The Effect of Bulk Viscosity on Bianchi Type V Cosmological Models with Varying Λ in General Relativity

Shilpa W. Samdurkar (Pahade)
Department Of Applied Mathematics,
P. I. E. T., Nagpur

Shoma S. Sen
Department Of Applied Mathematics,
P. I. E. T., Nagpur

ABSTRACT
In this paper, we discuss Bianchi type V cosmological model filled with viscous fluid in the presence of cosmological term Λ. The viscous coefficient of bulk viscous fluid is assumed to be a power function of mass density, whereas coefficient of shear viscosity is considered proportional to scale of expansion in the model. Exact solutions of the field equations are obtained by using scalar of expansion proportional to shear scalar $\theta \propto \sigma$ which leads to a relation between metric potential $A = B^r$ where r is constant. Some physical and geometrical properties of the model are also discussed.

1. INTRODUCTION
The cosmological models which are spatially homogenous and anisotropic play significant roles in the description of the universe at its early stages of evolution. Bianchi I-IX spaces are very useful to constructing special homogeneous cosmological models. (The importance of Bianchi type V model is due to the fact that the space of constant negative curvature is contained in it as a special case). These models can be used to analyze aspects of the physical universe which pertains or which may be affected by anisotropy in the rate of expansion, for example, the cosmic microwave background radiation, nucleosynthesis in the early universe and the question of isotropization of the universe itself (Mac Callum, [1]). Spatially homogeneous cosmologies also play an important role in the attempt to understand the structure and the properties of the space of all cosmological solutions of Einstein's field equations.

Most cosmological models assume that the mater in the universe can be described by dust (a pressure less distribution) or at the early stages of universe viscous effects do play a role (Israel and Vardalas [2], Kilmek [3], Weinberg [4]). For example, the existence of bulk viscosity is equivalent to slow process of restoring equilibrium states (Landau and Lipchitz [5]). The observed physical phenomena such as the large entropy per baryon and remarkable degree of isotropy of the cosmic microwave background radiation suggest analysis of dissipative effects in cosmology.

In the modern cosmological theories, the dynamic cosmological term Λ(t) remains a focal point of interest as it solves the cosmological constant problem in a natural way. There is significant observational evidence towards identifying Einstein's cosmological constant Λ or a component of material content of the universe that varies slowly with time and space and so acts like Λ. Recent cosmological observations by the High-z Supernova Team and the Supernova Cosmological Project [6–12] suggest the existence of a positive cosmological constant Λ with magnitude $\Lambda \left(\frac{Gh}{c^3}\right) \approx 10^{-123}$.

To consider more realistic models one must take into account viscosity mechanisms; and, indeed, viscosity mechanisms has attracted the attention of many researchers. Misner [13, 14] suggested the strong dissipation due to neutrino viscosity may considerably reduce the anisotropy of the black body radiation. Weinberg [15, 16] suggested that a viscosity mechanism in cosmology can explain the unusual high entropy per baryon in present events. Waga et al. [17], Pachel et al. [18], Guth [19], and Murphy [20] have shown that bulk viscosity associated with the grand unified theory Phase transition (see Langackar in [21]) may lead to an inflationary scenario.

A uniform cosmological model filled with fluid under pressure and with viscosity has been developed by Murphy [20]. A solution that we have found exhibits an interesting feature where the big bang type singularity appears in the infinite past. Exact solutions for isotropic homogeneous cosmology for open, closed and flat universes have been found by Santos et al. [22] with the bulk viscosity being a power function of energy density. The effect of bulk viscosity on cosmological evolution has been investigated by a number of authors in the context of general theory of relativity [23–31]. The nature of cosmological solution for homogenous cosmological model was investigated by Belinsky et al. [32] and shown that viscosity cannot remove the cosmological singularity but result in a qualitatively new behaviour of the solution near singularity. Huang [33] has studied Bianchi type models with bulk viscosity as a power function of energy density and when the universe is filled with stiff matter. The effect of bulk viscosity, with a time varying bulk viscous coefficient, on the evolution of isotropic FRW models in the context of open thermodynamics system was studied by Desikan [34]. Ray et al. [35] have studied anisotropic-charged fluid sphere with varying cosmological constant. Bianchi type I cosmological models with cosmological term Λ was studied by Singh et al. [36], Chakravarty and Biswas [37] and Belinchon [38].

