

# Application of Iterative Learning Control Strategy for SISO Process

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## ABSTRACT

Iterative Learning Control (ILC) has recently much attention for system, where reference commands are periodic signal that work in the repetitive mode. In this work, implementation of ILC for the speed control DC motor system is carried out. The second order transfer function model for DC motor system is derived and identified. The major key factors such as learning filter and robustness filter in the ILC are designed based on the model parameters of the DC motor system. Simulation test are executed in the DC motor system with ILC and conventional PID controller. The superiority of ILC is estimated by means of tracking error. The simulation results reveal the efficiency of the ILC.

## Keywords

DC motor system, ILCS, PID, Root locus technique.

## 1. INTRODUCTION

Iterative learning control (ILC) is a technique to control the systems doing a defined task repetitively and periodically in a limited and constant time interval. Since the iterative learning control concept was proposed [1] (widely credited to Arimoto), a very large number of approaches have been considered. Examples of such systems are robot manipulators that are required to repeat a given task with high precision, chemical batch processes or, more generally, the class of tracking systems. Detailed literature reviews and recent developments on ILC research can be found in [2,3]. Early research efforts on ILC schemes were mainly on their analysis, without explicit design or synthesis procedures. However, the convergence conditions found in the literature are typically not sufficient for actual ILC applications. Therefore, in recent years increasing efforts have been made on the design issue of ILC. There are some efforts in [4,5] to use the parametric optimization approach to design the ILC controller. In the case of conventional controllers, it is still the most commonly used controller in the process control industry [6,7]. In a conventional control problem, asymptotic control may be achieved but it is difficult to track a trajectory within a specified error bound for an entire given span. The time duration for the execution of an operational cycle is finite and tracking control is always difficult with conventional controllers like the PID controllers which are more suitable for set-point regulation. To achieve a better tracking performance, a feed forward controller is usually applied. In this paper, a new feed forward controller - ILC is proposed and developed as a learning enhancement to a PID feedback controller. The use of learning in control systems is an obvious approach to overcoming this difficulty. Iterative learning control (ILC) is well-recognized as an efficient method that offers significant performance improvement for systems that operate in an iterative or repetitive fashion.[8,9]

The main contributions of the work presented in this paper are Iterative Learning Control strategy in a DC motor system and analyzes the tracking performance. In section 2, the mathematical model of DC Motor system is summarized. Conventional controller design is discussed in section 3. The design and structure of Iterative Learning Control Strategy is detailed in section 4. Simulation results are analyzed in section 5 to exemplify the better performance of the ILCS in closed loop. Finally, section 6 concludes the paper.

## 2. MODELING OF A DC MOTOR

The DC motor system is an electro mechanical system. The electrical system consists of the armature and the field circuit but for analysis purpose only the armature circuit is considered because the field is excited by a constant voltage. The mechanical system consists of the rotating part of the motor and load connected to the shaft of the motor. The armature constructed DC motor speed control system is shown in (Figure 1).

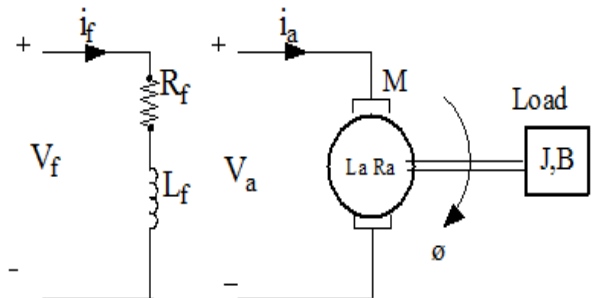


Fig 1: Schematic representation of DC motor

Where,

$R_a$  = Armature (Electric) resistance in  $\Omega$

$L_a$  = Armature (Electric) inductance in H

$i_a$  = Armature current in A

$V_a$  = Armature voltage (V)

$e_b$  = Back emf (V)

$T_q$  = Torque developed by motor (kg.cm)

