Plane Symmetric Space Time with Background Metric

S. D. Deo
Department of Mathematics
N. S. Sc. and Arts College, Bhadrawati,
Dist.-Chandrapur,(M.S.) 442902,India

ABSTRACT

Homogeneous plane symmetric cosmic strings and domain walls are respectively in Rosen's bimetric theory of relativity. As we know that, at the early stage of evolution of the universe domain walls as well as cosmic strings do appear which lead to formation of galaxies. Thus, it may be said that the bimetric relativity does not help to describe the early era of the universe. It is shown that the vacuum model represents Robertson-Walker flat metric which expands uniformly along space direction with time.

Keywords-Plane Symmetric , Bimetric Relativity, Cosmic Strings, Domain Walls

AMS Subject code -83C05

1. INTRODUCTION

In an attempt to get rid of the singularities appear in the Einstein's General Theory of Relativity (G.R.), Rosen [1-5]has proposed a modified theory of gravitation within the frame work of general relativity which is called Bimetric Theory of Relativity (B.R.).In this theory he has proposed a new formulation of the general relativity by introducing a background Euclidean metric tensor γ_{ij} in addition to the usual Riemannian metric tensor g_{ij} at each point of the four dimensional space-time. With the flat background metric γ_{ij} ,the physical content of the theory is the same as that of the general relativity .

Thus ,now the corresponding two line elements in a coordinate system x^i are –

$$ds^2=g_{ij}dx^idx^j \qquad (1.1)$$

And $$d\sigma^2= \gamma_{ij}dx^idx^j \qquad (1.2)$$

Where ds is the interval between two neighboring events as measured by means of a clock and a measuring rod. The interval dσ is an abstract or geometrical quantity not directly measurable . One can regard it as describing the geometry that would exist if no matter were present .

With the help of the metric tensor g_{ij} one can define the Christoffel symbols $\{^k{}_{ij}\}$and hence covariant differentiation (g-differentiation) denoted by a semicolon (;) and one can form the Riemannian Curvature Tensor $R^h{}_{ijk}$.

In recent years, cosmological model exhibiting plane symmetry have attracted the attention of various authors .Plane symmetric perfect fluid distribution was first discussed by Taub [6-7] in which the flow was taken to be isentropic in general relativity. As a special case of the plane symmetric cosmological models Bianchi type space-time has been extensively studied by Heckmann and Schucking [8] , Thorne [9] , Jacobs [10] , Singh and Singh [11]. These solutions have been obtained under different geometrical and physical conditions such as the priori stipulation of the equation of state or imposition of the Petrov type conditions . It is a well known fact that the free gravitational field affects the flow of the fluid by including shear in the flow lines.

Deo [12-13]also studied the Taub like plane symmetric metric with cosmic strings ,domain walls and Maxwell field etc. in the context of bimetric theory of relativity.

In this paper we have shown that plane symmetric space time does not accommodate cosmic string as well as domain walls in bimetric relativity. Further we have obtained vacuum model representing Robertson Walker flat metric which expands uniformly along space directions with time.

2. FIELD EQUATIONS

Rosen N has proposed the field equations of Bimetric Relativity as

$$K_i^j = N_i^j - \tfrac{1}{2} N g_i^j = - 8\pi\kappa\, T_i^j \qquad (2.1)$$

Where

$$N_i^j = \tfrac{1}{2}\, g^{\alpha\beta} [g^{hj} g_{hi\,|\alpha}]_{|\beta} \qquad (2.2)$$

$$N = N_\alpha{}^\alpha , \qquad \kappa = \sqrt{(g / \gamma)} \qquad (2.3)$$

And

$$g = \det g_{ij} \ , \quad \gamma = \det \gamma_{ij} \qquad (2.4)$$

And T_i^j = is the energy momentum tensor .

Equation (2.1) is obtained from Einstein field equations

$$G_i^j = R_i^j – \tfrac{1}{2} g_i^j R = - 8\pi\kappa\, T_i^j \qquad (2.5)$$

by replacing all the derivatives of g_{ij} by γ -derivatives .Using γ- derivatives does not change the physical contents of the field equations , but it has some advantages .One can derive the Einstein field equations for empty space from the variational principle .

3. COSMIC STRINGS

We consider here the plane symmetric line element of the form

$$ds^2= dt^2-A^2(dx^2+dy^2)-B^2dz^2 \qquad (3.1)$$

where A and B are functions of t only.