Bulk viscous models have prime roles in getting inflationary phases of the universe [39–45]. Bulk viscosity driven inflation is primarily due to the negative bulk viscous pressure giving rise to a total negative effective pressure which may overcome the pressure due to the usual gravity of matter distribution in the universe and provide an impetus to drive it apart. Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (Gron, [46]) for a review on cosmological models with bulk viscosity.

The effect of bulk viscosity on cosmological evolution has been investigated by a number of authors in the framework of general relativity (Pavon [47], Padmanabhan and Chitre [48], Johri and Sudarshan [49], Maartens [50] , Zimdahl [51], Santos et al. [52], Pradhan et al. [53], Kalyani and Singh [54], Singh et al. [55], Pradhan et al. [56–58]) This motivates to study cosmological bulk viscous fluid model. Banerjee and Sanyal [59] have considered Bianchi Type V cosmologies with viscosity and heat flow. It has also been shown that it is possible for dissipative Bianchi type V universe model not to be in thermal equilibrium in their early stages. Coley [60] have investigated Bianchi Type V spatially homogenous with perfect fluid cosmological model which contains both viscosity and heat flow. Recently, Kandalkar [61] have discussed the problem with cosmological constant in the presence of viscous fluid in evolution of the Kantowski - Sachs cosmological model.

Motivated by the situation discussed above, in this paper we discuss the problem with cosmological constant in the presence of viscous fluid in evolution of Bianchi type V cosmological model.

2. METRIC AND THE FIELD EQUATIONS

We consider metric in the form,

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x}\left(dy^2 + dz^2\right) \quad (1)$$

The Einstein field equations (in gravitational units

c = 1, G = 1) are

$$R_i^{\,j} - \frac{1}{2} R g_i^{\,j} = -(8\pi T_i^{\,j} - \Lambda g_i^{\,j}) \quad (2)$$

where $R_i^{\,j}$ is the Ricci tensor, $R = g^{ij} R_{ij}$ is the Ricci scalar and $T_i^{\,j}$ is the energy momentum tensor of viscous fluid given by,

$$T_i^{\,j} = (\rho + p')u_i u^j - p'\delta_i^j + \eta g^{j\beta}\begin{pmatrix} u_{i;\beta} + u_{\beta;i} - u_i u^\alpha u_{\beta;\alpha} \\ - u_\beta u^\alpha u_{i;\alpha} \end{pmatrix} \quad (3)$$

where $\rho' = p - \left(\xi - \frac{2}{3}\eta\right)u^i_{;i}$

Here ρ is the energy density, p is pressure and η and ξ are coefficients of shear and bulk viscosity respectively. The semicolon (;) indicates covariant differentiation. The shear and bulk viscosities are positively definite i.e. $\eta > 0,\ \xi > 0;$ and may be either constant or functions of time or energy as $\eta = |a|\rho^\alpha,\ \xi = |b|\rho^\beta$

where a and b are constants, u_i is the flow vector satisfying the relation,

$$g_{ij} u^i u^j = 1 \quad (4)$$

We choose the co-ordinates to be commoving so that

$$u^1 = u^2 = u^3 = 0, u^4 = 1 \quad (5)$$

The Einstein field equations (2) for the line element (1) has been set up as

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{3}{A^2} + \Lambda = 8\pi\rho \quad (6)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} + \Lambda = 8\pi\left(-p' + 2\eta\frac{\dot{A}}{A}\right) \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} + \Lambda = 8\pi\left(-p' + 2\eta\frac{\dot{B}}{B}\right) \quad (8)$$

where a dot (.) over a variable denotes ordinary differentiation with respect to time t.

