A Simple Method for Study of Radial Variation of Pump and Signal Intensities in Mono-Mode Erbium-Doped Graded Index Fiber Amplifier in the Low V Region

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ABSTRACT
An optically pumped doped fiber amplifier has emerged as an important device in all optical technology. It provides broad band amplification in case of multi-channel optical signals. Fibers belonging to low V number are important from the point of view of evanescent field coupling. Accordingly, for study of such fibers in all optical technology, one needs to investigate the response of doped fiber amplifier in the low V region. Therefore, knowledge of fundamental modal field in doped single-mode fiber of low V number is essential to extend the study in the field of variation of modal intensity with radial distance from the axis of the fiber. This study is extremely important in the context of processing information. Using the simple but accurate power series expression for fundamental mode of graded index fiber in the low V region, we predict how the modal intensity varies with radial distance in case of both signal and pump. Choosing some typical step and parabolic index fibers of low V number, we show that our estimations agree with the exact results. Our formalism involves prescription of analytical formulation of the concerned parameters and the execution involves little computation. Thus the present method will be extremely user friendly with the system users.

Keywords
Single-mode graded index fiber, Erbium-doped fiber amplifier, Fundamental modal intensity, Chebyshev technique, Low V region.

1. INTRODUCTION
In long-haul optical communication system, two factors namely attenuation loss and dispersion affect the transmission and accordingly, these need to be taken care of. The pulse dispersion can be minimized either by use of soliton pulses in data stream or by choice of operating wavelength corresponding to minimum dispersion. This type of dispersion managed system, however, involves loss of signal power due to attenuation. This is why compensation of the signal power loss is made by placing optical amplifiers at suitable positions.

Single-mode optical fiber has been considered as the most effective medium in the field of optical fiber communication. We maintain the operating wavelength for optical fiber in between 1.3μm and 1.6μm in order to get minimum attenuation loss as well as minimum material dispersion simultaneously [1]. This is because minimum attenuation loss (around 0.2 dB/km) occurs at the wavelength 1.55μm while the material dispersion vanishes at the wavelength 1.3μm. In the low V region, the study of single-mode graded index fibers has remarkable application in the field of directional coupler which involves evanescent field coupling. An exhaustive solution of vector wave equations relating to electromagnetic field theory is required to develop the general framework for study of propagation of light in an optical fiber. But the fibers, known as weakly guiding fibers, used for communication have extremely small relative core-cladding refractive index difference(less than 0.5%) and accordingly the vector wave equations can be approximated by scalar wave equations for such fibers. It is seen that the values of different propagation parameters obtained by scalar and vector wave equation are almost identical [2]. So one is justified to use scalar wave equations for the sake of simplicity and accuracy as well in case of weakly guiding fiber. Further, application of analytical expression for fundamental modal field of such type of fiber is still difficult owing to the involvement of Bessel functions. The two parameter variational technique [3-5] can predict the fundamental mode of graded index fiber accurately but its execution requires lengthy computation. An accurate but simple power series form of fundamental mode of graded index fiber based on Chebyshev technique has been reported [6]. This method contains prescription of linear relationship of $\frac{K_1(W)}{K_0(W)}$ with 1/W over a wide single-mode range $0.60 \leq W \leq 2.5$, where W denotes the cladding decay parameter. This formalism has been found to have predicted propagation parameters of such fibers excellently in the single-mode region [7-10].

Side by side, this series approximation of fundamental modal field has been extended for low V number corresponding to $W \leq 0.60$. Such extension involves linear approximation of

$\frac{K_1(W)}{K_0(W)}$ with 1/W for many short intervals [11]. The formalism for graded index fiber of low V number has resulted in estimation of concerned fiber parameters excellently in the low V region as well [12-13].

