

# Simulation and Modeling of Automatic Generation Control using PI and PID Controller in Deregulated Environment

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## ABSTRACT

Automatic Generation Control (AGC), a significant control process, which is responsible for frequency control, power interchange and economic dispatch operates constantly to balance the generation and load in power system at a minimum cost. AGC with control area concept, reflection of disco participation matrix (DPM) in AGC modeling and controller to obtain good output frequency response is studied in this paper. The tuning of PID controller is necessary to get an output with better dynamic and static performance. The output response of PID tuning is compared with PI and found reasonably good over conventional controller.

## Keywords

Automatic Generation Control, Bilateral contracts, deregulation, area participation factor, contract participation factor, Disco Participation Matrix

## 1. INTRODUCTION

To create a competitive market in power system, deregulation was introduced. Deregulated Power System requires innovation in planning and operation, keeping fundamental ideas intact. Ancillary Services of a power plant have a larger role to play in AGC. Hence they need to be formulated differently. It is applicable for generating companies (GENCO's), transmission companies (TRANSCO's), distribution companies (DISCO's). Management of large scale power system are done by control areas with appropriate interconnections between them. Each control area has to meet its own demand and scheduled power. Balancing between load and generation can be achieved by using AGC. Any mismatch between the two can be observed by means of deviation in frequency. Any changes in load of a Power System leads to a change in active power, which in turn cause system frequency change. Similarly any change in reactive power affects the magnitude of voltage. Objective of this paper is to compare the output of AGC with P, PI and PID controller and study the output response [1][4].

## 2. AUTOMATIC GENERATION CONTROL (AGC)

Frequency and Tie line power interchange vary in response to change in load. Due to disturbance, system dynamics also changes and becomes unstable. To make the system stable, frequency and tie line power variation should be made zero, which is achieved by supplementary controller that controls the whole system there by making it stable. The complete process is referred as AGC [6].

### A. Importance of AGC in Deregulated Environment

- i. To make static frequency nil.
- ii. To disperse generation among areas such that interconnected tie line flows are in line with prescribed schedule.
- iii. To accomplish equilibrium between the total power generation with net load and tie line power exchanges.

### B. Power System Frequency control

Any imbalance between the electrical load and the active power due to the connected generators resulted in frequency deviation. Constant frequency deviations directly impact the working of power system, along with its reliability, security and efficiency by deteriorating load performance, overloading transmission lines, triggering protection devices. In response to any frequency change primary control carries out local automatic control such that it delivers the reverse power in opposition. The supplementary loops gives feedback via the frequency deviation and add it to the primary control loop through a dynamic controller. The resulting signal is used to regulate the system frequency. To ensure system stability load shedding is performed to reduce system load. The load shedding will only be used if frequency falls below a specified frequency threshold [8].

## 3. TIE-LINE MODELLING IN TWO AREA

In normal operation, power on tie-line is:

$$P_{12} = \frac{|v_1||v_2|}{x} \sin(\delta_1 - \delta_2)$$

Where  $v_1, v_2$  are magnitude of end voltages of control areas 1 and 2.

$\delta_1, \delta_2$  are angles of the end voltage  $v_1$  and  $v_2$ .

For a small deviation in angles  $\delta_1$  and  $\delta_2$  tie-line power changes as:

$$\Delta P_{12} = \frac{|v_1||v_2|}{x} \cos(\delta_1 - \delta_2)(\Delta\delta_1 - \Delta\delta_2)$$

Analogous to concept of "electric stiffness" of synchronous machine, we define synchronizing co-efficient of a line:

$$T_{12} = \frac{|v_1||v_2|}{x} \cos(\delta_1 - \delta_2) \text{ Mw/radians}$$

The frequency deviation  $\Delta f$  is related to reference angle  $\Delta\delta$  by:

$$\Delta f = \frac{1}{2\pi} \frac{d(\delta + \Delta\delta)}{dt} = \frac{1}{2\pi} \frac{d(\Delta\delta)}{dt} \text{ Hz}$$

By expressing tie-line power deviations in terms of  $\Delta f$  rather than  $\Delta\delta$ , we get:

$$\Delta P_{12} = 2\pi T \left( \int_0^t \Delta f_1 dt - \int_0^t \Delta f_2 dt \right) \text{ MW}$$

Taking Laplace, it yields:

$$\Delta P_{12}(s) = \frac{2\pi T}{s} (\Delta f_1(s) - \Delta f_2(s))$$

$$T_{12} = \frac{1}{s} \text{ MW/radians.}$$

The transfer of power over Tie-line is :

$$\Delta P_{12} = -\Delta P_{21}$$

## 4. MATHEMATICAL MODELLING OF AGC

### a) Disco participation matrix (DPM)

For deregulated system having multiple GENCOs and DISCOs. Any DISCO may contract with any GENCO in another control area independently. This case is called as “bilateral transaction”. The transactions are to be implemented through an impartial entity called INDEPENDENT SYSTEM OPERATOR (ISO). In restructured environment, any DISCO has freedom to buy power at competitive prices from different GENCOs, which may or may not have contract in same area as the DISCO. This can be defined as Bilateral Transaction. DPM gives the participation of a DISCO in contract with which GENCO. Hence its name is DPM. It is a matrix in which the number of rows is equal to the number of GENCOs and the number of columns is equal to the number of the DISCOs in the system. Individual entry in this matrix can be taken as a fraction of total load contracted by a DISCO (column) concerning a GENCOs (row). Thus  $ij$ th entry corresponds to the fraction of the total load power contracted by DISCO, from a GENCO  $i$ . The sum of all the entries in a column in this matrix is unity. Hence  $\sum cpf_{ij}=1$  i.e. the sum of all  $cpf$ 's is unity.