3. SOLUTION OF THE FIELD EQUATIONS

Equations (6) – (8) are three independent equations in seven unknown A, B, $\rho, p, \eta, \xi\ and\ \Lambda$

First, we assume a relation in metric potential as,

$$A = B^r \quad (9)$$

Second, we assume that coefficient of shear viscosity is proportional to the scale of expansion i.e.

$$\eta \propto \theta \quad (10)$$

where r is a real number and θ is the scalar of expansion given by,

$$\theta = u^i_{;j} \quad (11)$$

From equations (7) and (8), we get,

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} = 16\pi\eta\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) \quad (12)$$

From equation (10), we obtain,

$$\eta = l\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \quad (13)$$

where l is the constant of proportionality.

Equation (12) together with (9) and (13) leads to,

$$B\ddot{B} + \beta\dot{B}^2 = 0 \quad (14)$$

which can be rewritten as,

$$\frac{df^2}{dB} + \frac{2\beta}{B} f^2 = 0 \quad (15)$$

where

$$\beta = [(r+1) + 16\pi l(r+2)] \quad (16)$$

and

$$\dot{B} = f(B) \quad (17)$$

From (15), we obtain,

$$\left(\frac{dB}{dt}\right)^2 = B^{-2\beta} \quad (18)$$

Integrating equation (18), we get

$$B = [(r+2)s(t+c)]^{\frac{1}{(r+2)s}} \quad (19)$$

where $s = 16\pi l + 1$

From (9), we obtain,

$$A = [(r+2)s(t+c)]^{\frac{r}{(r+2)s}} \quad (20)$$

The metric (1) reduces to,

$$ds^2 = dt^2 - [(r+2)s(t+c)]^{\frac{2r}{(r+2)s}} dx^2 - [(r+2)s(t+c)]^{\frac{2}{(r+2)s}} e^{2x}\left(dy^2 + dz^2\right) \quad (21)$$

The pressure and density for the model (21) are given by,

$$8\pi p = \frac{8\pi\xi}{s(t+c)} + \frac{[32\pi l(r+2)^2 + 3(2r+1)]}{3s^2(r+2)^2(t+c)^2} + \frac{1}{[(r+2)s(t+c)]^{\frac{2r}{(r+2)s}}} - \Lambda \tag{22}$$

$$8\pi\rho = \frac{(1+2r)}{s^2(r+2)^2(t+c)^2} - \frac{3}{[s(r+2)(t+c)]^{\frac{2r}{(r+2)s}}} + \Lambda \tag{23}$$

We assume that the fluid obeys an equation of state of the form,

$$p = \omega\rho \tag{24}$$

where $0 \le \omega \le 1$ is constant.

Bulk viscosity is assumed to be a simple power function of the energy density,

$$\xi(t) = \xi_0 \rho^n \tag{25}$$

where ξ_0 and n are constants.

On using equation (25) in (22), we obtain the following relation for pressure,

$$8\pi p = \frac{8\pi\xi_0\rho^n}{s(t+c)} + \frac{[32\pi l(r+2)^2 + 3(2r+1)]}{3s^2(r+2)^2(t+c)^2} + \frac{1}{[(r+2)s(t+c)]^{\frac{2r}{(r+2)s}}} - \Lambda \tag{26}$$

If n = 1, equation (25) may correspond to a radiative fluid. However, more realistic models (See Santos in [22] are based on n lying in the region $0 \le n \le \frac{1}{2}$.