Again, use of EDFA as repeater has removed the complicated process of conversion of optical signal to electrical signal and finally back to optical signal. In addition, EDFA is characterized by large gain (40-45 dB) and low noise (3-4dB) [14,15]. A wavelength division multiplexing (WDM) system implements the signal to be amplified (wavelength range 1530-1570nm) and the light emitted from a laser diode operated by high power pump (wavelength 980nm or 1480nm) into EDFA. The absorption of laser light by erbium ions produces amplification of the propagating optical signal (1530-1570 nm) by the erbium-doped fiber. Moreover, the excited state absorption (ESA) limits the gain obtainable from erbium-doped fiber amplifier (EDFA). It has been found in this context that signal wavelength ranging between 1530 and 1570nm along with pump wavelength at 980nm or at 1480nm lead to efficient performance of EDFA. Thus the study of the pump and the signal in the stated wavelengths emerges as an important matter for EDFA. Based on the simple series expression for fundamental mode of dispersion-managed fiber, the accurate prediction of power as a function of radial distance from the axis of doped step index fiber in case of both signal and pump has been reported [16]. Side by side, borrowing the series expression for first higher order mode (LP_{11}) for graded index fiber available in literature, such study for LP_{11} mode in pump and signal in case of EDFA has been reported [17]. Accordingly, we are motivated to extend the analysis of such fibers in the low V region [11].

In our present study, we employ the simple but accurate power series formulation for fundamental mode for graded index fiber in order to prescribe analytical expression for radial variation of pump and signal Intensities for different wavelength in mono-mode erbium-doped graded index fiber amplifier in the low V Region. Here, we restrict our analysis to step index and parabolic index fibers for some typical low V numbers. Our present work involves very little computation but provides accurate estimation of pump and signal intensity in case of EDFA involving graded index fiber in the low V region. Thus this analysis will be extremely important in all optical technology.

2. THEORY

The refractive index profile of optical fiber is expressed as

$$n^2(R) = \begin{cases} n_1^2(1-2\delta f(R)), & R \le 1 \\ n_2^2, & R > 1 \end{cases} \quad (1)$$

where, R=r/a with 'a' being the radius of the core and 'r' the radial distance from the axis of the fiber and $\delta = (n_1^2 - n_2^2)/2n_1^2$, n_1 and n_2 being refractive indices of the materials of core and cladding respectively. Here, $f(R)$ presents the shape of the refractive index profile of the fiber.

In case of graded index fiber, $f(R)$ can be expressed as

$$f(R) = R^q, \quad R \le 1 \quad (2)$$

where q denotes the profile exponent and its value for parabolic and step index profiles are respectively 2 and ∞.

Employing Chebyshev formalism, one can express the power series solution for modal field as below [6,11].

$$\psi(R) = a_0 + a_2R^2 + a_4R^4 + a_6R^6, \quad R \le 1$$

$$\psi(R) \sim K_0(WR), \quad R > 1 \quad (3)$$

In Appendix A, we describe how the value of cladding decay parameter W as well as values of a_2, a_4, a_6 are found in terms of a_0.

Further, using Eq (3), one can express modal intensity inside core and cladding as below

$$\begin{aligned}\psi^2(R) &= a_0^2 + 2a_0a_2R^2 + (a_2^2 + 2a_0a_4)R^4 \\ &+2(a_0a_6 + a_2a_4)R^6 + (a_4^2 + 2a_2a_6)R^8 \\ &+2a_4a_6R^{10} + a_6^2R^{12}, \quad R \le 1\end{aligned} \quad (4)$$

$$\psi^2(R) = (a_0 + a_2 + a_4 + a_6)^2 \frac{K_0^2(WR)}{K_0^2(W)}, \quad R > 1 \quad (5)$$

The above two Eqs. (4) and (5) predict the variation of modal intensity with normalized radial distance for graded index fiber in case of both signal and pump.

3. RESULTS AND DISCUSSIONS

In order to verify our formalism, we consider two types of step index fiber with different core radii 1.2765μm and 1.08μm and same numerical aperture (NA) of 0.24 corresponding to both pump wavelength (1480nm) and signal wavelength (1550nm) respectively. The normalized frequencies of step index fibers having radii 1.2765μm are 1.3, 1.24125 corresponding to pump wavelength (1480nm) and signal wavelength (1550nm) respectively and those in case of the other fiber of radius 1.08μm are 1.1, 1.05 with pump wavelength (1480nm) and signal wavelength(1550nm) respectively. At pump wavelength 1480 nm, we estimate modal intensity (ψ^2) for the said kinds of step index fiber from Eqs. (4) and (5) and graphically plot its variations with normalized radial distance (R) from the axis of the core of the aforesaid step index fibers in figs.1.1(a) and 1.2(a). Further, Eqs. (4) and (5) are also used to find the modal intensity for the said fibers at signal wavelength1550nm and present the variation of modal intensities with normalized radial distance for the same fibers in Figs.1.1 (b) and 1.2 (b).