Let us take a two area system in which a single area has two GENCOs and two DISCOs in it. Assume GENCO1, GENCO2, DISCO1, DISCO2 be in area1 and GENCO3, GENCO4, DISCO3, DISCO4 be in area 2 as shown in Fig 1.

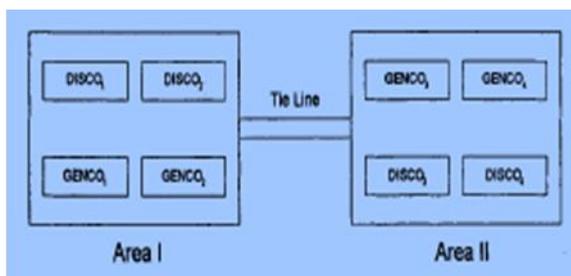


Fig. 1 Schematic of two area system in deregulated environment.

The DPM for this system will become:

$$\begin{bmatrix} cpf11 & cpf12 & cpf13 & cpf14 \\ cpf21 & cpf22 & cpf23 & cpf24 \\ cpf31 & cpf32 & cpf33 & cpf34 \\ cpf41 & cpf42 & cpf43 & cpf44 \end{bmatrix} \quad (1)$$

Where  $cpf$  refers to “contract participation factor”. The block diagonal of DPM refers to local demands while the demands of DISCOs in one area with GENCOs in another area represents off diagonal elements.

To explain the concept, suppose DISCO2 demands 0.04 PU MW power, out of which 0.01 PU MW power is demanded from GENCO1, 0.012 PU MW power from GENCO2, 0.014 PU MW power from GENCO3 and 0.004 PU MW power from GENCO4. Then the entries in column 2 of equation (1) are as follows:

$$Cpf_{12}=0.01/0.04=0.25, \\ cpf_{22}=0.012/0.04=0.30,$$

$$Cpf_{32}=0.014/0.04=0.35, \\ cpf_{42}=0.004/0.04=0.10. \quad (2)$$

### b) Block diagram formulation

In this portion, the block diagram for a two-area AGC system in the deregulated structure is originated. Whenever a load demanded by a DISCO fluctuates, it is manifested as a local load in the area to which this DISCO belongs. These local loads  $\Delta P_{L1}$  and  $\Delta P_{L2}$  should reveal at the point of input to power system block. It is possible that a DISCO breaks its contracts by asking more power than that which is mentioned in the contracts. This excess power is not contracted out to any GENCO and this uncontracted power demand must be provided by the GENCOs in the same area as the concerned DISCOs also manifest at the point of input to the power system block.

As several GENCOs are present in each area, hence ACE signal has to be distributed among them, equivalent to their involvement in AGC. This leads to formulation of ACE Participation Factor” (apf). apfs are the co-efficient that distributes ACE to several GENCOs.  $apf_{ij}$  represents the ACE participation factor of GENCO $_i$  in area ‘j’. Also the sum of all apfs in an area is unity.

In restructured environment, a DISCO demands a particular GENCO for power which can be known from DPM. Their demands must be reflected in dynamics of system. Turbine and governor units must react to this new power demand. Thus as a particular set of GENCOs are presumed to walk behind the load demanded by a DISCO, information signal should move from a particular DISCO to a particular GENCO identifying corresponding demands. The demands are particularised by  $cpf$ s and the power load of a DISCO. These signals carry information as to which GENCO has to follow a load, demanded by which DISCO.

The scheduled steady state power flow on the tie line is given as follows:

$$\Delta P_{TIE1-2, SCHEDULED} = (\text{demand of DISCOs in area II from GENCO in area I}) - (\text{demands of DISCOs in area I from GENCO in area II}) \quad (3)$$

At any instant the line power error  $\Delta P_{TIE1-2,error}$  can be expressed as

$$\Delta P_{TIE1-2,error} = \Delta P_{TIE1-2,actual} - \Delta P_{TIE1-2,scheduled} \quad (4)$$

$\Delta P_{TIE1-2,error}$  Disappears in the steady state as the actual tie line power flow attains the organized power flow. In the traditional structure, error signals are used to generate the respective ace signals as  $ACE1 = B_1 \Delta F_1 + \Delta P_{TIE1-2,error}$

$$ACE2 = B_2 \Delta F_2 + \Delta P_{TIE2-1,error} \quad (5)$$

Where

$$\Delta P_{TIE2-1,error} = -\frac{P_{r2}}{P_{r1}} \Delta P_{TIE1-2,error}$$

And  $P_{r1}, P_{r2}$  are the rated powers of areas 1 and 2 respectively

$$\text{Therefore, } ACE2 = B_2 \Delta F_2 + \alpha_{12} \Delta P_{TIE1-2,error}$$

$$\text{Where } \alpha_{12} = \frac{P_{r1}}{P_{r2}}$$

The block diagram for AGC in a deregulated system is shown in Fig 2. Lingually it is based upon the idea of [1] , [2]. Dashed lines show demand signals.  $\Delta P_{L1,LOC}$  and  $\Delta P_{L2,LOC}$  are the local loads in areas 1 and 2.