$\omega$  = Angular displacement (rad / sec)

$J$  = Moment of inertia of motor ( kg.m<sup>2</sup> )

$B$  = Frictional coefficient of motor and load (Nm.s)

$K_b$  = Back emf constant

$K_t$  = Torque constant

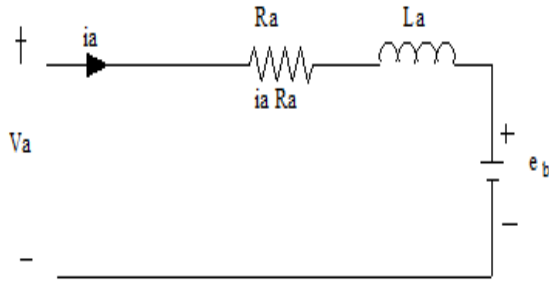


Fig 2: Equivalent circuit of armature

## 2.1 System Equation

The motor torque  $T$  is related to the armature current,  $i$ , by a torque constant  $K$  is shown in (Figure 3).

$$T = Ki \quad (1)$$

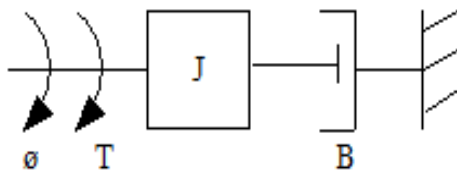


Fig 3: Mechanical system of DC motor

The generated voltage,  $e_b$ , is relative to angular velocity by

$$e_b = K\omega = K \frac{d\theta}{dt} \quad (2)$$

From (Figure 2), we can write the following equations based on the Newton's law combined with the Kirchhoff's law

$$J \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} = Ki \quad (3)$$

$$L \frac{di}{dt} + Ri = V - K \frac{d\theta}{dt} \quad (4)$$

Using the Laplace transform, equations (3) and (4) can be written as

$$Js^2 \theta(s) + b s \theta(s) = KI(s) \quad (5)$$

$$L s I(s) + R I(s) = V(s) - K s \theta(s) \quad (6)$$

Where  $s$  denotes the Laplace operator. From (6) we can express  $I(s)$

$$I(s) = \frac{V(s) - K s \theta(s)}{R + Ls} \quad (7)$$

And substitute it in (5) to obtain

$$Js^2 \theta(s) + b s \theta(s) = K \frac{V(s) - K s \theta(s)}{R + Ls} \quad (8)$$

The transfer function from the input voltage,  $V(s)$ , to the angular velocity,  $\omega$ , is

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K}{\{(R + Ls)(Js + b) + K^2\}} \quad (9)$$

The Closed-loop System that Representing the DC motor given in the (Figure 4).

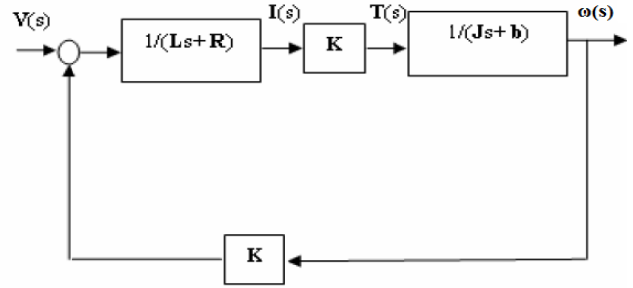


Fig 4: Closed-loop System that Representing the DC motor.

By considering the specifications of DC motor, the transfer function from the input voltage  $V(s)$ , to the angular velocity,  $\omega$ , is given as

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{1.01}{0.001025s^2 + 1.367s + 1} \quad (10)$$

## 3. CONVENTIONAL PID CONTROLLER

### 3.1 Control Design

Let us design a PID feedback controller to control the velocity of the DC motor. Recall that the transfer function of a PID controller.

$$C(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s = \frac{k_d s^2 + k_p s + k_i}{s}$$

Where  $u$  is the controller output,  $e = u_c + y$  is controller input and  $K_p$ ,  $K_i$ , and  $K_d$  are the proportional, derivative and integral gains, respectively.

