Topology based Radial Distribution Network and its Voltage Stability Analysis

M.Mohanty
M-tech Scholar
Centurion University

M.Senapati
M-tech Scholar
Centurion University

N.K.Ray
Assistant Professor
Centurion University

ABSTRACT
The stability index is a function of real & reactive power injected at that node as well as of node voltage.The report presents stability analysis of radial distribution networks. Two voltage stability indexes are used for identifying the node, which is most sensitive to voltage collapse. Therefore to calculate node voltages for varying operating conditions a simple load flow technique for solving radial distribution networks is used. This method involves only the evaluation of a simple algebraic expression of voltage magnitudes and no trigonometric functions as opposed to the standard load flow case. After load flow study, voltage at all nodes is known. The node, at which stability index is minimum, is more sensitive to voltage collapse

Keywords
Radial distribution networks;voltage stability analysis

1. INTRODUCTION

Electrical power is transmitted by high voltage transmission lines from sending end substation to receiving end substation. At the receiving end substation the voltage is stepped down to a low value (say 66KV or 33KV or 11KV). The secondary transmission system transfer power from the receiving end substation to secondary substation. The portion of the power network between a secondary substation and consumers is known as distribution system. The distribution system can be classified into primary and secondary system. Some large consumers are given high voltage supply from the receiving end substations or secondary substation.

The main difference between the transmission system and the distribution system shows up in the network structure. The former tends to be a loop structure and the latter generally, a radial structure.

The modern power distribution network is constantly being faced with an ever-growing load demand. Distribution networks experience distinct change from a low to high load level everyday. In certain industrial areas, it has been observed that under certain critical loading conditions, the distribution system experience voltage collapse.

Load flow analysis of distribution systems has not received much attention unlike load flow analysis of transmission systems. However, some work has been carried out on load flow analysis of a distribution network but the choice of a solution method for a practical system is often difficult. Generally distribution networks are radial and the R/X ratio is very high. Because of this, distribution networks are ill-conditioned and conventionalNewton-Raphson (NR) and fast decoupled load flow (FDLF) methods are inefficient at solving such networks.

Literature survey shows that a lot of work has been done on the voltage stability analysis of transmission systems have studied the voltage stability analysis of radial networks. They have represented the whole network by a single line equivalent. The single line equivalent derived by these authors [3,4] is valid only at the operating point at which it is derived. It can be used for small load changes around this point. However, since the power ¯ow equations are highly nonlinear, even in a simple radial system, the equivalent would be inadequate for assessing the voltage stability limit. Also their techniques [3,4] do not allow for the changing of the loading pattern of the various nodes which would greatly affect the collapse point.

2. ASSUMPTIONS
It is assumed that the three-phase radial distribution networks are balanced and can be represented by their equivalent single line diagrams. This assumption is quite valid for 11 kV rural distribution feeders in India and elsewhere.

2.1 Solution of Methodology
Figure 1 shows a single-line diagram of an existing rural distribution feeder. Branch number, sending-end and receiving-end nodes of this feeder are given in Table 1. Figure 2 shows the electrical equivalent of Figure 1

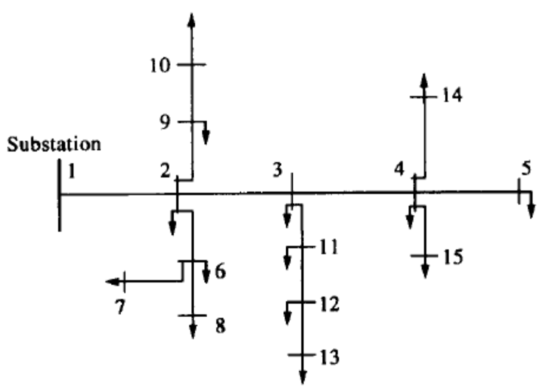


Fig 1.Single line diagram of a radial distribution feeder

Table 1. Branch number, sending end node and receiving end node of Figure 1

Branch number (j)	Sending-end node $IS(j)$	Receiving-end node $IR(j)$
1	1	2
2	2	3
3	3	4
4	4	5
5	2	9
6	9	10
7	2	6
8	6	7
9	6	8
10	3	11
11	11	12
12	12	13
13	4	14
14	4	15