### c) State space characterization of two area system in restructured environment

The closed loop system in fig. 2 is characterised in state space form as

$$dX/dt = A^{CL}x + B^{CL}u \quad (6)$$

Where x is the state vector and u is the vector of power demands of the DISCOs.  $A^{CL}$  and  $B^{CL}$  matrices are constructed from fig 2 . The state matrices are given by (7) and (8).

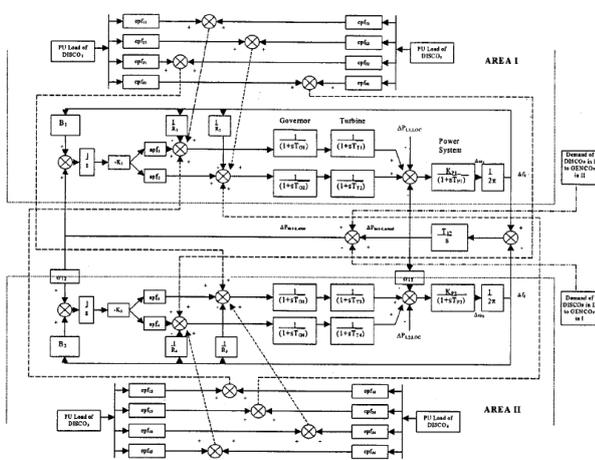


Fig. 2 Block Diagram Of Two-area AGC system.

## 5. SIMULATION RESULTS OF A TWO AREA SYSTEM IN THE RESTRUCTURED ENVIRONMENT

To explain the behavior of a two area proposed AGC scheme the following case are described. The data given in Table1 is used for simulations. Both areas are considered to be alike. The governor turbine units in the each area are supposed to be same.

A. 1<sup>st</sup> Case: Base case

Assume a case where the GENCOs in each area perform identically in AGC i.e., apfs are equal for each area i.e.  $apf_1=0.5, apf_2=1-apf_1=0.5, apf_3=0.5, apf_4=1-0.5=0.5$ .

Consider that the load change occurs only in area I. Thus the load involves only DISCO1 and DISCO2. Suppose the value of this load demanded be 0.1 pu MW for each of them . Referring to (1), DPM becomes,

$$DPM = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that  $DISCO_3$  and  $DISCO_4$  does not involve power from any GENCOs, which leads to zero cpfs for area 2.  $DISCO_1$  and  $DISCO_2$  demands identically from their local GENCOs, via,  $GENCO_1$  and  $GENCO_2$ . Fig 3 shows the responses of the load change: area frequency deviations, actual power flow on the tie line (in a direction from area I and area II) and the generated power of various GENCOs pursuing a step change in the load demands of DISCO1 and DISCO2. The frequency deviation in each area goes to zero in the steady state [fig 3(a)]. as only the DISCOs in area I, via DISCO1 and DISCO 2 have non zero load demands ,the transient dip in frequency of area I is larger than that of area II . Since the off diagonal blocks of DPM are zero i.e., there are no contracts of power between a GENCO in one area and a DISCO in another area the scheduled steady state power flow over the tie line is zero . the actual power on the tie line goes to zero shown in fig.3(b).

In the steady state, generation of GENCO must match the demands of DISCOs in contract with it .this desired generation of GENCO in pu MW can be expressed in terms of cpfs and the total demands of DISCOs as

$$\Delta P_{Mi} = \sum_j cpf_{ij} \Delta P_{Lj} \quad (9)$$

Where  $\Delta P_{Lj}$  is total demands of DISCO j and cpfs are given by DPM.

For the case under consideration ,we have ,

$$\Delta P_{M1} = 0.5X\Delta P_{L1} + 0.5X\Delta P_{L2} = 0.1puMW$$

Similarly ,

$$\Delta P_{M2} = 0.1puMW, \Delta P_{M3} = 0puMW, \Delta P_{M4} = 0puMW$$

As Fig 3(c) shows , the actual generated powers of the GENCOs reached the desired value in steady state. GENCO3 and GENCO4 are not contracted by any DISCO for transaction of power ,hence their change in generated power is zero in steady state .

B. 2<sup>nd</sup> CASE : Normal Case

Consider a case where all the DISCOs contract with the GENCOs for power as per the following DPM:

$$DPM = \begin{bmatrix} 0.5 & 0.25 & 0 & 0.3 \\ 0.2 & 0.25 & 0 & 0 \\ 0 & 0.25 & 1 & 0.7 \\ 0.3 & 0.25 & 0 & 0 \end{bmatrix}$$

It is assumed that each DISCO demands 0.1 pu MW power from GEMCOs as defined by cpfs in DPM matrix and each GENCO participates in AGC as defined by following apfs: apf1=0.75, apf2=1-apf1=0.25; apf3=0.5, apf4=1-apf3=0.5. ACE participation factors affect only the transient behavior of the system and not the steady state behavior when un-contracted loads are absent.