### 3.2 Root locus Technique

Closed-loop response depends on the location of closed-loop poles. If system has a variable design parameter (e.g., a simple gain adjustment or the location of compensation zero), then the closed-loop pole locations depend on the value of the design parameter. The poles that provide the desired closed-loop response are selected and the proper value of the design parameter is thereby established.

The rlocus and rlocfind matlab functions are used to select the overall gain of the PID controller, such that the controller is stable and has the desired location of the poles (within the defined ratio among the  $K_p$ ,  $K_i$  and  $K_d$  constants). From the (figure 5), the closed loop system is stable and then the PID values are identified and tabulated in (Table 1).

Table 1. PID controller settings

| $K_p$ | $K_i$ | $K_d$ |
|-------|-------|-------|
| 1     | 0.8   | 0.3   |

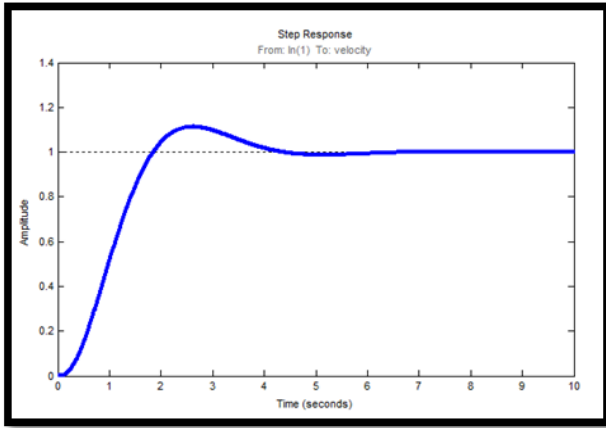


Fig 5: Closed-loop step responses with a PID controller

4.

## 5. ITERATIVE LEARNING CONTROL

Iterative learning control (ILC) is a method of tracking control for system that work in a repetitive mode. ILC is used data

Obtained during previous iteration to generate a new input which aims to reduce the tracking error at the iteration. ILC can reduce the tracking error to zero as the number of iteration tends to infinity.

### 5.1 Design Procedure and Guidelines

A block diagram of a system  $P(s)$  with feedback controller  $C(s)$  and ILC added is shown in (figure 6). The tracking error  $e(k)$  is defined as the difference between the reference signal  $r(k)$  and the measured position  $y(k)$ . The feed forward signal of ILC is denoted by  $f(k)$ . Iterations are denoted with an index,  $e(k)$  being the error of the  $k^{\text{th}}$  iteration. The iterations are all of equal finite time length,  $t \in [t_0, t_{\text{end}}]$ . ILC reduces the repetitive part of the error signal during several iterations. The tracking error is used off-line to calculate a feed forward signal that reduces the error from one iteration to the next. The error  $e(k)$  is filtered with a learning filter  $L$  and added to the feed forward  $f(k)$ .

## 5.2 Learning filter Design

The sum of  $f_k$  and the filtered error is applied to a robustness filter  $Q$ , resulting in the new feed forward signal  $f_{k+1}$ . The learning update of the feed forward signal equals

$$F_{(k+1)} = Q(f_k + Le_k) \quad (1)$$

Using the closed-loop transfer from the feed forward to the error signal, i.e. the process sensitivity  $S_p = P/(1+PC)$ , the propagation of the error from iteration to iteration can be written as

$$e_{(k+1)} = Q(1 - S_p L) e_k \quad (2)$$

The error reduces between successive iterations if

$$|Q(1 - S_p L)| < 1 \quad (3)$$

From (3) follows that a suitable choice for the learning filter  $L$  would be  $L = S_p^{-1}$ .