Model I: Solution for $\xi = \xi_0$

When n = 0 equation (25) reduces to $\xi = \xi_0$ = constant. Hence, in this case equation (26) with the use of (23) and (24) leads to,

$$8\pi(\omega+1)\rho = \frac{8\pi\xi_0}{s(t+c)} + \frac{[32\pi l(r+2)^2 + 6(2r+1)]}{3s^2(r+2)^2(t+c)^2} + \frac{1}{[(r+2)s(t+c)]^{\frac{2r}{(r+2)s}}} \tag{27}$$

Eliminating $\rho(t)$ between equations (23) and (25), leads to

$$(\omega+1)\Lambda = \frac{8\pi\xi_0}{s(t+c)} + \frac{[32\pi l(r+2)^2 + 3(2r+1)(1-\omega)]}{3s^2(r+2)^2(t+c)^2} + \frac{(3\omega+1)}{[(r+2)s(t+c)]^{\frac{2r}{(r+2)s}}} \tag{28}$$

The cosmic matter density parameter Ω_m and cosmic vacuum – energy density parameter Ω_Λ are given by

$$\Omega_m = \frac{\rho}{3H^2} = \frac{3s^2}{8\pi(\omega+1)}\left\{\begin{array}{l} \frac{8\pi\xi_0(t+c)}{s} \\ + \frac{[32\pi l(r+2)^2 + 6(2r+1)]}{3s^2(r+2)^2} \\ + \frac{(t+c)^{\frac{2r(s-1)+4s}{(r+2)s}}}{[(r+2)s]^{\frac{2r}{(r+2)s}}} \end{array}\right\} \tag{29}$$

$$\Omega_m = \frac{\Lambda}{3H^2} = \frac{3s^2}{(\omega+1)}\left\{\begin{array}{l} \frac{8\pi\xi_0(t+c)}{s} + \\ \frac{[32\pi l(r+2)^2 + 3(2r+1)(1-\omega)]}{3s^2(r+2)^2} \\ + \frac{(3\omega+1)(t+c)^{\frac{2r(s-1)+4s}{(r+2)s}}}{[(r+2)s]^{\frac{2r}{(r+2)s}}} \end{array}\right\} \tag{30}$$

Model II: Solution for $\xi = \xi_0\rho$

When n = 1, equation (25) reduces to $\xi = \xi_0\rho$. Hence in this case equation (26) with the use of (23) and (24) leads to

$$8\pi\rho = \frac{1}{\left[(1+\omega) - \frac{\xi_0}{s(t+c)}\right]}\left\{\begin{array}{l} \frac{32\pi l(r+2)^2 + 6(2r+1)}{3s^2(r+2)^2(t+c)^2} \\ - \frac{2}{[(r+2)s(t+c)]^{\frac{2r}{(r+2)s}}} \end{array}\right\} \tag{31}$$

$$\Lambda = \frac{1}{\left[(1+\omega) - \frac{\xi_0}{s(t+c)}\right]}\left\{\frac{32\pi l(r+2)^2 + 6(2r+1)}{3s^2(r+2)^2(t+c)^2} - \frac{2}{[(r+2)s(t+c)]^{\frac{2r}{(r+2)s}}}\right\} - \frac{(2r+1)}{s^2(r+2)^2(t+c)^2} + \frac{3}{[(r+2)s(t+c)]^{\frac{2r}{(r+2)s}}} \tag{32}$$

Some physical aspects of the models

The spatial volume V, Hubble parameter H, expansion factor θ, shear σ and deceleration parameter q of the fluid for the metric (21) leads to,

$$V = R^3 = [(r+2)s(t+c)]^{\frac{1}{s}} e^{2x} \tag{33}$$

$$H = \frac{1}{3s(t+c)} \tag{34}$$

$$\theta = \frac{1}{s(t+c)} \tag{35}$$

$$\sigma^2 = \frac{(r-1)^2}{3s^2(r+2)^2(t+c)^2} \tag{36}$$

$$\frac{\sigma}{\theta} = \frac{(r-1)}{\sqrt{3(r+2)}} \tag{37}$$

$$q=24\pi l+1 \tag{38}$$

The expansion factor θ decreases as a function of t and approaches zero. Also ρ and p approaches to zero as $t\to\infty$.