The system in fig 2 is simulated using data and the results are depicted in fig 4. The off diagonal blocks of the DPM corresponds to the contract of a DISCO in one area with a GENCO in another area. From (1) and (3), the scheduled power on the tie line in the direction from area I to area II is

$$\Delta P_{TIE1-2, SCHEDULED}$$

$$= \sum_{i=1}^2 \sum_{j=3}^4 cpf_{ij} \Delta P_{Lj} - \sum_{i=3}^4 \sum_{j=1}^2 cpf_{ij} \Delta P_{Lj} \quad (11)$$

Hence  $\Delta P_{TIE1-2, SCHEDULED} = 0.05$  pu MW fig.4(d) shows the actual power on the tie line. If it is to be observed that it settles to -0.05 pu MW, which is the scheduled power on the tie line in the steady state.

At given by (10), in the steady state, the GENCOs must generate

$$\Delta P_{M1} = 0.5(0.1) + 0.25(0.1) + 0 + 0.3(0.1) = 0.105 \text{ pu MW}$$

and

$$\Delta P_{M2} = 0.045 \text{ pu MW}, \Delta P_{M3} = 0.195 \text{ pu MW}, \Delta P_{M4} = 0.055 \text{ pu MW}$$

Fig 4(b),(c) shows actual generated powers of GENCOs. The responses meet respective desired generations in the steady state.

C. 3<sup>rd</sup> CASE : Contract fluctuation

It may happen that a DISCO fluctuates a contract by demanding more power than that specified in the contract. This excess power than that specified in the contract. This excess power is not contracted out of any GENCO. This uncontracted power must be supplied by the GENCOs in the same area as the DISCO. It must be reflected as the local load of the area but not as the contract demand. Consider case2 again with a modification that DISCO1 demands 0.1 pu MW of excess power.

The total local in area I ( $\Delta P_{L1, loc}$ )

$$= \text{load of DISCO1} + \text{load of DISCO2}$$

$$= (0.1 + 0.1) + 0.1 \text{ pu MW} = 0.3 \text{ pu MW}$$

Similarly, the total local load in area II ( $\Delta P_{L2, loc}$ )

$$= \text{load of DISCO3} + \text{load of DISCO 4} = 0.2 \text{ pu MW}$$

The frequency deviations vanish in the steady state (FIG.5(a)). A DPM is the same in case 2 and the excess load taken off by GENCOs in the same areas, the tie line power is the same as in case 2. In steady state, the generation of GENCOs 3 and 4 is not affected by the excess load of DISCO 1 (refer case2). The uncontracted load of DISCO1 is reflected in the generations of GENCO1 AND GENCO2. ACE participation factors decide the distribution of uncontracted load in the steady state, thus this excess load is taken by the GENCOs in the same area as that of DISCO making the uncontracted demands.

Discussion: In the proposed AGC implementation, contracted load is fed forward through the DPM matrix to GENCO set point this is shown in fig 2 by dotted lines the actual load affect system dynamic via the inputs to the power system blocks. Any mismatch between actual and contracted demands will result in frequency deviation that will drive AGC to re-dispatch GENCOs according to apfs.

It is assumed that each control area contains at least one GENCO that participates in AGC, i.e., has non zero apf.

The proposed AGC scheme does not require measurement of actual load the inputs in the block diagram of fig 2 are part of power system model not part of AGC.

## 6. MODEL USING PID CONTROLLER AND ITS COMPARISON WITH PI

Two single area systems having two generating units are connected in two areas shown in fig.2 and is interconnected via tie-line. It increases the system reliability. If any generating unit fails in one area, then the generating unit of other area compensates to meet its load demand by making the steady state error zero. To make the steady state error zero, we can make use of different controller such as P, PI, and PID. In this paper the use of PI controller is already shown. In this section we will observe the responses using PID controller and will see that PID controller gives better result than PI, hence making the frequency parameter optimized. The PID controller improves steady state error simultaneously transient response with little overshoots [8],[6]. The output response in the above three cases are depicted in Fig. 6(a),(b), Fig. 7(a),(b), Fig. 8(a),(b). The values used to eliminate steady state error are as follows:  $K_p = -0.1, K_i = -0.0002, K_d = -0.01$

