Choosing for instance, the parameter μ = 2, the effect of the function f_μ may be viewed as the result of the operation of folding the unit interval in two, then stretching the resulting interval [0,1/2] to get again the interval [0,1]. Iterating the procedure, any point x_0 of the interval assumes new subsequent positions as described above, generating a sequence x_n in [0, 1].

The μ = 2 case of the tent map is a non-linear transformation of both bit shift map and r = 4 case of the logistic map [7] [8].

Circuit implementation of Tent Map [9] can be done according to the flow diagram of the tent map as shown in fig. 2.

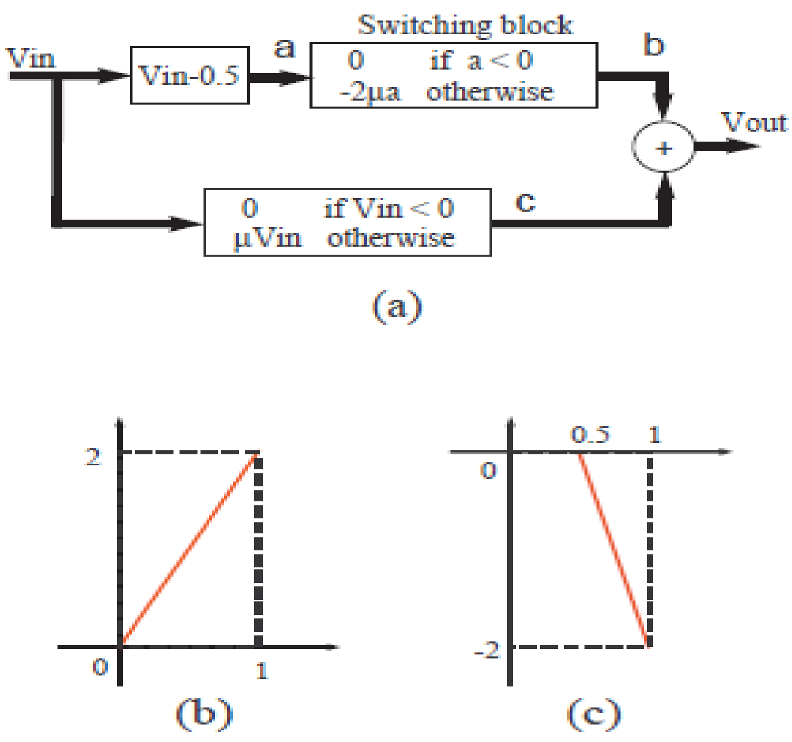

Fig 2 : (a) Block diagram of the tent map used to construct the electronic circuit, (b) Response of the lower branch of the block diagram and (c) Response of the upper branch of the block diagram.

4. UNSYNCHRONIZED TENT MAP SYSTEMS

Firstly, in this paper, 1 D [10] chaotic map is used to produce the chaotic sequence and to control the synchronization process. The chaos streams can be generated by using various chaotic maps like Tent Map, Logistic Map, Quadratic Map, Bernoulli Map, Bit Shift Map. But here, two Tent Maps have been chosen as the two chaotic systems required. The reason behind this is that Tent Map is a 1 D map which exhibits simple behaviour. This makes it simple and easy to use, as compared to the other chaotic systems.

So, here there are two Tent map systems, one is considered as the master system and the other as the slave system. Both the systems are controlled by a parameter μ. The two Tent map systems (master and slave) follow the Eq. (2) with different initial input values, x = 0.7 for master and y = 0.9 for slave, and with same value of the control parameter μ = 1.1. When both the systems operate independently, i.e. in unsynchronized state, their respective instantaneous outputs and the error between the outputs will be as shown in fig. 3.