4. PARTICULAR MODELS

If we set r = 2, the geometry of space time (21) reduces to,

$$ds^2=dt^2-\left[4s(t+c)\right]^{\frac{1}{s}}dx^2-\left[4s(t+c)\right]^{\frac{1}{2s}}e^{2x}\left(dy^2+dz^2\right) \tag{39}$$

The pressure and density for model (39) are given by

$$8\pi p=\frac{8\pi\xi}{s(t+c)}+\frac{[512\pi l+15]}{48s^2(t+c)^2}+\frac{1}{\left[4s(t+c)\right]^{\frac{1}{s}}}-\Lambda \tag{40}$$

$$8\pi\rho=\frac{5}{16s^2(t+c)^2}-\frac{3}{\left[4s(t+c)\right]^{\frac{1}{s}}}+\Lambda \tag{41}$$

Case I: Solution for $\xi=\xi_0$

When n = 0 equation (25) reduces to $\xi=\xi_0=\text{constant}$. Hence, in this case equation (27) leads to,

$$8\pi(\omega+1)\rho=\frac{8\pi\xi_0}{s(t+c)}+\frac{[256\pi l+15]}{48s^2(t+c)^2}+\frac{1}{\left[4s(t+c)\right]^{\frac{1}{s}}} \tag{42}$$

Eliminating $\rho(t)$ between equations (23) and (25), leads to

$$(\omega+1)\Lambda=\frac{8\pi\xi_0}{s(t+c)}+\frac{[512\pi l+15(1-\omega)]}{48s^2(t+c)^2}+\frac{(3\omega+1)}{\left[4s(t+c)\right]^{\frac{1}{s}}} \tag{43}$$

$$\Omega_m=\frac{3s^2}{8\pi(\omega+1)}\left\{\frac{8\pi\xi_0(t+c)}{s}+\frac{[512\pi l+30]}{48s^2}+\frac{(t+c)^{\frac{(2s-1)}{s}}}{[4s]^{\frac{1}{s}}}\right\} \tag{44}$$

$$\Omega_\Lambda=\frac{3s^2}{(\omega+1)}\left\{\frac{8\pi\xi_0(t+c)}{s}+\frac{[512\pi l+15(1-\omega)]}{48s^2}+\frac{(3\omega+1)(t+c)^{\frac{(2s-1)}{s}}}{[4s]^{\frac{1}{s}}}\right\} \tag{45}$$

Case II: Solution for $\xi=\xi_0\rho$

When n = 1, equation (25) reduces to $\xi=\xi_0\rho$. Hence in this case equation (31) and (32) leads to,

$$8\pi\rho=\frac{1}{\left[(1+\omega)-\frac{\xi_0}{s(t+c)}\right]}\left\{\frac{256\pi l+15}{48s^2(t+c)^2}-\frac{2}{\left[4s(t+c)\right]^{\frac{1}{s}}}\right\} \tag{46}$$

$$\Lambda=\frac{1}{\left[(1+\omega)-\frac{\xi_0}{s(t+c)}\right]}\left\{\frac{256\pi l+15}{48s^2(t+c)^2}-\frac{2}{\left[4s(t+c)\right]^{\frac{1}{s}}}\right\}-\frac{5}{16s^2(t+c)^2}+\frac{3}{\left[4s(t+c)\right]^{\frac{1}{s}}} \tag{47}$$

Some physical aspects of the models

The expansion factor θ and the shear σ of the fluid for the model(39) leads to,

$$\theta=\frac{1}{s(t+c)} \tag{48}$$

$$\sigma=\frac{1}{4\sqrt{3}s(t+c)} \tag{49}$$

$$\frac{\sigma}{\theta}=\frac{1}{4\sqrt{3}} \tag{50}$$

Note that the expansion factor θ is a decreasing function of t. Since $\lim_{t\to\infty}\frac{\sigma}{\theta}\neq 0$ isotropy is not approached for large values of t.