## 7. OUTPUT RESPONSE USING PI CONTROLLER

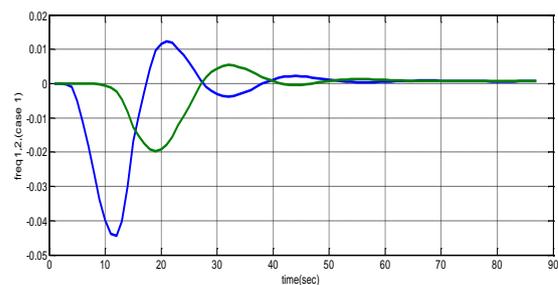
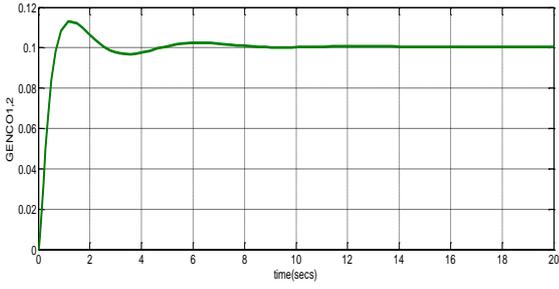
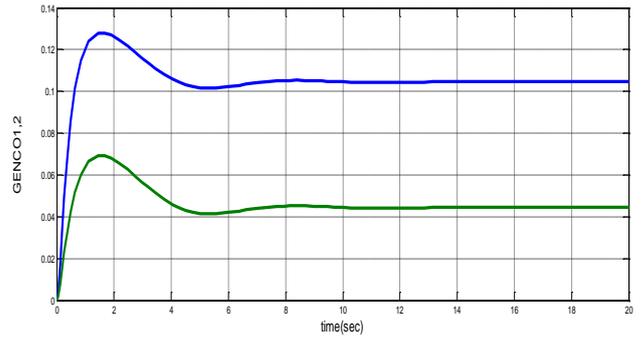