In most cases, especially in motion systems, the process sensitivity  $S_p$  is strictly proper and cannot be inverted. Furthermore, if  $S_p$  contains non-minimum phase zeros, the inverse will contain unstable poles. In these cases an approximate  $L \approx S_p^{-1}$  is determined by Zero Phase Error Tracking Control (ZPETC) [10] and the  $Q$  filter is required to provide robustness against modeling errors. The performance of ILC is determined to a great extent by the design of  $L$  and  $Q$ .

## 5.4 Low Pass Filter Design

In practice, there may be an insignificant deviation of the developed process model from the actual process. This deviation leads the  $L$  filter to cause some disturbances in stability condition of the control loop for high frequencies. To overcome this problem, the low pass filter is included in the control loop. A first order continuous time low pass filter is considered here. The cut-off frequency is identified in the frequency response of DC motor as shown in (figure 7).

$$Q = \frac{\omega_c}{s + \omega_c}$$

where  $\omega_c$  is the cut-off frequency in rad /s. The cut-off frequency is identified by frequency response of DC motor system.

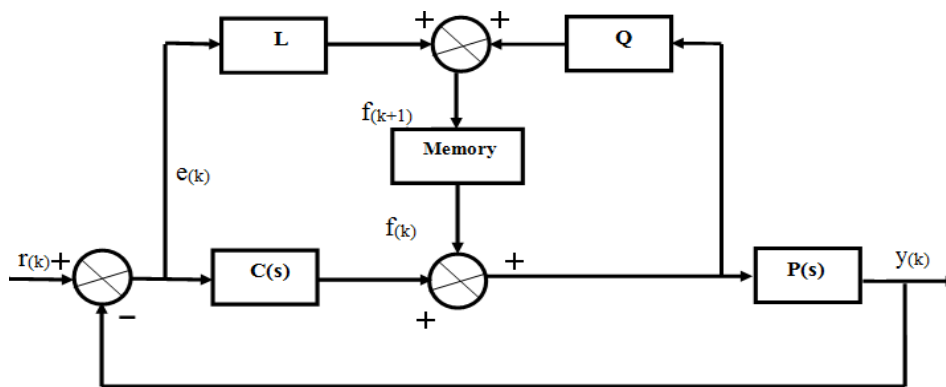


Fig 6: Block diagram of a system with feedback controller and ILC

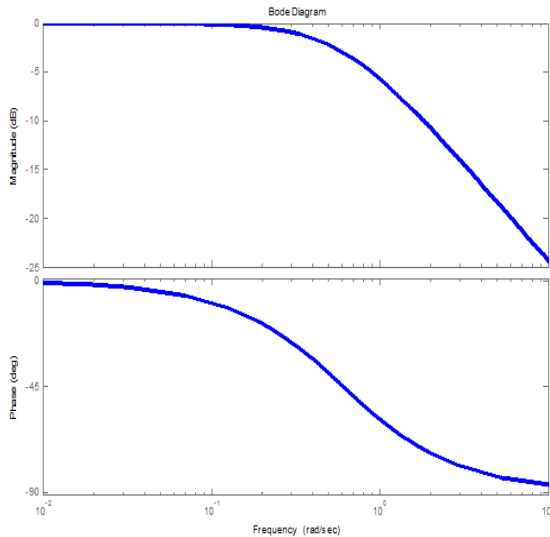


Fig 7: Bode plot of the DC motor

## 5. SIMULATION RESULT

An ILC controller is designed and implemented through simulation. The DC motor system with Iterative learning control is operated initially at 40% of steady state speed with (period  $T=70$ , amplitude  $A=5$ ). The tracking performances are traced in the (Figure 8). Similarly the conventional PID controller also carried out and their response is traced in the same figure. From the recorded data the performance index in terms of absolute tracking error at each iteration is computed and presented in the (Figure 12). To examine the flexibility of the ILCS simulations runs with another set of period and amplitude ( $T=60$ ,  $A=5$ ) are carried out in the DC motor system and their response are traced in the (Figure 9). In addition the different operating points (50%, 60% speed) are also carried out in the DC motor system and response are analyzed and is shown in (Figure 10 to 11). Also the absolute tracking errors are computed and plotted in (Figure 13 to 15).