We extend the analysis for parabolic index fiber with pump wavelength (1480nm) and signal wavelength (1550nm) for two typical core radii of 1.669μm and 1.4729μm having same numerical aperture (NA) of 0.24. The normalized frequencies are 1.7 and 1.6229 for the fiber of core radius1.669μm corresponding to the pump wavelength (1480nm) and signal wavelength (1550nm) respectively while these for the other fiber of radius 1.4729μm at pump wavelength (1480nm) and signal wavelength (1550nm) are 1.5 and 1.43222 respectively. At pump wavelength 1480 nm, we present graphically the modal intensities (ψ^2) as a function of normalized radial distance (R) for the parabolic index fibers for core radii of 1.669μm and 1.4729μm in Figs.2.1 (a) and

2.2 (a) after evaluating the modal intensities from Eqs. (4) and (5). Proceeding similarly, we plot the variation of modal intensity with normalized radial distance (R) for the parabolic index fiber corresponding to signal wavelength 1550nm for core radii of 1.669µm and 1.4729µm in the Figs.2.1 (b) and 2.2(b) respectively.

It is relevant to mention in this connection that in all the graphs, our calculated results have been presented by dots while the exact results have been shown by solid lines in the practical ranges of study. The excellent match between our results and the exact results verify the validity of our simple formalism involving study of EDFA.

4. CONCLUSION

Using the simple but accurate power series expression for fundamental mode of graded index fiber in the low V region, we prescribe analytical expressions of the modal intensity as a function of radial distance in case of both signal and pump. Choosing some typical step and parabolic index fibers of low V number, we apply our theoretical formulations to estimate radial variation of modal intensity for both pump and signal and show that our predictions match excellently with the exact results. Our formalism involves little computation in the process of execution. Thus the present method will be extremely user friendly with the system users. The excellent prediction by our theoretical formalism leaves enough scope for more investigations involving EDFA in the low V region.

5. APPENDIX A

In case of a weakly guiding mono-mode circular core fiber, the fundamental modal field Ψ (R) inside the core can be expressed by the following scalar wave equation [1,6,11].

$$\frac{d^2\psi}{dR^2}+\frac{1}{R}\frac{d\psi}{dR}+\left[V^2\left(1-f(\mathrm{R})\right)-W^2\right]\psi=0 \quad \text{(A1)}$$

together with the boundary condition

$$\left(\frac{1}{\psi}\frac{d\psi}{dR}\right)_{R=1}=-\frac{WK_1(\mathrm{W})}{K_0(\mathrm{W})} \quad \text{(A2)}$$

here, $V\left[=k_0\mathrm{a}(\mathrm{n}_1^2\text{-}\mathrm{n}_2^2)^{\frac{1}{2}}\right]$ and $W\left[=\mathrm{a}\left(\beta^2-n_2^2k_0^2\right)^{\frac{1}{2}}\right]$ represent the normalised frequency and cladding decay parameter respectively with k_0 and β denoting the wave number in free space and propagation constant respectively.

The fundamental modal field inside the cladding of the fiber can be given as

$$\psi(R) \square K_0(WR), \qquad R>1 \quad \text{(A3)}$$

Applying least square fitting technique over the interval $W \le 0.6$, we develop the linear relationship of $\frac{K_1(\mathrm{W})}{K_0(\mathrm{W})}$ with $1/W$ as given below

$$\frac{K_1(\mathrm{W})}{K_0(\mathrm{W})}=\alpha+\frac{\beta}{W} \quad \text{(A4)}$$

where, the values of α and β are evaluated by least square fitting technique for some short intervals of W as presented in Table 1A.

Table 1A. Values of α and β found for different ranges of cladding decay parameter W [11].