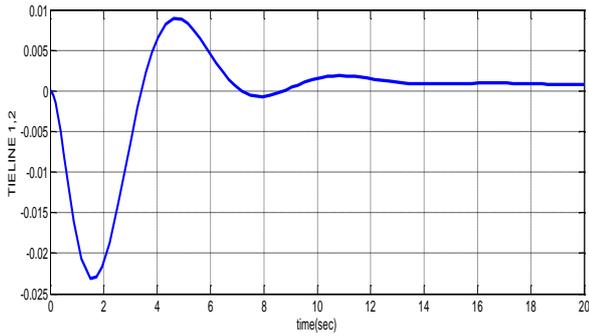
Fig. 3 (a) Frequency Deviation



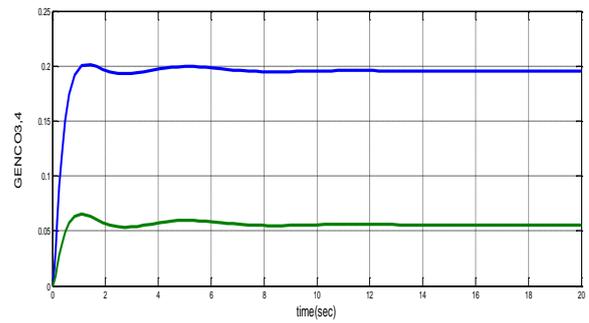
**Fig.3(b) GENCO power (1,2)**



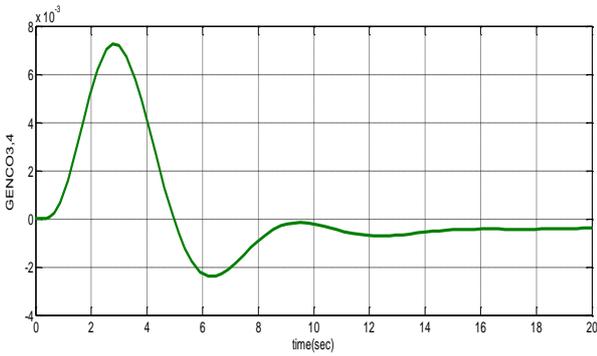
**Fig.4(b) GENCO power(1,2)**



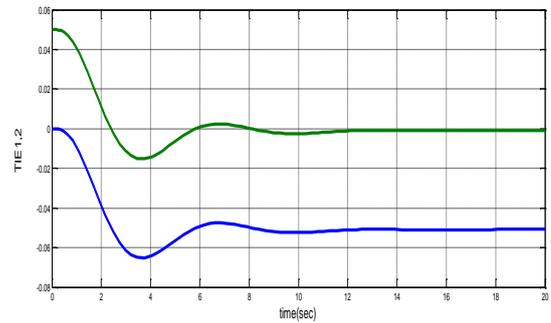
**Fig.3(c) TIE\_LINE power**



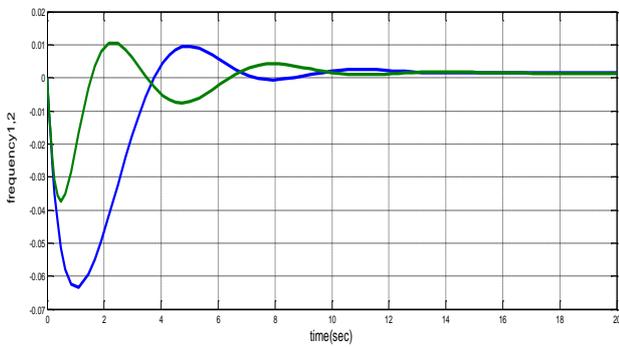
**Fig. 4(c) GENCO power(3,4)**



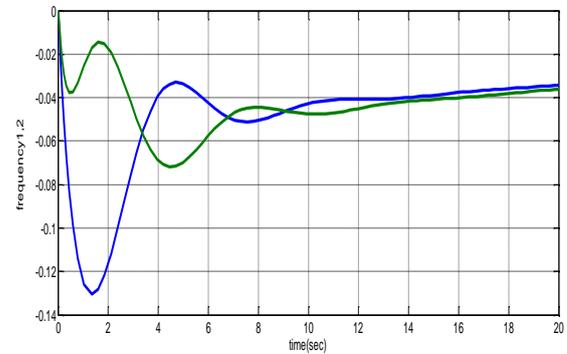
**Fig.3(d) GENCO power(3,4)**



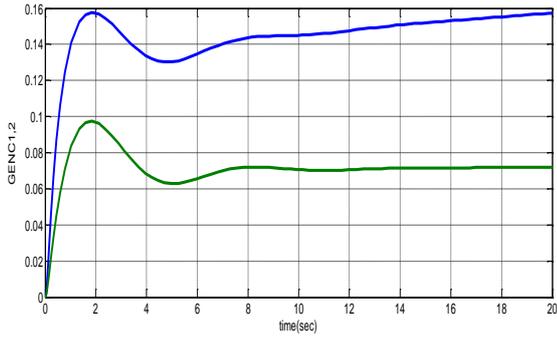
**Fig.4(d) TIE\_LINE power**



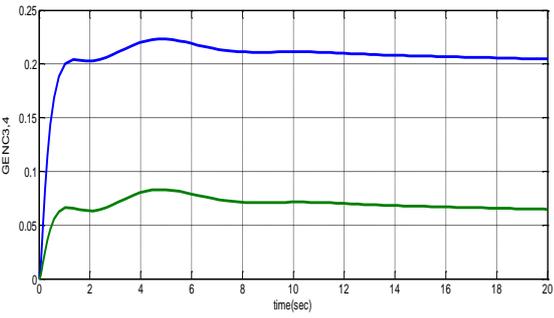
**Fig.4(a) Frequency Deviation**



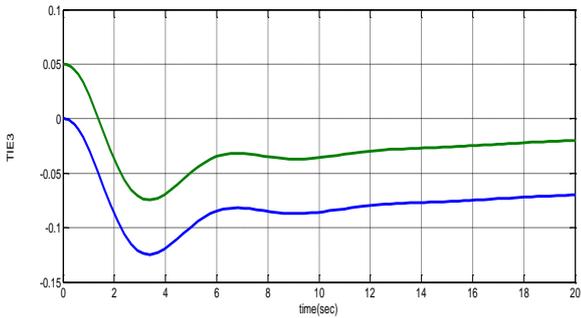
**Fig.5(a) Frequency Deviation**



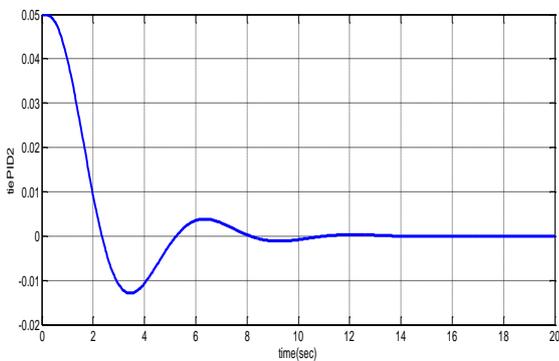
**Fig.5(b) GENCO power(1,2)**



**Fig.5(c) GENCO power(3,4)**

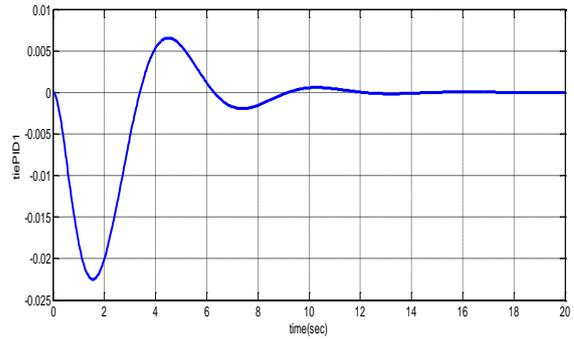


**Fig.5(d) TIE\_LINE power**

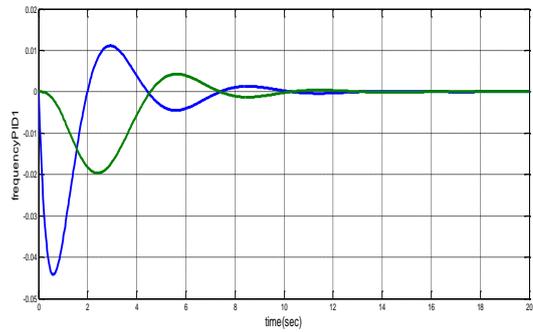


**Fig.7(a) TIE\_LINE power**

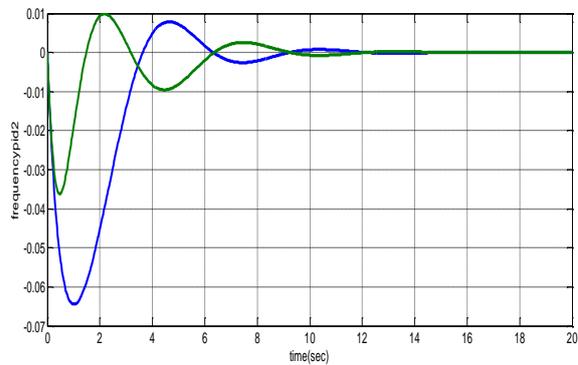
## 8. OUTPUT RESPONSE USING PID CONTROLLER:



**Fig. 6(a) TIE\_LINE power**



**Fig.6(b) Frequency Deviation**



**Fig.7(b) Frequency Deviation**

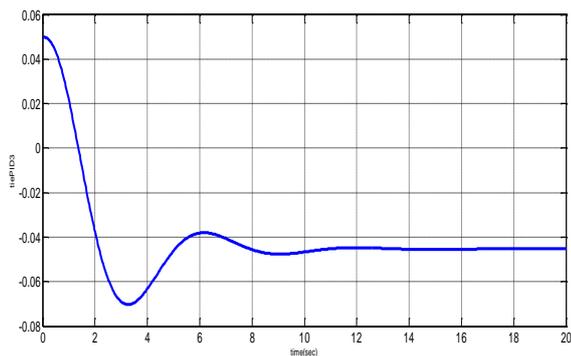


Fig.8(a) TIE\_LINE power

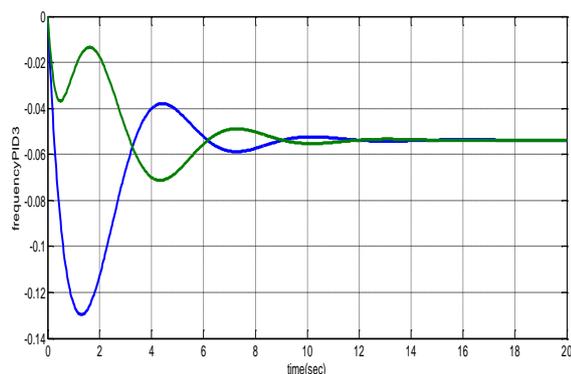


Fig.8(b) Frequency Deviation

## 9. CONCLUSION

This paper work gives an review of AGC in restructured domain which obtains a elementary role to allow power exchanges and to furnish better conditions for electricity trading. Bilateral contracts can prevail between DISCOs in one control area and GENCOs in other control area. DISCO participation matrix also concerns about bilateral contracts in this paper. Apart all this, the paper is first modeled using PI controller and then the steady state error elimination is done using PID controller, which eliminates the error upto a great extent.

## 10. NUMERICAL DATA

Pr1=Pr2	2000MW
H1=H2	5seconds
D1=D2	$8.33 \cdot 10^{-3}$
$T_{t1}=T_{t2}$	0.3 seconds
$T_{g1}=T_{g2}$	0.08 seconds
R1=R2	2.4Hz/pu MW
$T_{12}$	0.545 pu MW/Hz

## 11. REFERENCES

- [1] Donde V. Pai M.A., and Hiskens I.A.,2001. "Simulation and optimization in an AGC system after deregulation,"IEEE Transaction on Power Systems,vol.16,No.3.
- [2] J.Kumar,K.Ng and G.Sheble,"AGC simulator for price based operation:part 1,"IEEE Trans. Power Systems,vol.12,no. 2,May 1997.
- [3] J.Kumar,K.Ng and G.Sheble,"AGC simulator for price based operation:part 2,"IEEE Trans. Power Systems,vol.12,no. 2,May 1997.
- [4] O.I.Elgerd and C.Fosha,"Optimum Megawatt frequency control of multi-area electric energy systems," IEEE Trans. Power Apparatus &Systems,vol. PAS-89,no. 4,Apr. 1970.
- [5] C.Fosha and O.I.Elgerd,"The megawatt frequency control problem:A new approach via optimal control theory,"IEEETrans.Power Apparatus &Systems,vol.PAS-89,no. 4,Apr. 1970.
- [6] Karnavas and Dedousis,"Overall performance evaluation of evolutionary designed conventional AGC controllers for interconnected electric power system studies in a deregulated market environment,"IJEST, vol. 2,no. 3,2010.
- [7] I.J.Nagrath, D.P.Kothari, Power system Engineering, TATA McGrawHill, New York, 1994.
- [8] Aparajita Salgotra, Sumit Verma, "Modelling and simulation of automatic generation control in a deregulated environment and its optimization ULQR based integral controller," IJETAE, vol. 2, Issue 11, November 2012.
- [9] H. G. Kwatny, K. C. Kalnitsky, and A. Bhatt, "An optimal tracking approach to load frequency control," IEEE Trans. Power App. Syst., vol. PAS-94, no. 5, pp. 1635–1643, Sep./Oct. 1975.
- [10] T.M. Athay, "Generation Scheduling and Control", Proc. IEEE, Vol. 75, No. 12, pp. 1592-1606, December 1987.
- [11] R. Christie and A. Bose, "Load-frequency control issues in power systems operations after deregulation," IEEE Trans. Power Systems, vol. 11, pp. 1191–1200, Aug. 1996.
- [12] Christie, B. F. Wollenberg, and I. Wangenstein, "Transmission management in the deregulated environment," Proc. IEEE Special Issue on the Technology of Power System Competition, vol. 88, no. 2, pp. 170–195, Feb. 2000.
- [13] Ignacio Egido, Fidel Fernández-Bernal, Luis Rouco, Eloisa Porras, and Ángel Sáiz-Chicharro, "Modeling of Thermal Generating Units for Automatic Generation Control Purposes", IEEE Trans. control systems technology, vol. 12, no. 1, Jan. 2004.
- [14] P.V.Kokotovic and R.Rutman,"Sensitivity of automatic control system(survey), "Automation and remote control,vol. 26,1965.
- [15] N.Jaleeli, L.S. Vanslyck et al, "Understanding Automatic Generation Control,"IEEE Trans. On Power Systems, vol. 7,no. 3, August 1992.
- [16] K.RamaSudha,V.S.Vakula, R.VijayaShanthi, "PSO based Design of Robust Controller for Two Area Load Frequency Control with Non linearities", International Journal of Engineering Science and Technology, Vol. (5), 2010,1311-1324

