Optimization of Apodized Fiber Bragg Grating for Sensing Applications

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ABSTRACT
This paper presents the modeling and characterization of an Apodized optical fiber Bragg grating for maximum reflectivity and minimum side lobe power wastage and narrow spectral response. The modeling is based upon coupled mode theory together with transfer matrix method. This matrix approach is effective at treating a single grating as a series of separate gratings each having reduced overall length and different pitch lengths, and describing each with its own T-matrix.

FBG sensors are based on the fact that Bragg wavelength changes with change in pitch of the grating and the change in refractive index. Thus, any physical parameter which cause change in above mentioned parameters can be sensed using FBG. In optical sensing, the broad spectral response can result in poor sensitivity. In order to improve and to some extent to tailor the spectral response of FBG length, refractive index change, apodization and FWHM is optimized based upon maximum reflectivity criteria.

Index Terms: FBG, sensor, reflectivity, FWHM, coupled mode theory.

1. INTRODUCTION
Fiber and integrated optics technologies were primarily developed for telecommunication applications. However, the advances in the development of high quality and competitive price optoelectronic components and fibers have largely contributed to the expansion of guided wave technology for sensing as well. The main reasons which make guided wave optics attractive for sensing can be summarized as follows:
- Non-electrical method of operation, which is explosion-proof and offers intrinsic immunity to radio frequency and, more generally, to any kind of electromagnetic interference;
- Small size/weight and great flexibility, that allow access to otherwise restricted areas;
- Capability of resisting to chemically aggressive and ionizing environments;
- Easy interface with optical data communication systems and secure data transmission.

Many types of fiber gratings can be used in sensing applications including Bragg, long-period, and chirped gratings. The basic principle of FBG sensors is the measurement of an induced shift in the wavelength of an optical source due to a measured, such as strain or temperature, or chemical etc.[1], [2]. A basic reflective FBG sensor system is shown in Fig. 1. A broadband light source is used to interrogate the grating, from which a narrowband slice is reflected. The peak wavelength of the reflected spectrum can be compared to \( \lambda_B \). Narrower resonance peaks are more desirable for high accuracy wavelength measurement.

![Fig. 1. Basic reflective FBG sensor system](image)

2. MODELING
Please Refer Fig. 2, the fiber contains a Bragg grating, of length \( L \) and uniform pitch length \( \Lambda \). The electric fields of the propagating waves can then be expressed as

\[
E_a(z, t) = A(z) e^{i(\omega t - \beta z)} \quad (1)
\]

\[
E_b(z, t) = B(z) e^{i(\omega t + \beta z)} \quad (2)
\]

For the backward and forward propagating waves respectively [3-6].

![Fig. 2. Propagating waves in Bragg grating](image)

The coupled-mode equations describe their complex amplitudes, \( A(z) \) and \( B(z) \)

\[
\frac{dA(Z)}{dz} = i\kappa B(Z) e^{-i(\beta \Lambda z)} \quad 0 \leq z \leq L \quad (3)
\]

\[
\frac{dB(Z)}{dz} = -i\kappa^* A(Z) e^{i(\beta \Lambda z)}
\]

If we assume that both forward and backward waves enter the grating, then assume the boundary conditions \( B(0) = B_0 \) and \( A(L) = A_L \). Substituting these boundary conditions into equation 3, we can solve for the closed-form solutions and thus the z-dependence of the two waves.
\begin{align*}
a(z) &= A(z)e^{-i\beta z} \\
b(z) &= B(z)e^{i\beta z}
\end{align*}

The reflected wave, \(a(0)\), and the transmitted wave, \(b(L)\) can be expressed by means of the scattering matrix

\[
\begin{bmatrix}
a(0) \\ b(0)
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
a(L) \\ b(L)
\end{bmatrix}
\]

Substituting \(a(L)\) and \(b(0)\) from equation 4 into equation 5 we get

\[
S_{11} = S_{22} = \frac{iS}{\Delta \beta \sinh (SL) + iS \cosh (SL)}
\]

\[
S_{12} = \frac{\kappa}{i} S_{21} e^{i\Delta \beta L} = \frac{\kappa \sinh (SL)}{-i \beta \sinh (SL) + iS \cosh (SL)}
\]

Based on equations 5 and 6, the scattering matrix, we can obtain the transfer-matrix, or

\[
T \text{-matrix equation}
\]

\[
\begin{bmatrix}
a(0) \\ b(0)
\end{bmatrix} =
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
a(L) \\ b(L)
\end{bmatrix}
\]

Where

\[
T_{11} = T_{22} = \frac{\Delta \beta \sinh (SL) + iS \cosh (SL)}{\Delta \beta \sinh (SL) + iS \cosh (SL)} e^{-i\Delta \beta L}
\]

\[
T_{12} = T_{21} = \frac{\kappa \sinh (SL)}{iS} e^{-i\Delta \beta L}
\]