W	α	β
0.000008 – 0.00001	1784.03559000	0.06519083
0.00001 – 0.00005	397.14916110	0.08228378
0.0003 – 0.0007	36.29120034	0.11094628
0.001 – 0.005	11.89030264	0.13114227
0.01 – 0.10	2.12331170	0.20702490
0.10 – 0.20	1.36850086	0.27248694
0.20 – 0.30	1.22199127	0.30182093
0.30 – 0.40	1.15243299	0.32309927
0.40 – 0.50	1.11884710	0.33673703

Further, taking into consideration that the fundamental modal field $\psi(R)$ is an even function of R together with $\psi'(0)$ and $\psi(0)$ being zero and nonzero respectively, one can express $\psi(R)$ in terms of a Chebyshev power series expansion as presented below [18-19]

$$\psi(R)=\sum_{j=0}^{j=M-1}a_{2j}R^{2j} \quad \text{(A5)}$$

The Chebyshev points are given as

$$R_m=\cos\left(\frac{2m-1}{2M-1}\frac{\pi}{2}\right),\ m=1,\ 2,\ 3,\ldots(\text{M-1}). \quad \text{(A6)}$$

Following [6,11], we take $M=4$ and thereby (A5) can be put as

$$\psi(R)=\sum_{j=0}^{j=a}a_{2j}R^{2j} \quad \text{(A7)}$$

and the corresponding Chebyshev points are found as below [6, 11]

R_1=0.9749, R_2=0.7818 and R_3=0.4338 (A8)

The power series expression for $\psi(R)$ in (A7) is employed in (A1). As a result, we obtain the following three equations corresponding to three Chebyshev points given in (A8)

$$a_0[(V^2(1-f(R_i))-W^2)]+a_2[4+R_i^2(V^2(1-f(R_i))-W^2)]$$
$$+a_4[16R_i^2+R_i^4(V^2(1-f(R_i))-W^2)]$$
$$+a_6[36R_i^4+R_i^6(V^2(1-f(R_i))-W^2)]=0 \quad \text{(A9)}$$

where i=1,2 and 3.

Further, use of (A7) and (A4) in (A2) leads to the following equation

$$a_0\left(\alpha W+\beta\right)+a_2\left(\alpha W+2+\beta\right)+a_4\left(\alpha W+4+\beta\right)$$
$$+a_6\left(\alpha W+6+\beta\right)=0 \quad \text{(A10)}$$

Further, a_0, a_2, a_4 and a_6 contained in the three equations of (A9) and (A10) will give non-trivial solution if

$$\begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 & \delta_1 \\ \alpha_2 & \beta_2 & \gamma_2 & \delta_2 \\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3 \\ \alpha_4 & \beta_4 & \gamma_4 & \delta_4 \end{vmatrix}=0 \quad \text{(A11)}$$

where,

$$\alpha_i=V^2(1-f(R_i))-W^2$$
$$\beta_i=4+R_i^2(V^2(1-f(R_i))-W^2)$$
$$\gamma_i=16R_i^2+R_i^4(V^2(1-f(R_i))-W^2) \quad \text{(A12)}$$
$$\delta_i=36R_i^4+R_i^6(V^2(1-f(R_i))-W^2)$$

with i being 1, 2 and 3 together with the corresponding Chebyshev points R_1=0.9749, R_2=0.7818, R_3=0.4338 .

also,

$$\alpha_4=\alpha W+\beta; \qquad \beta_4=2+\alpha_4;$$
$$\gamma_4=4+\alpha_4; \qquad \delta_4=6+\alpha_4 \quad \text{(A13)}$$

One can find cladding decay parameter (W) for a given value of V by solving (A11).