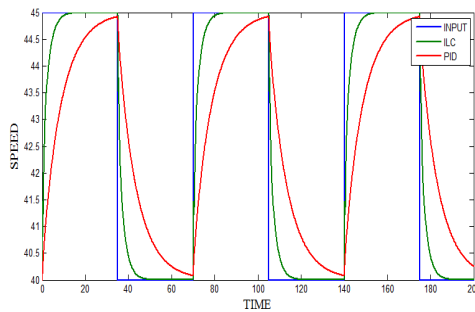


Fig 8: Simulation tracking responses of square reference trajectories {Period =70 Amp= 5 OP=40}

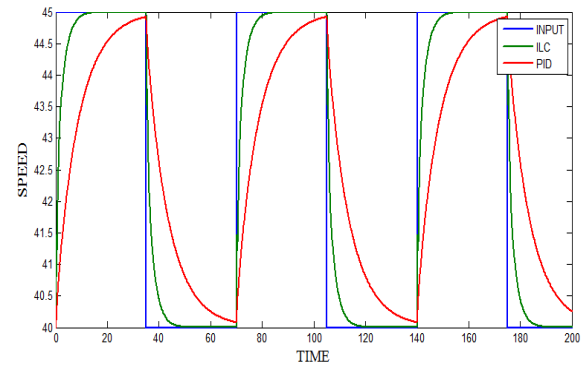


Fig 9: Simulation tracking responses of square reference trajectories {Period =60 Amp= 5 OP=40}

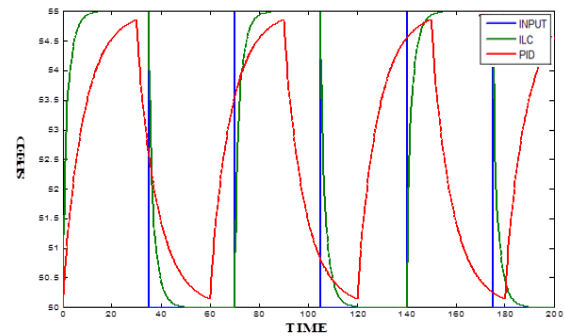


Fig 10: Simulation tracking responses of square reference trajectories {Period =70 Amp= 5 OP=50}

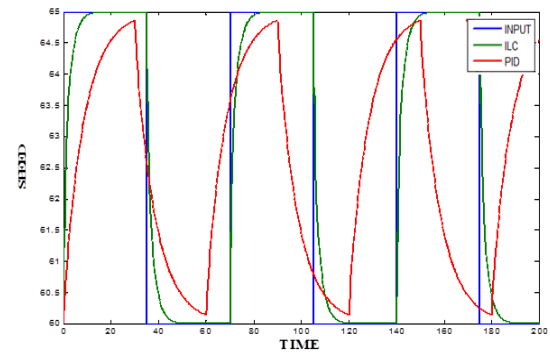


Fig 11: Simulation tracking responses of square reference trajectories {Period =70 Amp= 5 OP=60}

