Application of an Optimization Algorithm for Channel Assignment in Mobile Communication

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ABSTRACT
The channel assignment problem is a complex problem where a minimum number of channels have to be assigned, under several constraints, to the calls requested in the cellular system. Several approaches have been proposed to solve the dynamic channel assignment (DCA). In this paper, DCA has been modeled as a combinatorial optimization problem. Genetic Algorithm (GA) is a simple tool that can be used to solve such optimization problems in a fast and effective manner. It selects the best option from all the possible solutions, thus making it very different from all the other existing approaches. Several constraints like cochannel and adjacent channel interferences have been considered while solving the channel assignment problem. The performance of the proposed GA-DCA model has been evaluated by a computer simulation tool under the effective of varying cellular capacity.

General Terms
Channel assignment problem, optimization, simulation

Keywords
Cellular communication, Dynamic channel assignment, Genetic algorithm, interference constraints

1. INTRODUCTION
Over the last few years, the number of cellular users and the popularity of mobile computing have grown tremendously. This is mainly due to the different technological advances that support mobile communications such as providing high-quality voice communications and high-speed data services. Voice calls, SMS and internet access are services currently provided to customers. The new generations of wireless access system promises to offer wideband services, such as multimedia As the trend moves towards a larger number of cellular users, the reality is that the available frequency spectrum for allocation to these users becomes limited. Consequently, the effective use of channel frequencies becomes more and more significant and has stipulated critical research in this area. The allocated spectrum is divided into a number of channels, depending on the service requirements. In order to minimize communication interference and to satisfy the large demand of mobile telephone services, channels need to be assigned and reused. In turn, these channels increase the traffic-carrying capacity of the system. This is referred to as the Channel Assignment Problem (CAP). CAP is classified as an NP-complete problem, which means that as the size of the problem increases, the time required to solve the problem does not increase in a polynomial manner, but rather in an exponential one. The channels in a cellular network can be assumed to be a frequency slot in FDMA systems, time slot in TDMA systems or a specific code in CDMA based systems. These channels must be placed some distance apart in order to avoid interference. The cellular structure approach is adopted to increase the frequency spectrum usage. By this approach the wireless service area is divided into a large number of hexagonal cells. Users located in these cells request calls through their portable applications. The system then assigns channels to each request and provides the service.

Genetic algorithm (GA) plays an important role among evolutionary algorithms that exhibit parallelism along with the ability to effectively explore information over all search spaces. During the last three decades, there has been a growing interest in algorithms that rely on analogies to natural processes. The emergence of massively parallel computers and faster computers in general, has made these algorithms of practical interest. GA is one of the best known algorithms in this class. It is based on the principle of evolution, operations such as crossover and mutation, and the concept of fitness. In this paper, the problem of dynamic channel assignment has been portrayed as an optimization problem and solved using genetic algorithm. The next section gives a brief overview on the cellular system. Section 3 explains the concept of channel assignment. Section 4 describes genetic algorithm. Section 5 formulates the GA-DCA model and Section 6 presents the simulation results. Section 7 concludes the paper.

2. CELLULAR SYSTEM
2.1 Overview
In cellular networks, the existing geographical area is partitioned into smaller regions called cells. A base station (BS) is located at the centre of each cell. Each local base station assigns a channel to the mobile host (MH) through which it communicates with the BS. The main objective of channel allocation is to minimize failure rate while assigning channels to a new call in a cell or while assigning channels to an on-going call that moves to a new cell. When the cell fails to assign a channel to new call, the call is blocked. When the cell fails to assign a channel to a call that has been handed over to a new cell, it leads to call dropping. An ideal cellular system minimizes the call blocking and call dropping probabilities of the system, thus maximizing the number of users. Proper power regulation should be done at the base station to attain required signal strength so as to reduce interference at the cell borders. Spatially isolated cells can use the same frequency for communication. The frequency reuse ratio should be determined such that the interference between base stations is brought to a required level. Frequency reuse factor is obtained by frequency planning. For the cells to reuse all the frequencies, the frequency reuse factor should be set to1. If f= 1/3 then a particular frequency should be reused one of every three cells.