- [17] N. Cohn, Control of Generation and Power Flow on Interconnected Power Systems, John Wiley & Sons, New York, 1961.
- [18] Gjerde, I. Glende, G. Nilssen, L. Nesse, "Coordination of Power Sys- tem Operation in a Competitive Power Market Environment", CIGRE Paper 39-204, CIGRE, 28 August-September 3, 1994, Paris, France.
- [19] Jeffries, "Changes and Challenges for the National Grid", Modern Power Systems, July 1992, suppl. pp. 23, 25. L.R.
- Day, "CPC Revisited", IEEE Computer Applications in Power, Vol. 7, No. 4, October 1994 pp. 40-45.
- [20] R.P. Schulte, W.R. McReynolds and D.E. Badley, "Modified Automatic Time Error Control and Inadvertent Interchange Reduction for the WSCC Interconnected Power Systems", IEEE Transactions on Power Apparatus and Systems, Vol. 6, No. 3, August 1991, pp. 904-913v

**APPENDIX**

$B_{CL} =$

$$\begin{bmatrix}
 -\frac{K_{P1}}{T_{P1}} & -\frac{K_{P1}}{T_{P1}} & 0 & 0 \\
 0 & 0 & -\frac{K_{P2}}{T_{P2}} & -\frac{K_{P2}}{T_{P2}} \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 \frac{cpf11}{T_{G1}} & \frac{cpf12}{T_{G1}} & \frac{cpf13}{T_{G1}} & \frac{cpf14}{T_{G1}} \\
 \frac{cpf21}{T_{G2}} & \frac{cpf22}{T_{G2}} & \frac{cpf23}{T_{G2}} & \frac{cpf24}{T_{G2}} \\
 \frac{cpf31}{T_{G3}} & \frac{cpf32}{T_{G3}} & \frac{cpf33}{T_{G3}} & \frac{cpf34}{T_{G3}} \\
 \frac{cpf41}{T_{G4}} & \frac{cpf42}{T_{G4}} & \frac{cpf43}{T_{G4}} & \frac{cpf44}{T_{G4}} \\
 cpf31 + cpf41 & cpf32 + cpf42 & -(cpf13 + cpf23) & -(cpf14 + cpf24) \\
 \frac{cpf41}{T_{G4}} & \frac{cpf42}{T_{G4}} & \frac{cpf43}{T_{G4}} & \frac{cpf44}{T_{G4}} \\
 0 & 0 & 0 & 0
 \end{bmatrix}$$

$A_{CL} =$

$$\begin{bmatrix}
 -\frac{1}{T_{P1}} & 0 & \frac{K_{P1}}{T_{P1}} & \frac{K_{P1}}{T_{P1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{K_{P1}}{T_{P1}} \\
 0 & -\frac{1}{T_{P2}} & 0 & 0 & \frac{K_{P2}}{T_{P2}} & \frac{K_{P2}}{T_{P2}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_{P2}}{T_{P2}} \\
 0 & 0 & -\frac{1}{T_{T1}} & 0 & 0 & 0 & \frac{1}{T_{T1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & -\frac{1}{T_{T2}} & 0 & 0 & 0 & \frac{1}{T_{T1}} & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -\frac{1}{T_{T3}} & 0 & 0 & 0 & \frac{1}{T_{T3}} & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{T4}} & 0 & 0 & 0 & \frac{1}{T_{T4}} & 0 & 0 & 0 \\
 -\frac{1}{2\pi R_1 T_{G1}} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{G1}} & 0 & 0 & 0 & -\frac{K1apf1}{T_{G1}} & 0 & 0 \\
 -\frac{1}{2\pi R_2 T_{G2}} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{G2}} & 0 & 0 & -\frac{K1apf2}{T_{G2}} & 0 & 0 \\
 0 & -\frac{1}{2\pi R_3 T_{G3}} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{G3}} & 0 & 0 & -\frac{K2apf3}{T_{G3}} & 0 \\
 0 & -\frac{1}{2\pi R_4 T_{G4}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{G4}} & 0 & -\frac{K2apf4}{T_{G4}} & 0 \\
 \frac{B1}{2\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
 0 & \frac{B2}{2\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
 \frac{T12}{2\pi} & -\frac{T12}{2\pi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix} \quad (7)$$