5. SPECIAL MODELS

If we set r = 2 and $l=-\frac{1}{32\pi}$ equation (18) leads to,

$$B\,dB=dt \tag{51}$$

This on integration gives,

$$B^2=2(t+c) \tag{52}$$

where c is the constant of integration. Hence, we obtain

$$A=B^2=2(t+c) \tag{53}$$

The geometry of space time (21) reduces to,

$$ds^2=dt^2-\left[2(t+c)\right]^2dx^2-\left[2(t+c)\right]^2e^{2x}\left(dy^2+dz^2\right) \tag{54}$$

The pressure and density for the model (54) are given by,

$$8\pi p=\frac{16\pi\xi}{(t+c)}+\frac{1}{6(t+c)^2}-\Lambda \tag{55}$$

$$8\pi\rho=\frac{1}{2(t+c)^2}+\Lambda \tag{56}$$

Case I: Solution for $\xi=\xi_0$

$$8\pi(\omega+1)\rho=\frac{16\pi\xi_0}{(t+c)}+\frac{1}{12(t+c)^2} \tag{57}$$

$$(\omega+1)\Lambda=\frac{16\pi\xi_0}{(t+c)}+\frac{(1-3\omega)}{6(t+c)^2} \tag{58}$$

$$\Omega_m=\frac{3}{32\pi(\omega+1)}\left\{32\pi\xi_0(t+c)+\frac{17}{12}\right\} \tag{59}$$

$$\Omega_\Lambda=\frac{3}{4(\omega+1)}\left\{16\pi\xi_0(t+c)+\frac{(1-3\omega)}{6}\right\} \tag{60}$$

Case II: Solution for $\xi=\xi_0\,\rho$

$$8\pi\rho=\frac{1}{\left[(1+\omega)-\frac{2\xi_0}{(t+c)}\right]}\left\{\frac{1}{12(t+c)^2}\right\} \tag{61}$$

$$\Lambda=\frac{1}{\left[(1+\omega)-\frac{2\xi_0}{(t+c)}\right]}\left\{\frac{1}{12(t+c)^2}\right\}-\frac{1}{2(t+c)^2} \tag{62}$$

Some physical aspects of the models

The expansion factor θ and the shear σ of the fluid for the model (54) leads to,

$$\theta=\frac{2}{(t+c)} \tag{63}$$

$$\sigma=\frac{1}{2\sqrt{3}(t+c)} \tag{64}$$

$$\frac{\sigma}{\theta}=\frac{1}{4\sqrt{3}} \tag{65}$$

6. CONCLUSION

In this paper, we discuss Bianchi type V models with viscous fluid in the presence of cosmological term Λ. We have assumed that the fluid obeys an equation of state of the form $p=\omega\rho$ .Also, the viscous coefficient of bulk viscous fluid is assumed to be a power function of mass density. It is observed that the expansion factor θ is a decreasing function of t and approaches to zero as $t\to\infty$. Also from equation (23), the energy density ρ approaches zero as $t\to\infty$. Since $\lim_{t\to\infty}\frac{\sigma}{\theta}=\text{constant}$, the model is not isotropic for large values of t. The behavior of the universe in models will be determined by the cosmological term Λ. It is noticed that Λ is positive. Also, we observe that for n = 0, the cosmic matter density parameter Ω_m and vacuum energy density parameter Ω_Λ exist which approaches to zero for $t\to 0$ but for n = 1 Ω_m and Ω_Λ does not exist.

If we set r = 2, it is observed that density ρ and pressure p approaches to zero as $t\to\infty$. Thus, we have obtained a physically relevant decay law for the cosmological constant without considering an assumption for variation.

In special models, we observed that the density ρ and pressure p are valid for t > 0.

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