The background flat space time corresponding to the metric (3.1) is

$d\sigma^2 = dt^2 - (dx^2 + dy^2 + dz^2)$ (3.2)

The energy momentum tensor T_i^j for cosmic cloud strings is given by

$T_i^j = T_{i\,strings}^j = \rho\, v_i v^j - \lambda\, x_i x^j$ (3.3)

Here ρ is the rest energy density for a cloud with particle attached along the extension, thus $\rho = \rho_p + \lambda$, where ρ_p is the particle energy density, λ is the tension density of the strings and v^i the flow vector of matter. The flow of the matter is taken orthogonal to the hyper – surface of homogeneity so that $v_4 v^4 = -1$ and x^i representing the direction vector of anisotropy,

i.e. z – axis => $x_3 x^3 = 1$ and $v_i x^i = 0$.

Using commoving coordinate system the field equations (2.1)-(2.4) for the metrics (3.1)and (3.2)corresponding to the energy momentum tensor (3.3) in bimetric theory can be written explicitly as

$$\left(\frac{B_4}{B}\right)_4 = 0 \qquad (3.4)$$

$$2\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 = 8\pi\kappa\lambda \qquad (3.5)$$

$$2\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 = 8\pi\kappa\rho \qquad (3.6)$$

Where $\kappa = A^2 B$

And hereafter words the suffix 4 after field variables stands for ordinary differentiation with respect to coordinate t .

Using the equation (3.4)in (3.5)and(3.6) we obtain

$$\rho - \lambda = 0 \qquad (3.7)$$

i.e. $\rho_p = 0$ (3.8)

by equation (3.8) we come to know that particle energy density vanishes .Thus we can say that there is no contribution from cosmic strings tn a plane symmetric space time in bimetric relativity and hence only vacuum model exists.

For $\lambda = 0 = \rho$ (vacuum case) using equation (3.4) in (3.5) gives -

$$\left(\frac{A_4}{A}\right) = 0 \qquad (3.9)$$

Solutions of equations (3.4)and (3.5) can be easily obtained as

$$A = c_1 e^{n_1 t} \qquad (3.10)$$

$$B = c_2 e^{n_2 t} \qquad (3.11)$$

Where c_1 , c_2 n_1, n_2, are constants of integration.

Thus in view of equations (3.10)and (3.11), the metric (3.1) takes the form

$$ds^2 = dt^2 - e^{2n_2 t}\left(dx^2 + dy^2\right) - e^{2n_1 t} dz^2 \qquad (3.12)$$

Which for $n_1 = n_2 = n$ reduces to

$$ds^2 = dt^2 - e^{2nt}\left(dx^2 + dy^2 + dz^2\right) \qquad (3.13)$$

This vacuum model represents Robertson – Walker flat model, which expands uniformly along space directions with time .The rate of expansion depends on the signature of the parameter n.

4. DOMAIN WALLS

In this section we consider the region of the space –time with thick domain walls whose energy momentum tensor is given by

$T_i^j = \rho\,(g_i^j + \omega_i \omega^j) + p\, \omega_i \omega^j$ (4.1)

With $\omega_i \omega^j = -1$

Were ρ is the energy density of the wall , p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space like vector in the same direction .

The explicit form of the field equations (2.1)for the metrics (3.1)and (3.2)with the energy momentum tensor (4.1) are obtained as

$$\left(\frac{B_4}{B}\right) = -16\pi\kappa\, p \qquad (4.2)$$

$$\left(\frac{B_4}{B}\right) = 16\pi\kappa\rho \qquad (4.3)$$

$$2\left(\frac{A_4}{A}\right) - \left(\frac{B_4}{B}\right) = 16\pi\kappa\rho \qquad (4.4)$$

$$2\left(\frac{A_4}{A}\right) + \left(\frac{B_4}{B}\right) = 16\pi\kappa\rho \qquad (4.5)$$

Solving the equations (4.2)and (4.3) , we have

$$\rho + p = 0 \qquad (4.6)$$

In view of reality conditions i.e. $p > 0$, $\rho > 0$, the equation (4.6) is true only when

$p = 0 = \rho$. Thus in bimetric theory of relativity the plane symmetric cosmological thick domain wall does not survive and hence only vacuum model exists.

For $p = 0 = \rho$ (vacuum case) we will get the same results which are obtained in previous section.

5. CONCLUSION

As **we** study the cosmic cloud strings and thick domain walls with the plane symmetric cosmological space – time in the context of bimetric relativity, and we have obtained the nil contribution of cosmic strings and domain walls respectively. As we know that, at the early stage of evolution of the universe domain walls as well as cosmic strings do appear which lead to formation of galaxies. Thus, it may be said that the bimetric relativity does not help to describe the early era of the universe. And further we obtain the vacuum solutions which represents the resulting space – time reduce to well known Robertson – Walker flat space – time which expands uniformly along space direction with time.

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