2.2 Interference in Cellular Systems

Cellular systems mainly experience three types of interferences. They are:

2.2.1 Cochannel Interference

When channels used at one cell site are repeated in other cell sites, the capacity of the system increases. But if this repeated use is followed in cells that lie close to each other, then they would experience cochannel interference. The cells that use the same frequency are called cochannel cells. Thus, in order to avoid cochannel interference, the cells must be placed atleast three cell units away. Cells may only use the same channels provided that the distance between their centres is equal to or a multiple of this minimum distance (reuse distance). If the above condition is satisfied, then the cells are said to belong to the same reuse scheme. Cells that lie at a distance less than the reuse distance and use different frequencies forms a cluster. If R be the radius of each cell and D be the reuse distance, the cochannel interference factor can be measured as D/R, such that:

$$D/R = (3N)^{1/2} \tag{1}$$

where N is the number of cells in the cluster. Cochannel interference is the prime source of noise in cellular systems and depends on cellular traffic. During busy hours of a cellular system the possibility of cochannel interference appears to be greater.

2.2.2 Co-Site Interference

A single cell may contain many users and would thus require more than one channel. But while assigning channels in the same cell, it should be noted that adjacent channels should not be assigned. In this context, adjacent channels do not refer to exact neighbours in the frequency spectrum. They could also be nearby channels. If adjacent channels are placed in the same cell, it will lead to cosite interference. This forms the second source of noise in cellular radio. Thus the minimum spacing between channels in the same cell must be five, to avoid this interference.

2.2.3 Adjacent Channel Interference

Cells that are placed close to each other also should not use adjacent channels. This will lead to adjacent channel interference. Though this interference is not as severe as the previous two, it has a major role in controlling the performance of the cellular system. Thus all the three types of interferences must be suppressed through proper cellular planning.

3. CHANNEL ASSIGNMENT STRATEGIES

Cellular mobile communication systems are expected to have a high degree of capacity, i.e., they have to serve the maximum number of calls even though the number of channels per cell is limited. Moreover, cells in the same cluster cannot use the same channel because of an increased possibility of cochannel interference that occurs mainly during the busy hours of the system. Hence the process of channel assignment, that determines the channels that are to be used in each cell, is very important for the operation and reliability of cellular systems. There are mainly three channel assignment strategies:

1. *Fixed Channel Assignment (FCA)*: In such systems, the channels are assigned to the cells initially during system design itself. Thus the total number of channels in every cluster will be equal to the total number of channels in the cellular system. Every cell in the system uses the same set of predetermined set of channels. This classic scheme is very simple in design and efficient for uniform traffic distributions. But it is not considered as a good option for real nonuniform type of traffic distributions, as the number of users keep varying. It also leads to poor spectrum utilization.

2. *Dynamic Channel Assignment (DCA)*: It is the opposite of FCA. In this scheme, all the available channels are placed in a common pool. Whenever a call arrives at a particular cell, it requests a channel from the common pool and returns it back to the pool after the call is terminated. In this way the system works efficiently even during nonuniform traffic and promotes effective spectrum utilization. An important property of DCA is channel reassignment, which further raises its quality of service. But all these advantages have to be balanced with the high implementation complexity associated with DCA.

3. *Hybrid Channel Assignment (HCA)*: HCA techniques are designed by combining the FCA and DCA schemes. In HCA, channels are divided into two disjoint sets: one set of channels is assigned to each cell on an FCA basis, while the others are kept in a central pool for dynamic assignment. In this way, the merits of FCA and DCA can be combined in a single scheme. But the complexity of implementation increases further.