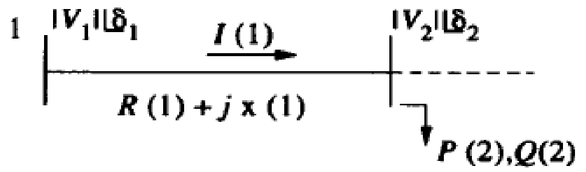


Figure 2. Electrical equivalent of figure 1

Consider branch 1. From figure 2 receiving end node voltage can be written as

$$V(2) = V(1) - I(1)Z(1) \qquad (1)$$

Similarly for branch 2

$$V(3) = V(2) - I(2)Z(2) \qquad (2)$$

As the substation voltage V (1) is known, so if I (l) is known, i.e. current of branch 1, it is easy to calculate V (2) from eqn. 1. Once V (2) is known, it is easy to calculate V (3) from eqn. (2), if the current through branch 2 is known. Similarly, voltages of nodes 4, 5, NB can easily be calculated if all the branch currents are known. Therefore, a generalized equation of receiving-end voltage, sending-end voltage, branch current and branch impedance is

$$V(m2) = V(m1) - I(jj)Z(jj) \qquad (3)$$

Where,

m1: Sending end node

m2: Receiving end node

jj: Branch number

Current through branch 1 is equal to the sum of the load currents of all the nodes beyond branch 1 i.e.

$$I(1)=I_L(1)+I_L(2)+I_L(3)+\ldots+I_L(15) \qquad (4)$$

Similarly the current through branch 3 is equal to the sum of the load currents of all the nodes beyond branch 3 i. e.

$$I(3)=I_L(4)+I_L(5)+I_L(14)+I_L(15) \qquad (5)$$

Therefore, if it is possible to identify the nodes beyond all the branches, it is possible to compute all the branch currents. Identification of the nodes beyond all the branches is realized through an algorithm as explained in Section **V**.

The load current of node i can be computed as

$$(i) = \frac{P_L(i) - jQ_L(i)}{V^*(i)} \qquad (6)$$

Load currents are computed iteratively. Initially, a flat voltage of all the nodes is assumed and load currents are computed using eqns. (6). A detailed load-flow-calculation procedure is described in the later section.

The real and reactive power loss of branch jj are given by

$$P_l(jj) = I(jj)^2 R(jj) \qquad (7)$$

$$Q_l(jj) = I(jj)^2 X(jj) \qquad (8)$$

Before giving the detailed algorithm, we will discuss the methodology of identifying the nodes beyond a particular branch which will help in finding the exact current flowing through each branch.

2.2 Identification of Nodes

BEYOND A PARTICULAR BRANCH

Step 1: For the given branch find the corresponding receiving end node from line data table.

Step 2: Check the sending end node(s) which matches the above receiving end node.

Step 3: Store the sending end nodes obtained in Step 2 in an array **IE** & the corresponding receiving end nodes in an array **IB.**

Step 4: Repeat Step 1, Step 2 & Step 3 for each entry in **IE**

2.3 LOAD FLOW SOLUTION

Once all nodes beyond each branch are identified, it is very easy to calculate the current flowing through each branch as described in Section **IV**. For this purpose, the load current of each node are calculated by using eqns. (6). Once the nodes are identified beyond each branch, the expression of branch current is given as

$$I(jj) = \sum_{i}^{N(jj)} I_L(i) \qquad (9)$$

Initially, a constant voltage of all the nodes is assumed and load currents are computed using eqns. (6). After load currents have been calculated, branch currents are computed using eqn. (9). The voltage of each node is then calculated by using eqn. (3). Real and reactive power loss of each branch is calculated by using eqns. (7) and (8), respectively. Once the new values of the voltages of all the nodes are computed, convergence of the solution is checked. If it does not converge, then the load currents are computed using the most recent values of the voltages and the whole process is repeated. The convergence criterion of this method is that if, in successive iterations the maximum difference in voltage magnitude (DVMAX) is less than 0.000l p.u., the solution has then converged.

3.VOLTAGE STABILITY ANALYSIS

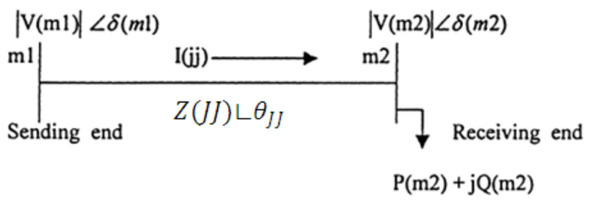


Fig 3. Generalized Electrical equivalent of figure 1

From Fig. 3, the following equations can be written

$$I(jj) = \frac{V(m1) - V(m2)}{r(jj) + jx(jj)} \qquad (10)$$

$$P(m2) - jQ(m2) = V^*(m2)I(jj) \qquad (11)$$

From Eqs. (10) and (11), we ge

$$|V(m2)|^4 - \{|V(m1)|^2 - 2P(m2)r(jj) - 2Q(m2)x(jj)\}|V(m2)|^2 + \{P^2(m2) + Q^2(m2)\}\{r^2(jj) + x^2(jj)\} = 0 \qquad (12)$$

