Modeling for Flow through Unsaturated Porous Media with Constant and Variable Density Conditions using Local Thermal Equilibrium

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ABSTRACT
The Flow through saturated-unsaturated porous media is extremely important in various natural and industrial based applications. While the Darcy's law with various modifications are used to model the flow through a porous media, the flow through unsaturated porous media is largely based on conservation of mass and modified Darcy's law where non-linear relationship exists between the pressure head and the fluid saturation coupled with fluid density variations. This paper represents mathematical modelling of flow through unsaturated porous media using constant and variable fluid density. The variable density model is further split into thermal and Isothermal models. The mathematical model is applied to an unsaturated porous media filled with water and oil having immiscible flow. The variables describing the models like permeability, capillary pressure, fluid saturation and their constituent relations are considered. The models are extremely important for different industrial applications like enhancing oil recovery, sea water filtration, nuclear waste disposal, chemical clean-up of soil, underground hydrology, soil physics etc.

Keywords
Unsaturated porous media flow, Mathematical modeling, porous media, variable density, local thermal equilibrium, immiscible flow

1. INTRODUCTION
Numerous experimental, numerical and analytical studies have been conducted in past for better understanding of fluid flow through a porous media which has a complex internal structure pattern. The studies are largely based on experimental work conducted by Darcy [1] for incompressible and isothermal creeping flow.

$$u = -\left(\frac{K}{\mu}\right)\Delta p \tag{1-1}$$

The permeability K, m^2 is a hydraulic property characterizing a porous media and the absolute viscosity μ, $kgm^{-1}s^{-1}$ is a fluid property. Various modifications to the above equation are suggested by Dupuit [2] and Hazen [3] further developed Darcy's equation by considering fluid temperature effect. Muskat [4] further extended model by considering fluid viscosity, μ. Brinkman [5] worked on the calculation of viscous force, occurred due to flow through dense swarm particles like spherical particles embedded porous media for evaluating relationship between particle size, density and permeability. Irmay [6] worked on mathematical formulation based on the mass balance equation.

Extension to Darcy's law along with capillary pressure-saturation-relative permeability relationships explained multiphase flow through porous media are considered by Richards [7] and Gardner [8] where the voids contain air in the form of bubble were considered. Pressure head based upon water content and permeability functions are important aspect of unsaturated flow through porous media. Many empirical relationships were used to identify the relationship of parameters like Brooks-Corey model, Van Genuchten model [9]. Edlefsen et al. [10] extend Darcy law by adding mechanical potential *E* and capillary pressure. Several researchers presented the different form of a pore-scale model like Nieber et al. [11] a bundle of tubes model Dahle et al. [12] and Yang et al. [13] a bed of having multiple diameter spheres. Despite their extremely simple nature, these models were able to describe the major features of a porous media.

The Darcy's model, equation 1-1, is suitable for low value of particle Reynolds number. As the Reynolds number increases, the result is a transition from laminar to turbulent flow. This transitional flow is called non-Darcy (non-laminar) flow. Non-Darcy behaviour was important for describing fluid flow in porous media in situations where high velocity occurs. A criterion to identify the beginning of non-Darcy flow was needed. Various non-Darcy terms like presence of solid wall boundary, tortuosity, fluid-fluid friction, Forchheimer quadratic velocity term, channelling coupled with inertia, drag and vortex shedding describe a non-Darcy flow as discussed by various researchers, Muralidhar et al. [14], Teruel et al. [14], Chaia et al. [15], Pamuk et al. [16], Wang et al. [17], Hsu et al. [18], Zeng et al. [19], Su et al. [20] etc.

The term multiphase flow is used to refer to any fluid flow consisting of more than one phase or component .In other words interaction flow of two or more phase (solid, liquid, gas) where the interface between the phases is influenced by their motion. One could classify them according to the state of the different phases or components and therefore refer to gas/solids flows, or liquid/solids flows or gas/particle flows or bubbly flows and so on; many texts exist that limit their attention in this way. Some treatises are defined in terms of a specific type of fluid flow and deal with low Reynolds number suspension flows, dusty gas dynamics and so on. Others focus attention on a specific application such as slurry flows, cavitation flows, aerosols, debris flows, and fluidized beds and so on.