4. GENETIC ALGORITHM

GA is a search algorithm that is based on the mechanics of natural selection, genetics and evolution. GAs have been used in a variety of applications. In [5] Sancho Salcedo-Sanz, et. al., uses hybrid genetic algorithms for optimal switch location in mobile communication networks. The optimal positioning of switches in a mobile communication network is an important task, which can save costs and improve the performance of the network. Chandralekha [6] used genetic algorithm to minimize the number of handoffs in heterogenous wireless networks. They work with a large *population* of solutions. Each solution is represented as a *chromosome*. A collection of chromosomes forms a population. Just as in genetics, a chromosome contains several *genes*. A chromosome is usually represented in binary numbers. Each binary bit corresponds to a gene. Chromosomes are also known as individuals or strings. From the population of solutions, GA selects the best possible solution on the basis of a threshold or *fitness function*. This fitness function is unique for every optimization problem. The fitness of each chromosome in the population is measured, and the best chromosome is selected. GA ensures a quicker convergence to the near-optimal solution. Any problem that can be represented as an optimization problem can be solved using GA. The process keeps repeating in an iterative manner till a particular termination criterion has reached.

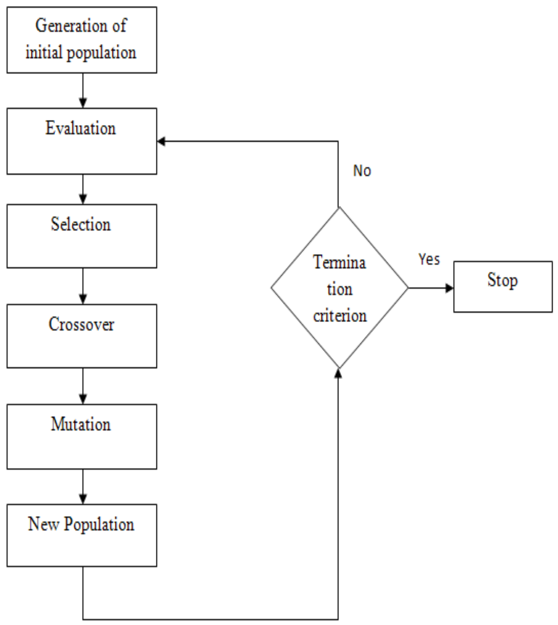

Fig 1: General flowchart of GA

4.1 Initialization

As it can be seen from the figure, an initial population is randomly generated. The population size can be designed by the user. This population contains many chromosomes, and should be present in a binary form. The number of chromosomes in a population forms the population size. This parameter plays an important role in the performance of GA.

4.2 Evaluation

Before starting with the GA, a fitness function has to be formulated first. This fitness function is the most crucial part of the algorithm, and varies depending upon the application GA is used in. The fitness function must be designed such that the best chromosome corresponds to the one with the least fitness value. In the evaluation phase, the fitness functions of all the chromosomes present in the initial population are calculated. The fitness function should be formulated such that the individual chromosome be its variable input parameter. In certain applications, real numbers have to be converted to binary form, to be applied in GA. For simplification at the later stages, all these values can be included in a new matrix:

$$\text{pop} = \begin{pmatrix} \text{binary string 1} & f(x1) \\ \text{binary string 2} & f(x2) \\ : & : \\ \text{binary string popsize} & f(xpopsize) \end{pmatrix}$$

where binary string represents each chromosome and $f(x_i)$ represents the corresponding fitness value. Population size is denoted as *popsize*.

4.3 Selection

Once the fitness values have been calculated, the chromosomes that correspond to the lesser fitness values can be selected. Usually, selection probability is 0.5, i.e., half of the initial population is selected based on their fitness values. For this purpose, the chromosomes can be arranged in their increasing order of fitness values and the first half can be selected.