Let,

$$b(jj) = |V(m1)|^2 - 2P(m2)r(jj) - 2Q(m2)x(jj) \qquad (13)$$

$$c(jj) = \{P^2(m2) + Q^2(m2)\}\{r^2 + x^2(jj)\} \qquad (14)$$

From Eqs. (12)- (14) we get,

$$|V(m2)|^4 - b(jj)|V(m2)|^2 + c(jj) = 0 \qquad (15)$$

Therefore, the solution of Eq. (15) is real if

$$|V(m2)| = 0.707[b(jj) + \{b^2(jj) - 4.0c(jj)\}^{1/2}]^{1/2} \qquad (16)$$

From Eq. (16), it is seen that, a feasible load flow solution of radial distribution networks will exist if

$$b^2(jj) - 4.0c(jj) \geq 0 \qquad (17)$$

From Eqs. (13), (14) and (17) we get

$$\{|V(m1)|^2 - 2P(m2)x(jj) - 2Q(m2)r(jj)\}^2 - 4.0\{P^2(m2) + Q^2(m2)\}\{r^2(jj) + x^2(jj)\} \geq 0 \qquad (18)$$

After simplification we get

$$|V(m1)|^4 - 4.0\{P(m2)x(jj) - Q(m2)r(jj)\}^2 - 4.0\{P(m2)r(jj) + Q(m2)x(jj)\}|V(m1)|^2 \geq 0$$

Let's define the voltage stability index as

$$SI(m2) = |V(m1)|^4 - 4.0\{P(m2)x(jj) - Q(m2)r(jj)\}^2 - 4.0\{P(m2)r(jj) + Q(m2)x(jj)\}|V(m1)|^2 \qquad (19)$$

Where,
SI (m2) = Voltage stability index of node m2 (m2=2, 3 … NB)

For stable operation of the radial distribution networks, SI (m2)>=0 for m2=2, 3, ….. NB

After the load flow study, the voltages of all the nodes are known, the branch currents are known, therefore P(m2) and Q(m2) for m2 = 2, 3,4… NB can easily be calculated using Eq. (11) and hence one can easily calculate the voltage stability index of each node (m2 = 2, 3, 4… NB). The node, at which the value of the stability index is minimum, is more sensitive to the voltage collapse.

3.1 voltage stability analysis[12]

From figure 3, real & reactive power at the receiving end can be written as

$$P(m2) = \frac{|V(m2)||V(m1)|}{|Z(jj)|}\cos(\theta_{jj} - \delta_i) - \frac{|V(m2)|^2}{|Z(jj)|}\cos(\theta_{jj}) \qquad (20)$$

$$Q(m2) = \frac{|V(m2)||V(m1)|}{|Z(jj)|}\sin(\theta_{jj} - \delta_i) - \frac{|V(m2)|^2}{|Z(jj)|}\sin(\theta_{jj}) \qquad (21)$$

Rearranging eqn (20) & simplifying for V (m2) we get

$$|V(m2)|^2 = \left[\frac{-P(m2)}{\cos\theta_{jj}} + \frac{|V(m2)|\,|V(m1)|}{Z(jj)}\frac{\cos(\theta_{jj} - \delta_i)}{\cos\theta_j}\right]Z(jj)$$

(22)

In order to obtain real values of V (m2) in terms of P (m2) the equation above must have real roots. Thus the following condition needs to be satisfied

$$\{V(m1)\cos(\theta_{jj}-\delta_{jj})\}^2 - 4R(jj)P(m2) \geq 0$$

Or

$$\frac{4R(jj)P(m2)}{\{|V(m1)|\cos(\theta_{jj}-\delta_i)\}^2} = L_p \leq 1 \qquad (23)$$

Similarly rearranging eqn (21) & simplifying for V (m2) we get

$$|V(m2)|^2 = \left[\frac{-Q(m2)}{\sin\theta_{jj}} + \frac{|V(m2)||V(m1)|}{Z(jj)}\frac{\sin(\theta_{jj}-\delta_i)}{\sin\theta_j}\right]Z(jj)$$

(24)

In order to obtain real values of V (m2) in terms of Q (m2) the equation above must have real roots. Thus the following condition needs to be satisfied

$$\{V(m1)\sin(\theta_{jj}-\delta_{jj})\}^2 - 4X(jj)Q(m2) \geq 0$$

(25)

Or

$$\frac{4X(jj)Q(m2)}{\{|V(m1)|\sin(\theta_{jj}-\delta_i)\}^2} = L_Q \leq 1 \qquad (26)$$

L_P & L_Q are designated as stability indexes. They show how close the operating point is to the limit of instability. For any values of L_P & L_Q greater than one, the system is considered unstable and if it is loaded further voltage collapse occurs.