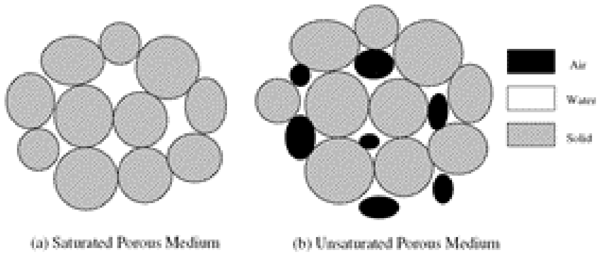

Fig 1: **Porous Media Saturated & Unsaturated**

A flow through a porous structure is largely a question of distance-the distance between the problem solver and the actual flow structure [21]. When the distance is short, the observer sees only one or two channels, or one or two open or closed cavities.

Multiphase (Unsaturated) flow in the porous material as shown in **Fig 1**, is an important process in a number of industrial applications. Hence, it is necessary to understand multiphase flow and analysis of boundary conditions for unsaturated two phase flow in porous media. Examples for two phase flow in porous media are the simultaneous transport of water and oil, water and air, or any liquid together with any gas. Such flow processes occur for example in groundwater modelling, enhance oil recovery, and environmental problems. In addition, they are of great importance in industrial applications, such as filter processes, flow through catalysts, or the gas and water flow within the diffusion media of a fuel cell.

Nieber et al. [11] explained the dynamic capillary pressure effect. They analysed a few alternative formulations of unsaturated flow that account for dynamic capillary pressure. Dahle et al. [12] presented the simplest form of a pore-scale model, namely a bundle of tubes model. They use their model to demonstrate the pore-scale process that underlies dynamic capillary pressure effects. Manthey et al. [22] discussed dynamic two phase flow in porous media. Starting with the Darcian description of two-phase flow in a (heterogeneous) porous media, they perform fine-scale simulations and obtain macro-scale effective properties through averaging of numerical results.

Valvatne et al. [23] employed static pore-scale network models to obtain hydraulic properties relevant to single, two- and three-phase flow for a variety of rocks. They consider single-phase flow of non-Newtonian as well as Newtonian fluids. Eichel et al. [24] presented an upscaling method for two-phase in a heterogeneous porous media. The approach was based on a percolation model and volume averaging method.

Zeng et al. [19] revised the Forchheimer number and recommended as a criterion for identifying non-Darcy flow in porous media. It equals the ratio of pressure drop caused by liquid–solid interactions to that by viscous resistance. The critical Forchheimer number for non-Darcy flow was expressed in terms of the critical non Darcy effect. Teruel et al. [14] presented a large set of microscopic flow simulations in the Representative Elementary Volume (REV) of a porous media formed by staggered square cylinders. The Reynolds number was varied from $Re = 10^3$ to 10^5, covering the Stokes flow regime, the laminar flow regime and the turbulence flow regime. Numerical results allow the investigation of the microscopic features of the flow as a function of the porosity and Reynolds number. Based on these microscopic results, the permeability of the porous media was computed and a porosity-dependent correlation was developed for this macroscopic parameter. Yedder et al. [25] investigated the natural convection in an air-filled (Prandtl number = 0.7) porous cavity with profiled side cooling and constant bottom heating over the Rayleigh number range of 1×10^4 to 1×10^8 at two Darcy numbers: 1×10^4 and 1×10^6. The general non-Darcy model adopted in this work was validated against experimental and theoretical results in the literature and Nusselt number was predicted within less than 3% in the worst case. Yang et al. [13] reported that a bundle-of-tubes construct was used as a model system to study ensemble averaged equations for multiphase flow in a porous material. Momentum equations for the fluid phases obtained from the method were similar to Darcy’s law, but with additional terms diffusion Chaia et al. [15] investigated numerically the non-Darcy effect on incompressible flows through disordered porous media. It is well known that the Darcy law is insufficient for describing high-rate flows in porous media. However, it is still an open problem to establish a universal form for the nonlinear correction to Darcy law. Cortez et al. [26] worked on Brinkman equations model for fluid flows in a porous media and developed the exact solution of the Brinkman equations for three-dimensional incompressible flow driven by regularized forces. Pamuk et al. [16] experimentally studied oscillatory and steady flows of water through two different porous media consisting of mono-sized stainless steel balls. The friction factors, permeability and inertial coefficients were determined experimentally for steady and oscillating flows. It was experimentally shown that the permeability and inertial coefficient of oscillating flows were greater than those of steady flow in the same range of Reynolds number. Su et al. [20] defined dimensionless geometry factor as the ratio of the product of the microscopic length scale and the solid fluid interface area to the solid volume in a representative elementary volume (REV), connects the macroscopic and microscopic drag and heat flux between the solid and fluid phases in a porous media. The geometry factor represents how widely the solid was distributed in the REV. Also relationships between the microscopic drag coefficients and permeability, Forchheimer coefficient, and Ergun constants were obtained based on the geometry factor.