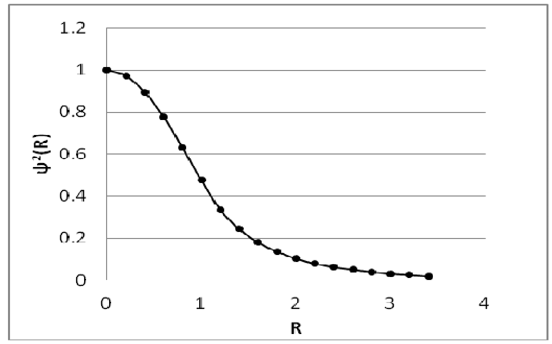
Further, the found value of W can be used in any three of the four equations given by (A9) and (A10) to find the constants a_{2j} (j =1,2,3) in terms of a_0. Thus the field inside the core and cladding are found as

$$\psi^2(R)=a_0^2[1+2A_2R^2+\left(A_2^2+2A_4\right)R^4$$
$$+2(A_6+A_2A_4)R^6+(A_4^2+2A_2A_6)R^8 \quad \text{(A14)}$$
$$+2A_4A_6R^{10}+A_6^2R^{12}], \qquad R\le 1$$

$$\psi^2\left(R\right)=a_0^2(1+A_2+A_4+A_6)^2\,\frac{K_0^{\,2}(WR)}{K_0^{\,2}(W)}, \qquad R>1 \quad \text{(A15)}$$

where, $A_{2j}=\dfrac{a_{2j}}{a_0}$, j = 1, 2, 3 and W is found by the present method.

Therefore, A_2, A_4, A_6 in the expression for $\psi(R)$ in Eqs (A14) and (A15) are values of a_{2j} normalized in terms of a_0.



.Fig.1.1(a): Variation of $\psi^2(\mathrm{R})$ with R for pump (1480 nm) in case of step index fiber (N.A=0.24, a=1.2765µm, W=0.4972) having V=1.3; • our results, − exact results

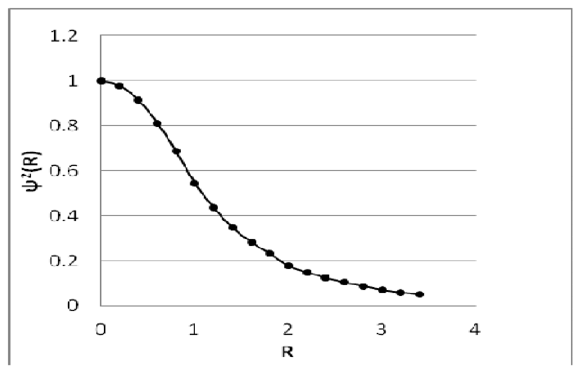


Fig.1.1(b): Variation of $\psi^2(\mathrm{R})$ with R for signal (1550 nm) in case of step index fiber (N.A=0.24, a=1.2765, W=0.434975) having V=1.24125; • our results, − exact results

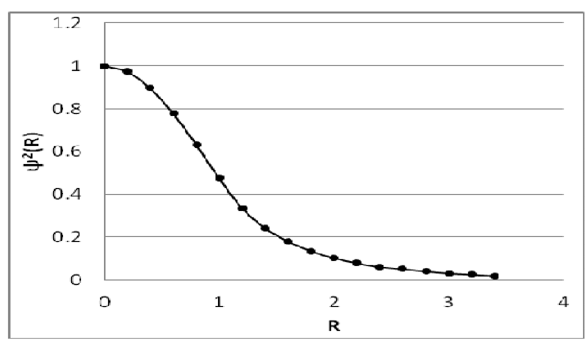


Fig.1.2(a): Variation of $\psi^2(\mathrm{R})$ with R for pump (1480 nm) in case of step index fiber (N.A=0.24, a=1.08µm, W=0.292715) having V=1.1; • our results, − exact results

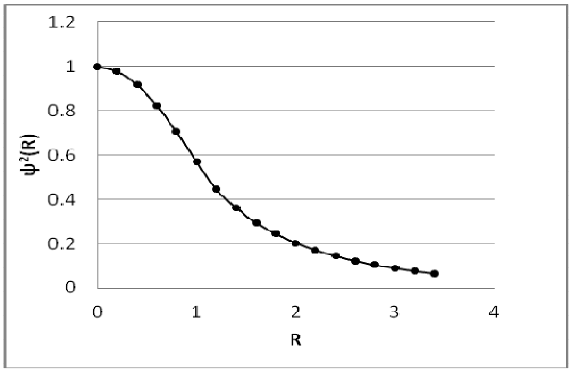


Fig.1.2(b): Variation of $\psi^2(\mathrm{R})$ with R for signal (1550 nm) in case of step index fiber (N.A=0.24, a=1.08, W=0.246499) having V=1.05; • our results, − exact results