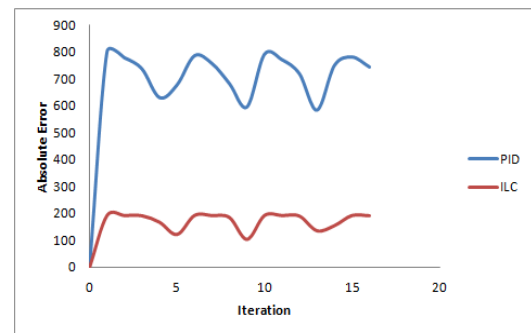
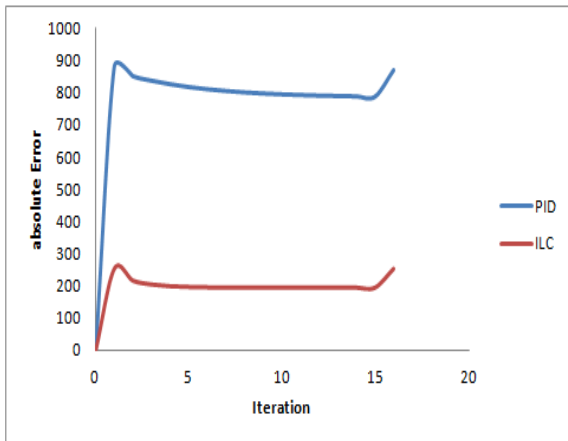
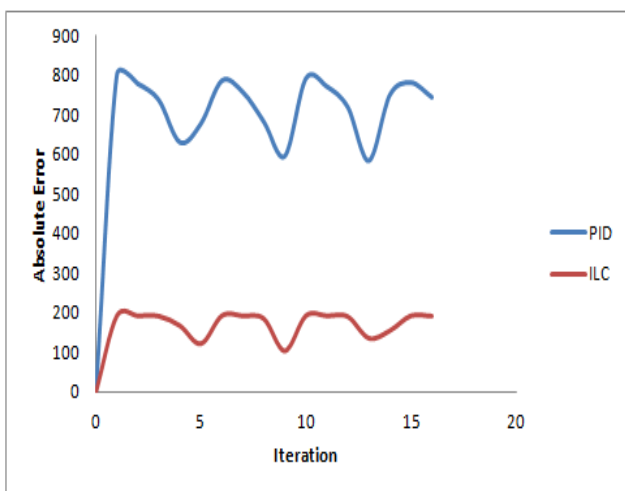


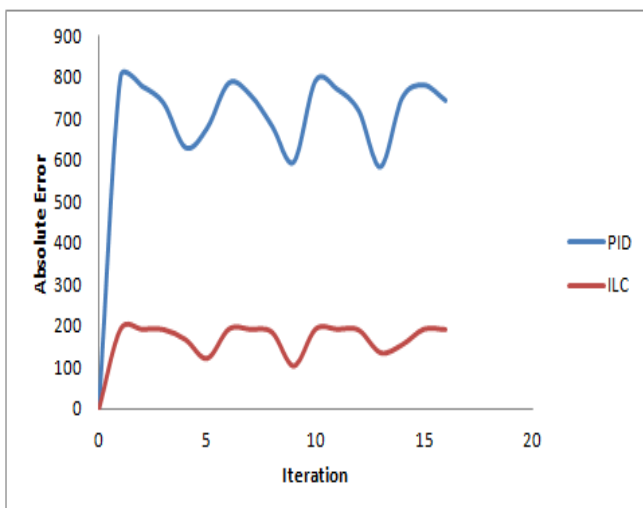
Fig 12: ILC and PID: Absolute tracking error responses of square reference trajectories {Period =70 Amp= 5 opp=40}



**Fig 13: ILC and PID: Absolute tracking error responses of square reference trajectories {Period =60 Amp= 5 OP=40}**



**Fig 14: ILC and PID: Absolute tracking error responses of square reference trajectories {Period =70 Amp= 5 OP=50}**



**Fig 15: ILC and PID: Absolute tracking error responses of square reference trajectories {Period =70 Amp= 5 OP=60}**

From the (Figure 12 to 15), it seems that, ILC gives better performance than the conventional PID controller. In all the operating conditions the result clearly indicates that the ILC gives minimum tracking error with lesser number of iteration as compared to conventional PID controller.

## 6. CONCLUSION

Iterative Learning Control is implemented in DC motor system and their performance is analyzed. The rlocus and rlocfind functions are used to select the overall controller parameters of the PID controller. Simulations are carried out with conventional PID controller and ILC controller. Performance analysis are also done. The results illustrate the superiority of ILC in speed control of DC motor system.

## 7. REFERENCES

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