The above cited literature covered many aspects / investigation related to modelling of flow through saturated and unsaturated porous media. Some investigation focus on theoretical aspect where as some investigation focus on practical aspect. The literature coved many aspect like extension of Darcy’s law, geometry aspect, model consideration like bundle of tubes, skin effect, viscosity, fluid-fluid viscous effect, capillary pressure variation, saturation-pressure head relation etc. However most of the studies focus on considering density driven function as constant on initial stage. Very limited studies covered density variation in flow through unsaturated porous media. This paper represents approach to model for flow through unsaturated porous media with variable and constant density conditions.

2. MATHEMATICAL MODELLING

2.1 Conceptual model

Unsaturated flow through porous media include properties of porous media, fluids properties, fluid-fluid interaction along with porous media relationships etc. The porous media considered in this paper must processes following properties

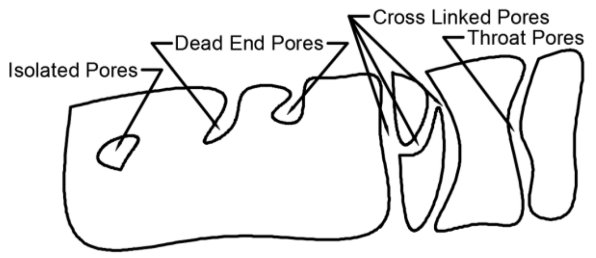

Fig 2: Classification of pores

1. It must contains relatively small spaces or voids, free of solids, imbedded in the solid or semisolid matrix.
2. It must be permeable to a variety of fluids.
3. The velocity of solid phase with respect to the boundary of the system is zero than the velocity of the fluid that can flow with in the porous system.

Above qualified porous media domain containing fluid 1 i.e. oil at a prescribed initial saturation and temperature is considered for modelling of unsaturated flow modelling.

2.2 Local volume averaging

The fluid contained in a porous media has a large number of molecules that move about colliding with each other. The geometry of the pore space is complex. For the large number of molecules in fluid contained in the pores, it is very difficult to determine their initial and final positions, velocities and momentum. Hence for the treatment of porous media the concept of representative volume (REV) is used. REV is an ensemble of many molecules contained in a small volume. Its size should be much larger than mean free path of a single molecule and it is sufficiently small as compared to fluid domain such that by averaging fluid and flow properties over the molecules included in it, bulk fluid properties will be obtained. Such an approach is called local volume averaging.