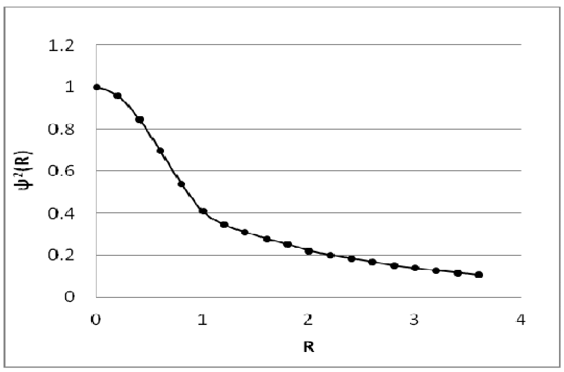


Fig.2.1(a): Variation of $\psi^2(\mathrm{R})$ with R for pump (1480 nm) in case of parabolic index fiber (N.A=0.24, a=1.669, W=0.48855) having V=1.7; • our results, − exact numerical results

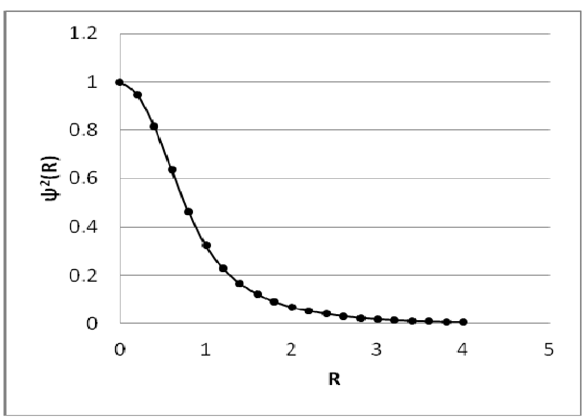


Fig.2.1(b): Variation of $\psi^2(\mathrm{R})$ with R for signal (1550 nm) in case of parabolic index fiber (N.A=0.24, a=1.669, W=0.419759) having V=1.6229; • our results,−exact numerical results

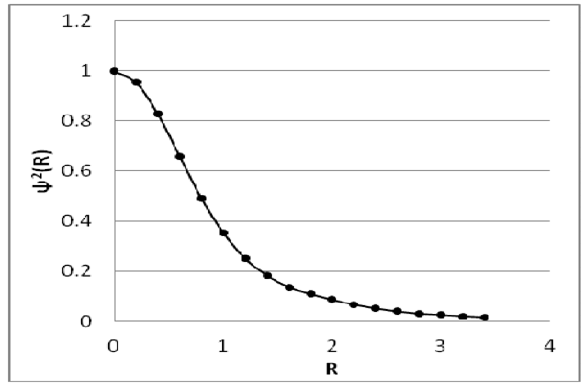


Fig.2.2(a): Variation of $\psi^2(\mathrm{R})$ with R for pump (1480 nm) in case of parabolic index fiber (N.A=0.24, a=1.4729, W=0.31538) having V=1.500; • our results, − exact numerical results

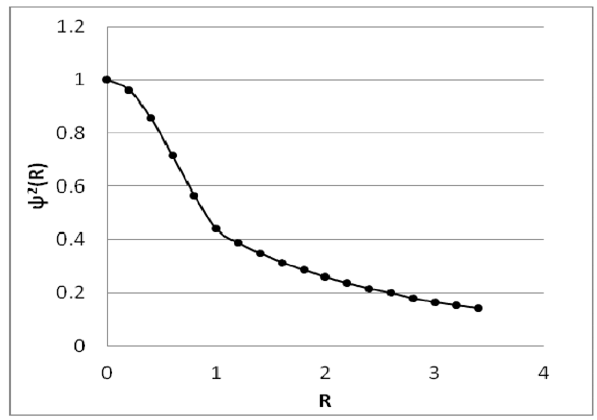


Fig.2.2(b): Variation of $\psi^2(\mathrm{R})$ with R for signal (1550 nm) in case of parabolic index fiber (N.A=0.24, a=1.4729, W=0.260847) having V=1.43222; • our results,– exact numerical results

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