4.4 Crossover

Once a portion of the population has been selected, the number of chromosomes in the initial population decrements. But the population size must be maintained throughout. For this purpose, new chromosomes have to be generated from the existing ones. This is done with the help of two functions: *crossover* and *mutation*. Crossover is the process where two chromosomes are combined to form two new chromosomes. The strings that are selected from the selected population for this purpose are called parent chromosomes. The offsprings produced are called child chromosomes. Crossover can be either one-point or two-point crossover. Two parents and a random crossover point, *cpoint*, is selected.. A general algorithm for one-point crossover can be shown as:

```
function [child1, child2]=crossover(parent1,parent2,pc);
if (rand<pc)
cpoint=round(rand*(stringlength-2))+1;
child1=[parent1(:,1:cpoint) parent2(:,cpoint+1:stringlength)];
child2=[parent2(:,1:cpoint) parent1(:,cpoint+1:stringlength)];
child1(:,stringlength+1)=fun(child1(:,stringlength+1));
child2(:,stringlength+1)=fun(child2(:,stringlength+1));
else
child1=parent1;
child2=parent2;
end
end
```

At cpoint, the two chromosomes are split. Then the first part of parent1 and second half of parent2 forms the first child chromosome. Similarly, the second part of parent1 and first part of parent2 forms child2. p_c is the probability of crossover. In the case of two-point crossover, two crossover points are selected in each chromosome. The part between the two points is interchanged between the two parents to form two new chromosomes. *Stringlength* represents the length of each chromosome.

4.5 Mutation

Mutation is the process where only one parent is involved to form a new chromosome. Some random genes are selected for mutation or change. Usually the probability of mutation is chosen to be less than the probability of crossover. Let p_m be the probability of mutation. A general algorithm for mutation can be shown as:

```
function [child]=crossover(parent,pm);
if (rand<pm)
mpoint=round(rand*(stringlength-1))+1;
child=parent;
child[mpoint]=abs(parent[mpoint]-1);
child(:,stringlength+1)=fun(child(:,stringlength+1));
else
child=parent;
end
end
```

A mutation point, or *mpoint*, is selected as the point where mutation occurs. At that point, 0 changes to 1 and vice versa. The fitness value has to be calculated for the new individuals. Thus a new population will be formed, by maintaining the population size.

4.6 Termination

These processes take place in an iterative manner. But certain terminating criterions are provided. It could either be the number of iterations, or a particular threshold value that has to be attained, or else the time taken for implementation. When any of these criterion reaches, the iteration automatically stops, and the first chromosome in the current population is selected as the best individual, or as the optimum solution to the problem.

5. DCA USING GA

For performing dynamic channel assignment using genetic algorithm, the DCA problem has to be formulated as an optimization problem first. A cellular topology consists of several hexagonal shaped cells placed in a parallelogram format. Assume a cellular system of 49 cells and 70 channels. For simulation purposes, only the nine central cells are considered. The three interference constraints are mainly considered in this model. For every incoming call in a cell, a channel is selected such that there is no cochannel, cosite and adjacent channel interferences. As mentioned above, cochannel interference does not allow the same channel to be allotted to cells that are at a distance less than the minimum reuse distance. Similarly, the channels assigned to users within the same cell or adjacent cells must satisfy the cosite and adjacent channel constraint. These three conditions are referred to as hard conditions and must not be violated in any way. The general block diagram of the GA-DCA model is shown in fig 3. The fitness function has to be evaluated first before performing the genetic algorithm.

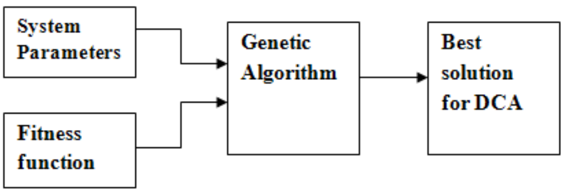

Fig 2: Proposed GA-DCA Model

System parameters like number of cells in the network and total number of channels must be given as input to the genetic algorithm. The fitness function, which determines the quality of each possible solution, must also be designed and given as input to the algorithm. At the output, the best solution corresponding to the channels that must be assigned to each cell is obtained.

Representation: A chromosome represents a cell from the cellular system to which a call is referred, and a binary gene corresponds to a channel. The number of bits in a chromosome is the number of channels that the cell may serve. If the gene is 0, it means the channel is free and if it is 1, it means the channel is occupied.