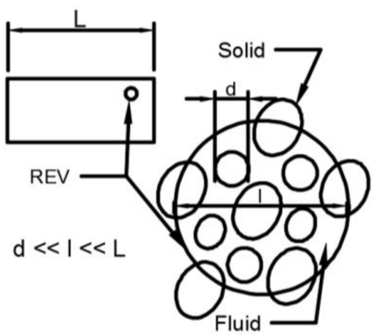

Fig 3. Schematic representation of control volume in porous media

Fig 3 shows representative elementary volume contained in a flow domain. The size of the pores or the solid particles is represented by d and size of the representative elementary volume, REV (V) is represented by l. The size of flow domain L is much larger than the size of the REV l. Thus, macroscopic transport equations are obtained by averaging the pore scale transport over this volume REV. Consider a property Ψ associated with β phase in REV. we define two different averages as:

Phase Average:

$$\langle \Psi_\beta \rangle = \frac{1}{V} \int_{V_\beta} \Psi_\beta dV \qquad 2\text{-}1$$

Intrinsic phase Average

$$\langle \Psi_\beta \rangle^\beta = \frac{1}{V_\beta} \int_{V_\beta} \Psi_\beta dV \qquad 2\text{-}2$$

And

$$\langle \Psi_\beta \rangle \; = \epsilon_\beta \langle \Psi_\beta \rangle^\beta \qquad 2\text{-}3$$

In which V_β represents the volume of β phase contained within REV and ε_β is volume fraction of β phase.

The volume averaging method relates the volume average of a spatial derivation to the spatial derivative of the volume average, and makes the transformation from microscopic equations to macroscopic equations. [27], [18] simplified the volume averaging method for making it widely useful for engineering application.

The Richard suggested that the Darcy's Law used for saturated flow in porous media is also applicable to unsaturated flow in porous media equation can also write as [28]

$$\frac{\partial \theta}{\partial t} = \nabla[D\nabla\theta + K] \qquad 2\text{-}4$$

Above equation is solved for flow through one dimensional horizontal column having length L in horizontal direction for x position and t time interval. Water Diffusivity, D and Permeability, K is assumed to be constant throughout the rig.

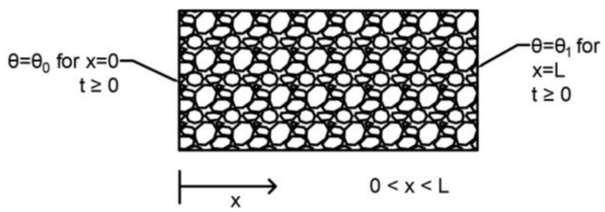

Fig 4. One Dimensional Horizontal Column

The theory of separated flow through the pore channels are distinguished two kinds of displacement:

1. The wetting phase is the displacing fluid (imbibition)
2. The wetting phase is the displaced fluid (drainage)

The phase whose saturation has been increased after displacement is always considered the displacing phase.

2.3 Permeability

The ease with which a fluid will travel through a pore space is known as permeability. Auriault et al. [29] presented that the permeability is a function of the properties of the solid media as well as of the flowing fluid. Experimental evidence has provided the insight that the permeability is proportional to the fluid weight per unit volume, ρg, and inversely proportional to the dynamic viscosity of the fluid, μ.

$$K = \frac{k\rho g}{\mu} \qquad 2\text{-}5$$

Where
k = function (ϕ and some measure of pore structure)

2.4 Capillary pressure

In capillary systems, mechanical equilibrium (i.e. the absence of a net mechanical force action on the system) is determined not only by the hydrostatic pressure and gravitational attraction but also by forces associated with surface tension. Capillary pressure (P_c) is difference between the concave and the convex sides of the meniscus is synonymous with the pressure difference between the nonwetting phase and the wetting phase. In simple word P_c is a function of saturation and is also equal to the pressure difference between the nonwetting and wetting phase (Pinder et al. [30]).

$$P_c = P'' - P' \qquad 2\text{-}6$$

2.5 Capillary pressure head

As discuss above, the capillary pressure is simply the difference between the higher one and the lower one of the two pressure reading (Bear, 1972 [31]). Often the symbol ψ is used for the capillary pressure head, that is

$$\psi \equiv h_c = \frac{P_c}{\rho_w g} \qquad 2\text{-}7$$

2.6 Saturation

The degree of saturation s relates to portion of pore volume filled with water as:

$$s = \frac{\text{Volume of liquid in the pores}}{\text{Pore volume}} \qquad 2\text{-}8$$