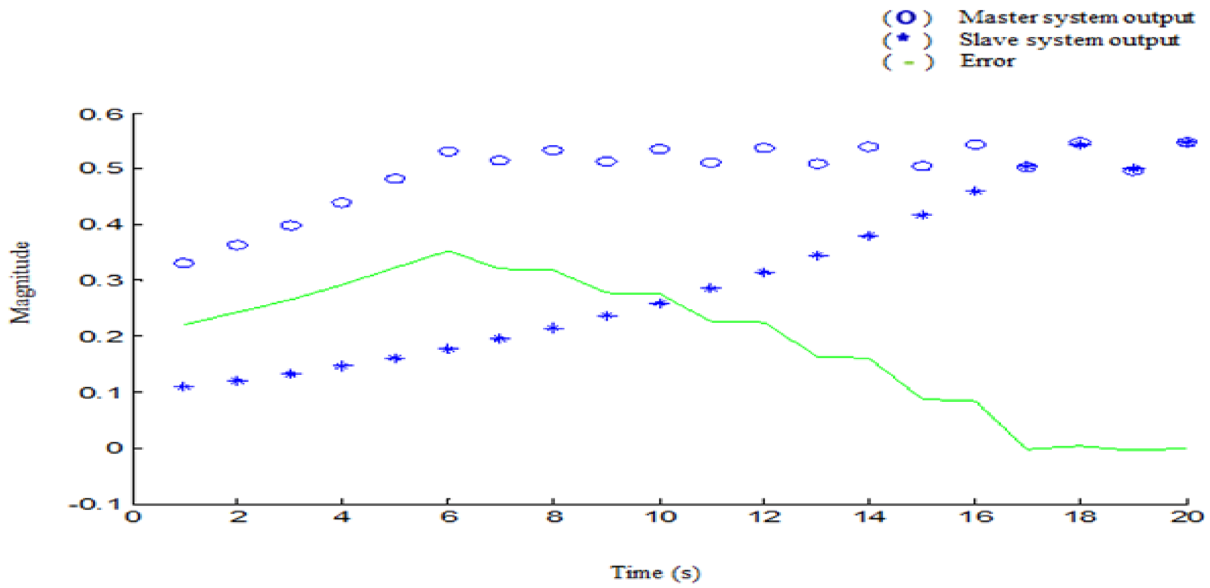

Fig 3: The Tent Map Systems before Synchronization.

5. SYNCHRONIZED TENT MAP SYSTEMS

The functional block diagram of Feedback and Cascading Method (FCM) has been shown in fig. 4. In this method, two systems have been considered, master system and slave system. To make them synchronized, here Feedback and Cascading Method (FCM) has been used. The same value of μ, which is the control parameter, has been considered for the master system as well as for the slave system.

For t = 1s, first initialize the system parameters as x = 0.7, y = 0.9 and μ = 1.1 (as done in case of fig. 3), where x is the input to the master system, y is the input to the slave system and μ is the control parameter for both the systems.

The first output from the master system is obtained as:

$$f_x = \mu \min\{x, 1 - x\} \tag{3}$$

The first output (also for t > 2s) from the slave system is obtained as:

$$f_y = \mu \min\{y, 1 - y\} \tag{4}$$

Now both the outputs are compared and error is evaluated. If the error between the outputs of the master and the slave systems is positive then the control parameter μ is decreased by 0.1 for generation of further sequence of random numbers. If the error between the outputs of the master systems is negative then the control parameter μ is increased by 0.1 for generation of further sequence of random numbers. If there is no error, i.e. both the systems are synchronized, then the control parameter μ is kept as it is.

Now the past output of the Master system becomes the present input for the Master system to achieve tent map characteristics by the feedback mechanism.

For t = 2s, the past output of the Master syatem is divided by the new value of the control variable μ as:

$$y(n+1) = f_x(n) / \mu(n+1) \tag{5}$$

and is fed to the present input of the slave system to achieve synchronization by cascading the two tent map systems.

For t > 2s, the past output of the slave becomes the present input of the slave system to achieve tent map characteristics by feedback mechanism. Further outputs of both the Master system and the Slave system are generated by iterating the above process.

Finally, against the time axis, the instantaneous outputs of both the Master system and the Slave system have been plotted, along with the instantaneous values of the error between the Two Tent Map Systems, using MATLAB.