Evaluation function: The evaluation function that determines the fitness of the chromosomes is the energy function of the model. The fitness function formulated in this paper is:

$$E = C_1 . \sum_k\sum_j V(k,j)*CCC(k,j) + C_2 . \sum_k\sum_j V(k,j)*CSC(k,j) + C_3 . \sum_k\sum_j V(k,j)*ACC(k,j) \quad (3)$$

In this function, E represents the fitness value, which is to be minimized. V is the individual chromosome or string that goes in as input to this function. Suppose the total number of cells in the network is CE, and total number of channels is CH, then *k* represents the cells and varies from 1 to CE. Similarly *j* represents the channels and varies from 1 to CH. Thus V_k corresponds to the chromosome of the k^{th} cell. CCC represents the cochannel constraint matrix. Whenever there is cochannel interference at a particular channel, the corresponding value of CCC(k,j) will be 1, else 0. CSC corresponds to cosite constraint matrix and ACC refers to adjacent channel constraint matrix. The values of CSC(k,j) and ACC(k,j) will be 1 if there is cosite or adjacent channel interference experienced at that particular channel *j* of cell *k*. C_1, C_2 and C_3 are constants that determine the magnitude of each interference while calculating the fitness factor. Usually adjacent channel interference is given less importance compared to the other two interferences and thus C_3 will be less than C_1 and C_2. As the value of interference increases, the corresponding value of E also increases, which directly reflects that it becomes a less preferred option.

6. SIMULATION RESULTS

The simulation model for GA-DCA has been implemented using MATLAB programming language. For simulation simplicity, a cellular system of four cells has been considered. The total number of channels in the system is initially assumed to be seven. Thus in the population matrics, the stringlength becomes number of cells multiplied by number of channels, i.e., 28. The (stringlength +1)st column is used to represent the fitness value of that particular individual or string. The population size can be assumed to be any value initially. Later, simulations were performed with various values to find out the optimized values of population size, number of channels and maximum number of iterations to be performed. With population size 16, number of cells 4 and number of channels 7, the graph shown below was obtained for a random set of input values. The number of channels is represented by 'nch' and number of cells is represented by 'nce'.

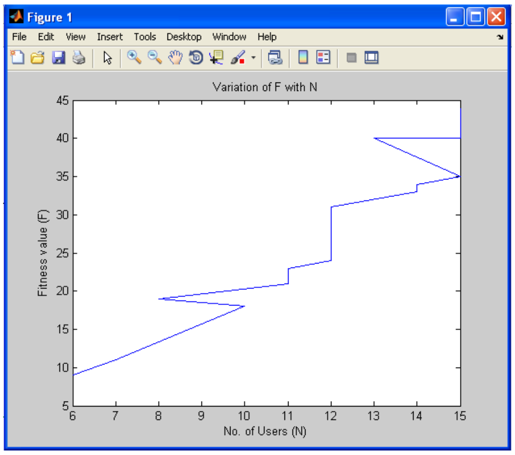

Fig 3: Variation of F with N with popsize=16, nch=7, nce=4

As the number of users in the cellular system increases, the interference in the network also increases. This increase in interference is directly reflected in a rise in the fitness function, as F is a function of interference. The main objective is to minimize the fitness function and maximize the number of users. For this purpose, the various parameters have to be optimized.

6.1 Optimization Of Population Size

Assuming the number of channels and number of cells to be fixed in the network, as 7 and 4 respectively, the population size (popsize) alone was varied to compare the results of the various cases and thus find out the most optimized value.

Table 1: Variation with popsize = 16

Parameters	Fitness Function (F)	No. of Users (N)
Values	9	7
	18	9
	19	11
	10	8
	9	4
	17	11
	14	8
	16	10
	13	8
	16	12
Average	**14.1**	**8.8**

The program was run 10 times to evaluate the results on the basis of an average value. As the input metrics is set to be random numbers, the values keep changing in each case. The number of ones in each individual represents the number of users that were assigned channels in that particular string. The fitness function represents the interference in the system. Hence fitness value should be as low as possible and number of users must be maximized. Thus the N/F ratio determines the goodness of the values obtained. In this case, N/F ratio=0.6241.