It varies between zero and unity. In addition, the relation between saturation and water content

$$\theta = \frac{\text{Liquid volume in the pores}}{\text{Volume of REV}} = s\phi \qquad 2\text{-}9$$

The relationship for pressure head and water content is available in literature, Van Genuchten [9]. The commonly used relationship for water content and pressure head is

$$\Theta = \frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}} = \left[\frac{1}{1 + (\alpha\Psi)^n}\right]^m \qquad 2\text{-}10$$

Where $\boldsymbol{\theta}$ = water content, $\boldsymbol{\theta}_{res}$ = Residual water content, $\boldsymbol{\theta}_{sat}$ = porosity, $\boldsymbol{\alpha}$ = constant = $\mathbf{y_b}$**-1**, **n**= constant = **λ+1**, **m** = constant = **1-(1/n)**, **λ** = pore size distribution index, $\mathbf{y_b}$ = bubbling pressure, **Ψ** = pressure head = **h-z**, **h** = total hydraulic head, **z** = elevation head,
Θ = Effective saturation.

The relationship between the relative permeability (K_r) and pressure head (h) as a function of the dimensionless water content (Θ) as derived by Mualem [32]is given as

$$K_r = \Theta^{1/2}\left[\int_0^\Theta \frac{1}{h(x)}dx \Big/ \int_0^1 \frac{1}{h(x)}dx\right]^2 \qquad 2\text{-}11$$

By solving for water content, using the Equation for dimensionless water content, we get

$$K_r(h) = \frac{\{(1-(\alpha h)^{n-1}[1+(\alpha h)^n]^{-m}\}^2}{[1+(\alpha h)^n]^{m/2}} \qquad 2\text{-}12$$

2.7 Conservation of Mass

The mass balance equation for each of the phases is given below Bear [31]:

$$\frac{\partial(\phi s_i \rho_i)}{\partial t} + \nabla(\rho_i u_i) = 0 \qquad 2\text{-}13$$

Where . i refered to water or oil phase, $\boldsymbol{\phi}$ is porosity, **s** the phase saturation, **u** the Darcy velocity and **ρ** the density.

2.8 Modified Momentum Equation

The momentum equation in the form of the generalized Darcy's Law is written by neglecting gravitation effect as

$$u_i = -\frac{K\, k_{ri}}{\mu_i}\nabla p_i \qquad 2\text{-}14$$

Where K, the absolute permeability of formation can be a space-dependent property (m^2) , k_r is relative permeability, p the pressure (N/m^2) and μ is the viscosity ($N\text{-}s/m^2$) of the i^{th} phase. Phase i^{th} stand for oil/ water phase.

Equation 2-13 can be solved using modified momentum equation for oil and water pressure equations with the help of Constitutive relationship.

3. CONSTANT DENSITY MODEL

By considering density as constant, equation 2-13 becomes

$$\frac{\partial}{\partial t}(\emptyset\, s_i) + \nabla u_i = 0 \qquad (i = \text{water, oil}) \qquad 3\text{-}1$$

By simplifying the equation can be written as

$$\frac{\partial s_i}{\partial t} + \frac{\nabla u_i}{\emptyset} = 0 \qquad (i = \text{water, oil}) \qquad 3\text{-}2$$

For Water System:

$$\frac{\partial s_w}{\partial t} = -\frac{\nabla u_w}{\emptyset} \qquad 3\text{-}3$$

By putting value of u_i from equation 2-14 the equation can be write for water system

$$\frac{\partial s_w}{\partial p_{cap}}\frac{\partial p_{cap}}{\partial t} = -\frac{\nabla}{\emptyset}\left[-\frac{K\, k_{rw}}{\mu_w}\nabla p_w\right] \qquad 3\text{-}4$$