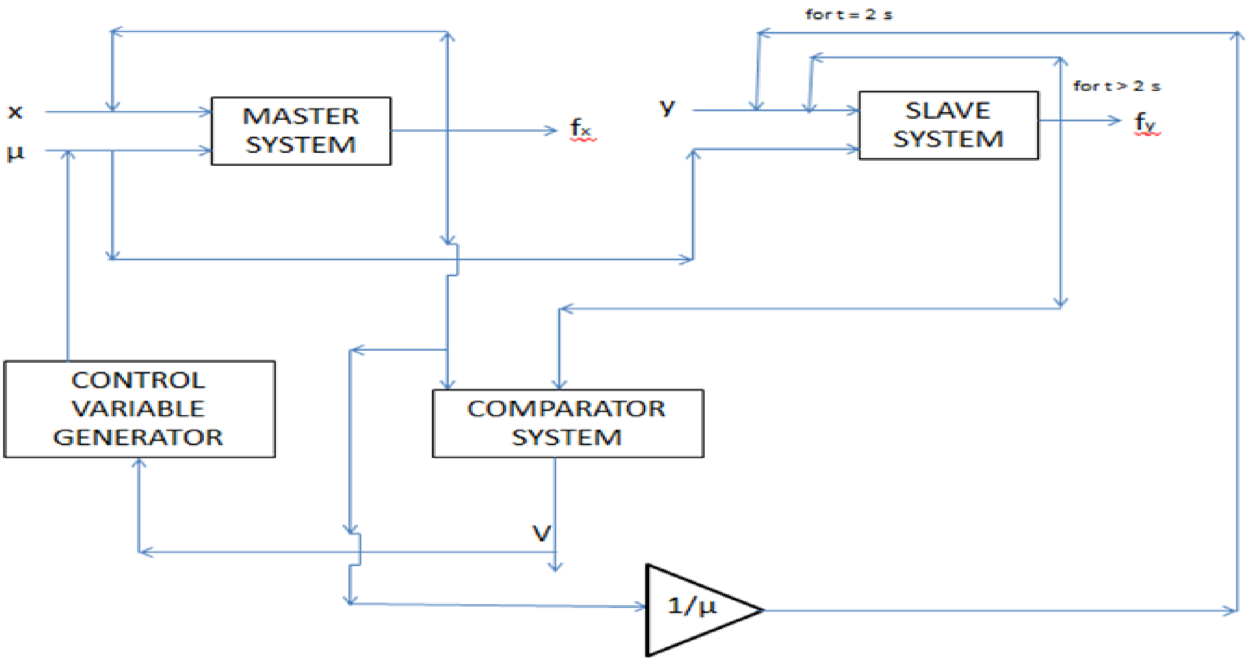

Fig 4: Synchronization of Two Tent Map Systems from FCM

6. RESULT AND ANALYSIS

Before implementing any external modifications, it is observed that both the tent maps function independently and are absolutely unsynchronized, as shown in fig. 3. But when the systems are modified using the proposed Feedback and Cascading Method as explained in fig. 4, the simulation output in MATLAB, as shown in fig. 5, shows both the systems to become gradually synchronized. It can be observed that initially, i.e. for $t < 2$s, both the systems are not synchronized and error is present (although it has a decreasing trend). But, from $t = 2$s onwards, the Slave system starts following the Master system and zero error is obtained, leading to the synchronization of both the Tent map systems. Also, it is observed that the outputs gradually become completely stable, thus enhancing the stability of any system in which the proposed method is applied.

The utmost benefits of applying the method (FCM) of synchronization that this paper presents, are the simplicity of the proposed method and its comparatively very quick response to synchronization.