This matrix approach is effective at treating a single grating as a series of separate gratings each having reduced overall lengths and different pitch lengths, and describing each with its own T-matrix. Combining all the matrices yields the properties of the initial non-uniform grating. The resultant system of matrices is treated as an individual matrix

\[
[T] = [T_1] [T_2] ... [T_M]
\]

Light passing through successive optical elements can be calculated by series of matrices, as such

\[
\begin{bmatrix}
a(0) \\ b(0)
\end{bmatrix} = [T_M] [T_{M-1}] ... [T_2] [T_1]
\begin{bmatrix}
a(L) \\ b(L)
\end{bmatrix}
\]

The characteristics response from Bragg Grating can be fully described by

- The center wavelength of Grating \(\lambda B\)
- Peak reflectivity \(R_{\text{max}}\) of grating which occur at \(\lambda B\)
- Physical length of Grating \(L\)
  1. Refractive index of core of optical fiber \(n_o\)
  2. Amplitude of induced core index perturbation \(\Delta n\)
- For a grating with uniform index modulation and period the reflectivity is given by

\[
R(L, \lambda) = \frac{\kappa^2 \sinh^2 (SL)}{\Delta \beta^2 \sinh^2 (SL) + \kappa^2 \cosh^2 (SL)}
\]

3. Where

4. \(R\): Grating reflectivity as a function of both grating length and wavelength

5. \(L\): total length of grating

6. \(\kappa\): coupling constant, given by \(\kappa = \pi \Delta n / \lambda\)

7. \(\Delta \beta\): wave vector detuning, given by \(\Delta \beta = \beta - \beta_c (\pi / \Lambda)\)

8. \(\beta\): fiber core propagation constant, given by \(\beta = 2 \pi n_0 / \lambda\)

9. \(S = \sqrt{\kappa^2 - \Delta \beta^2}\)

10. For light at the Bragg grating center wavelength, \(\lambda B\), there is no wave vector detuning and so \(\Delta \beta = 0\). The reflectivity function then becomes

\[
R(L, \lambda) = \tanh^2 (SL)
\]

3. RESULT AND ANALYSIS

The parameters chosen for simulation [7]-[10]:

- Profile type: Step index, single mode
- Modulation depth: 0.0012
- Cladding index: 1.45
- Period: 0.5 \(\mu m\)
- Free space wavelength: 1.55 \(\mu m\)

The simulation is performed for different values of index difference and length of FBG as indicated in fig. 3, and 4.

![Grating Spectral Response for L=0.5mm, Δn=0.008](image1)

![Grating Spectral Response for L=2mm, Index diff.: 0.008](image2)

The observations for reflectivity of FBG and side lobe strength are taken and it is seen that , the reflectivity increases with...
increase in grating length as well as index difference. Almost after a length of 2.5mm, 100% reflectivity is achieved. Side lobe strength also increases with increase in grating length, so we have to make a compromise between maximum reflectivity & minimum power wasted in side lobes.

To study the bandwidth changes of FBG, FWHM (Full width half maximum) values with different grating lengths were measured and plotted in fig. 5, the FWHM Bandwidth reduces with increase in length and reduction in index difference of FBG. Bandwidth is almost constant after a length of 5mm.

Fig. 5. Relationship between FBG bandwidth and grating length

The power wasted in side lobes can be minimized by applying different index profiles, called as Apodization. Doping concentration variation limits the index variation to maximum value $\Delta n_o$. Index inside core after FBG has been printed can be expressed by

$$n(z) = n_{co} + \Delta n_o A(z)n_0(z) \quad (11)$$

where $n_{co}$ - core refractive index,
$\Delta n_o$ - maximum index variation
$A(z)$ - Apodization function

For uniform FBG with no apodization index variation function $n_0(z)$ is given as

$$n_0(z) = \frac{2\pi z}{\Lambda}$$

Where $\Lambda$: constant grating period, $\Lambda$=1

A uniform fiber grating has two ends, thus it begins abruptly and ends abruptly. The Fourier transform of such a rectangular function yields the well-known sinc function, with its associated side lobes structure apparent in the reflection spectrum. The transform of Gaussian function is also Gaussian, with no side lobes. A grating with a similar refractive modulation amplitude profile diminishes the side lobes substantially. The suppression of the side lobes in the reflection spectrum by gradually increasing the coupling coefficient with penetration into, as well as gradually decreasing on existing from, the grating is called apodization [11], [12]. Apodization function $A(z)$ in equation (11) can be changed to reduce the side lobes. The side lobes are fully suppressed for raised cosine apodization as shown in fig. 6.

For sinc Apodization

$$A(z) = \exp \left( -4 \left( \frac{z-L/2}{L} \right)^2 \right)$$

For Raised cosine Apodization

$$A(z) = a \left( 1 + \cos \left( \frac{\pi (z-L/2)}{L} \right) \right)$$

Where $a$ is raised-cosine parameter

Fig. 6. No chirp, Length: 1.0 mm, Apodization Raised Cosine

5. REFERENCES


[5] Sunita Ugale, Dr. V. Mishra, “Fiber Bragg Grating Modeling, Characterization and Optimization with different