By arranging value we get

$$\frac{\partial s_w}{\partial p_{cap}}\left[\frac{\partial p_o}{\partial t} - \frac{\partial p_w}{\partial t}\right] = \frac{\nabla k}{\emptyset \mu_w}[k_{rw}\nabla p_w] \qquad 3\text{-}5$$

For Oil System

Similar as equation 3-2 the oil system considering constant density unsaturated flow through porous media can be write as

$$\frac{\partial s_o}{\partial p_{cap}}\left[\frac{\partial p_o}{\partial t} - \frac{\partial p_w}{\partial t}\right] = \frac{\nabla k}{\emptyset \mu_o}[k_{ro}\nabla p_o] \qquad 3\text{-}6$$

By considering well known constitutive relation i.e.

$$s_w + s_o = 1 \qquad 3\text{-}7$$

We get

$$\partial s_o = -\partial s_w \qquad 3\text{-}8$$

So 3-6 can be write as

$$-\frac{\partial s_w}{\partial p_{cap}}\left[\frac{\partial p_o}{\partial t} - \frac{\partial p_w}{\partial t}\right] = \frac{\nabla k}{\emptyset \mu_o}\left[k_{ro} \nabla p_o\right] \qquad 3\text{-}9$$

By using equation 3-5 & 3-9 the couple model can be simulate.

4. VARIABLE DENSITY MODEL

Model which consider spatial variability in their input and output variables are known as spatially distributed variability model where a variety of constraints become operative. Constitutive relationship based on laws of nature or material behaviour helps to identified material properties or act as leverage along boundary condition.

By considering density as variable, equation 2-13 becomes

$$\frac{\partial}{\partial t}(\emptyset\, s_i \rho_i) + \nabla(\rho_i u_i) = 0 \qquad (i = \text{water, oil}) \qquad 4\text{-}1$$

To solve equation 4-1, constitutive relationships for density, ρ variation based on pressure and temperature is considered in this paper as compressibility and expansivity respectively.

4.1 Density variation considering compressibility only

Equation 4-1 can be further solved using compressibility term, which is based on pressure variation of i^{th} phase considering temperature as constant

$$\mathbb{C}_i = \frac{1}{\rho_i}\left(\frac{\partial \rho_i}{\partial p_i}\right)_T \qquad 4\text{-}2$$

By considering compressibility term only equation 4-1 becomes as

$$\frac{\partial}{\partial t}(\emptyset\, s_i \rho_i[\mathbb{C}_i]) + \nabla(\rho_i[\mathbb{C}_i]u_i) = 0 \qquad 4\text{-}3$$

Where, i^{th} represent water or oil phase. Term ρ_i is modified as $\rho_i[\mathbb{C}_i]$ as function of variation of density due to compressibility only.

Density variation considering expansivity only

Expansivity relationship considered only temperature variation effect considering pressure term as constant.

$$\ddot{E}_i = -\frac{1}{\rho_i}\left(\frac{\partial \rho_i}{\partial T}\right)_{p_i} \qquad 4\text{-}4$$

Similarly considering only expansivity terms only we get

$$\frac{\partial}{\partial t}(\emptyset\, s_i \rho_i[\ddot{E}_i]) + \nabla(\rho_i[\ddot{E}_i]u_i) = 0 \qquad 4\text{-}5$$

Where, i^{th} represent water or oil phase. Term ρ_i is modified as $\rho_i[\ddot{E}_i]$ as function of variation of density due to expansivity only.