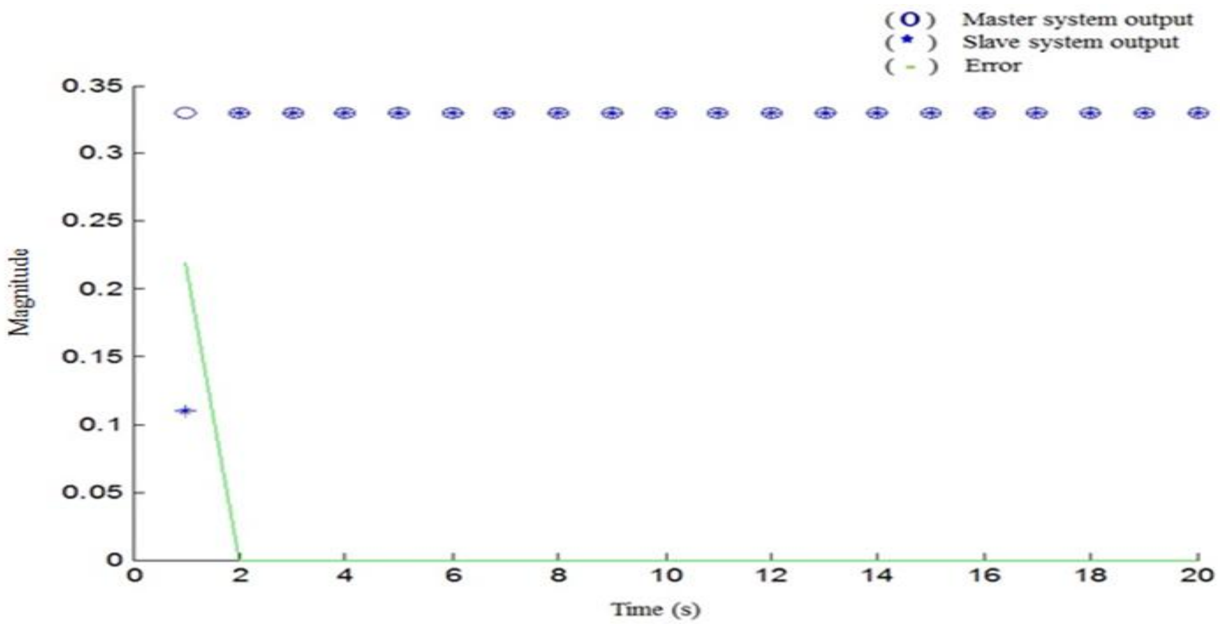

Fig 5: The Tent Map Systems after Synchronization from FCM

7. ACKNOWLEDGMENTS

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8. REFERENCES

[1] RULKOV, NF, and AR VOLKOVSKII. "EXPERIMENTING WITH CHAOS IN ELECTRONIC CIRCUITS."Nonlinear Dynamics in Circuits (1995): 139.

[2] Pecora, Lou, et al. "Synchronization stability in coupled oscillator arrays: solution for arbitrary configurations." International Journal of Bifurcation and Chaos 10.02 (2000): 273-290.

[3] Pikovsky, Arkady, Michael Rosenblum, and Jürgen Kurths. Synchronization: a universal concept in nonlinear sciences. Vol. 12. Cambridge university press, 2003.

[4] Yamada, Tomoji, and Hirokazu Fujisaka. "Stability theory of synchronized motion in coupled-oscillator systems.ii the mapping approach." Progress of Theoretical Physics 70.5 (1983): 1240-1248.

[5] Fujisaka, Hirokazu, and Tomoji Yamada. "Stability Theory of Synchronized Motion in Coupled-Oscillator Systems.IV Instability of Synchronized Chaos and New Intermittency."Progress of theoretical physics 75.5 (1986): 1087-1104.

[6] G. A. Sathishkumar, Dr. K. Bhoopathybagan and Dr. N. Sriraam, 2011. "IMAGE ENCRYPTION BASED ON DIFFUSION AND MULTIPLE CHAOTICMAPS", International Journal of Network Security & Its Applications (IJNSA), Vol.3, No.2.

[7] May, Robert M., 1976. "Simple mathematical models with very complicated dynamics", Nature 261(5560):459-467. (1976).

[8] Weisstein, Eric W., "Logistic Equation", MathWorld.

[9] Campos-Cant´on, E. Campos-Cant´on, J. S. Murgu´ıa and H. C. Rosu, " A Simple Electronic Circuit Realization of the Tent Map", Chaos, Solitons & Fractals 42 (2009) 12-16.

[10] W.Wu .,N. F. Rulkov., :Studying chaos via 1-Dmaps—a tutorial. IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications, vol. 40, no. 10, pp. 707–721 (1993).