4. SIMULATION RESULTS

To validate the effectiveness of proposed method load flow solution is carried out on following three rural distribution networks.

Example 1: 15 node radial distribution network

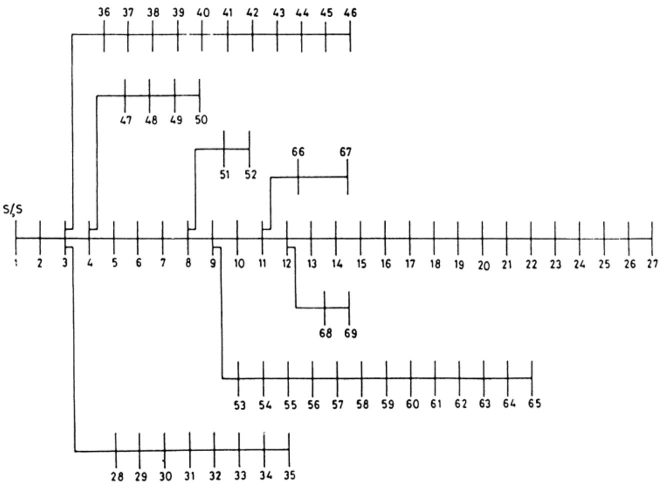
Line data & load data of this network is given in [1].It took three iterations to converge with real & reactive power losses of 61.508 KW & 57.032 KVAR respectively.

Table 2: Load flow solution of Example 1

Node no.	$\|V\|$
1 (substation)	1.00000
2	0.97128
3	0.95667
4	0.95090
5	0.94992
6	0.95823
7	0.95601
8	0.95695
9	0.96797
10	0.96690
11	0.94995
12	0.94583
13	0.94452
14	0.94861
15	0.94844

Example 2:69 node radial distribution network

Line data & load data of this network is given in [2].It took four iterations to converge with real & reactive power losses of 224.65 KW & 102.011 KVAR respectively as shown in fig 4



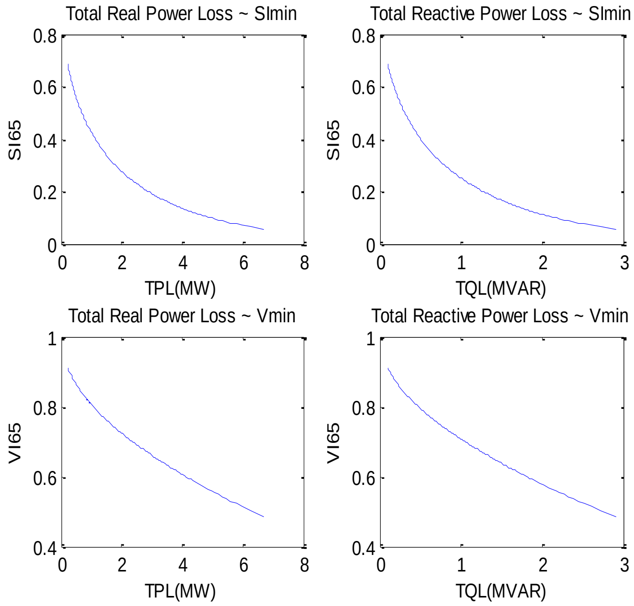


Figure 5: Plots of TPL vs SI_{65}, TQL vs SI_{65}, TPL vs VI_{65n} for Vs=1.0

Table 3: (Critical Loading conditions of 69 node Radial Distribution System for different source Voltages)

Critical Loading conditions without capacitor placement					
Vs (pu)	Alpha	TPL (Mw)	TQL (MVar)	SI	V (pu)
1.0	3.21	6.69	2.90	0.0559	0.4843
1.025	3.37	6.8805	2.9877	0.0647	0.5025
1.05	3.54	7.4639	3.2379	0.0662	0.5053
Critical Loading conditions with 600 KVar capacitor placed at node 61					
1.0	3.6	5.7014	2.5029	0.0739	0.5194
1.025	3.8	6.3049	2.7637	0.0737	0.5190
1.05	4.0	6.9125	3.0260	0.0741	0.5197

5. CONCLUSION

A simple and efficient load flow technique has been used for solving radial distribution networks. It completely exploits the radial feature of the distribution network. This method always guarantees convergence of any type of practical radial distribution network with a realistic R/X ratio. Computationally this method is extremely efficient as it solves simple algebraic expression of voltage magnitude only. Voltage stability analysis has also been carried out considering three different voltage stability indices. Analysis reveals that critical loading limits for all the three cases are the same. Analysis also reveals that in the presence of shunt capacitor critical loading limit increases. Sensitivity analysis has also been carried out to find best location for shunt capacitor placement.

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