Density variation considering both compressively as well as expansivity effect

By considering both compressibility and expansivity combining effects on density, ρ from equation 4-2 & 4-4 we get or simplified write as

$$\frac{\rho_i}{\rho_{ref}} = 1 + \mathbb{C}_i(p_i - p_{ref}) - \ddot{E}_i(T_i - T_{ref}) \qquad 4\text{-}6$$

Where ref term used as reference to initial porous media condition. Phase i^{th} represents presentence of water or oil phase. Here we consider change in density due to compressibility as positive, than effect of expansivity can be consider as negative. By solving equation 4-1 by putting value of density from equation 4-6 and neglecting small quantity effects we get

$$s_i \mathbb{C}_i \rho_{ref}\frac{\partial p_i}{\partial t} - s_i \ddot{E}_i \rho_{ref}\frac{\partial T_i}{\partial t} + \rho_i \frac{\partial s_i}{\partial t} = \nabla\left[\frac{K\, kr_i \rho_i}{u_i \emptyset}\right]\nabla p_i \qquad 4\text{-}7$$

Equation 4-7 further can be solved as

$$s_i \mathbb{C}_i \rho_{ref}\frac{\partial p_i}{\partial t} - s_i \ddot{E}_i \rho_{ref}\frac{\partial T_i}{\partial t} + \rho_i \frac{\partial s_i}{\partial p_{cap}}\frac{\partial p_{cap}}{\partial t} = \nabla\left[\frac{K\, kr_i \rho_i}{u_i \emptyset}\right]\nabla p_i \qquad 4\text{-}8$$

Further simplified as

$$s_i \mathbb{C}_i \frac{\partial p_i}{\partial t} - s_i \ddot{E}_i \frac{\partial T_i}{\partial t} + \frac{\partial s_i}{\partial p_{cap}}\frac{\partial p_{cap}}{\partial t} = \frac{\nabla}{\rho_i}\left[\frac{K\, kr_i \rho_i}{u_i \emptyset}\right]\nabla p_i \qquad 4\text{-}9$$

By putting value of $p_{cap} = p_o - p_w$, and further simplified we get

$$\left(s_i \mathbb{C}_i - \frac{\partial s_i}{\partial p_{cap}}\right)\frac{\partial p_i}{\partial t} - s_i \ddot{E}_i \frac{\partial T_i}{\partial t} + \frac{\partial s_i}{\partial p_{cap}}\frac{\partial p_r}{\partial t} = \frac{\nabla}{\rho_i}\left[\frac{K\, kr_i \rho_i}{u_i \emptyset}\right]\nabla p_i \qquad 4\text{-}10$$

Equation 4-10 is governing equation for variable density unsaturated porous media flow.

For water system

Equation 4-10 can be written for water phase as

$$\left(s_w \mathbb{C}_w - \frac{\partial s_w}{\partial p_{cap}}\right)\frac{\partial p_w}{\partial t} - s_w \ddot{E}_w \frac{\partial T_w}{\partial t} + \frac{\partial s_w}{\partial p_{cap}}\frac{\partial p_o}{\partial t} = \frac{\nabla}{\rho_w}\left[\frac{K\, kr_w \rho_w}{u_w \emptyset}\right]\nabla p_w \qquad 4\text{-}11$$

For oil system

Equation 4-10 can be written for oil phase as

$$\left(s_o \mathbb{C}_o - \frac{\partial s_o}{\partial p_{cap}}\right)\frac{\partial p_o}{\partial t} - s_o \ddot{E}_o \frac{\partial T_o}{\partial t} + \frac{\partial s_o}{\partial p_{cap}}\frac{\partial p_w}{\partial t} = \frac{\nabla}{\rho_o}\left[\frac{K\, kr_o \rho_o}{u_o \emptyset}\right]\nabla p_o \qquad 4\text{-}12$$

Equation 4-11 & 4-12 can be coupled using constitutive equation to find out simulation data.

5. CONCLUSION

Conservation of mass and momentum equation are used for pressure variation measurement. Based on pressure variation, properties based on capillary pressure, pressure head variation, saturation variation of water and oil can be calculated to plot/ evaluate relationship to better understanding of unsaturated flow problem deals in porous media. Above approach can be extended to develop a reliable laboratory scale model to investigate the flow related problem characteristic through porous media.

Further work can be extended to discretize the model for validation to real life problem by specifying the boundary conditions. Exploration of factor based on compressibility and expansivity are further scope/challenges for researchers.

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