Table 2: Variation with popsize = 24

Parameters	Fitness Function (F)	No. of Users (N)
Values	9	8
	3	7
	11	6
	4	5
	12	11
	5	7
	15	9
	5	5
	1	7
	13	9
Average	**7.8**	**7.4**

As it can be seen from the obtained values, the fitness function has reduced when compared to the earlier case. The net N/F ratio was found to be 0.9487

Table 3: Variation with popsize = 36

Parameters	Fitness Function (F)	No. of Users (N)
Values	6	7
	18	10
	14	8
	7	7
	13	9
	10	8
	12	7
	10	8
	11	9
	9	6
Average	**11**	**7.9**

In this case, even though the number of users has slightly increased, the fitness value also increased greatly, thereby degrading the performance of the system. Here N/F = 0.7182.

Table 4: Variation with popsize = 48

Parameters	Fitness Function (F)	No. of Users (N)
Values	5	7
	11	8
	7	7
	11	9
	2	5
	9	9
	7	7
	6	7
	14	8
	9	7
Average	**8.1**	**7.4**

Table 5: Variation with popsize = 64

Parameters	Fitness Function (F)	No. of Users (N)
Values	11	9
	12	9
	12	9
	8	7
	10	9
	13	9
	11	9
	10	5
	12	8
	2	7
Average	**10.1**	**8.1**

Tables 5.4 and 5.5 represent the values obtained when population size was set to 48 and 64 respectively. The N/F ratios were found to be 0.9136 and 0.8020 respectively. The corresponding changes of N/F ratio with population size have been represented in a tabular form as table 5.6.

Table 6: Variation of N/F with population size

Population size	Fitness function (F)	No. of Users (N)	N/F
16	14.1	9	0.6241
24	7.8	7	0.9487
36	11	8	0.7182
48	8.1	7	0.9136
64	10.1	8	0.8020

This variation has been plotted graphically also.

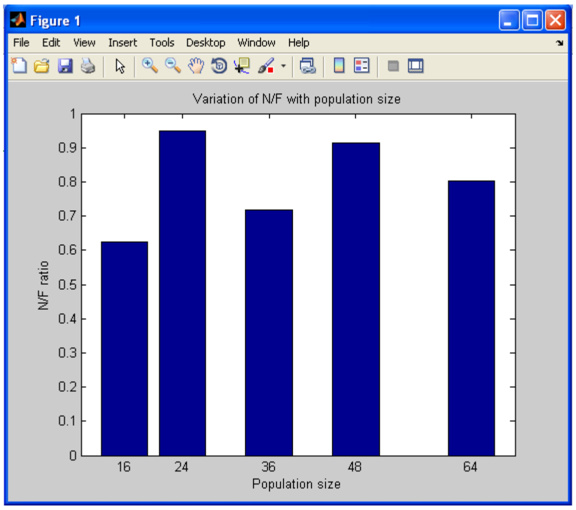

Fig 4: Variation of N/F with population size

6.2 Optimization of Number of Channels

Assuming the population size to be 16, number of cells 4 and maximum iterations 500, the program was simulated for various values of 'nch'. For each value of nch, corresponding values of F and N were obtained.

Table 7: Variation of N/F with number of channels (nch)

Number of channels	Fitness function (F)	Number of users (N)	N/F
7	14.1	9	0.6241
10	26.1	14	0.5211
15	52.8	24	0.4545
20	75.7	31	0.4135
25	105.4	41	0.3928
30	122.9	50	0.4036

The values of N/F with varying values of number of channels have been represented in a tabular form. The performance of the system with the corresponding number of channels is directly reflected in the N/F ratio. Thus higher the ratio, better is the performance. These values have been represented graphically also.

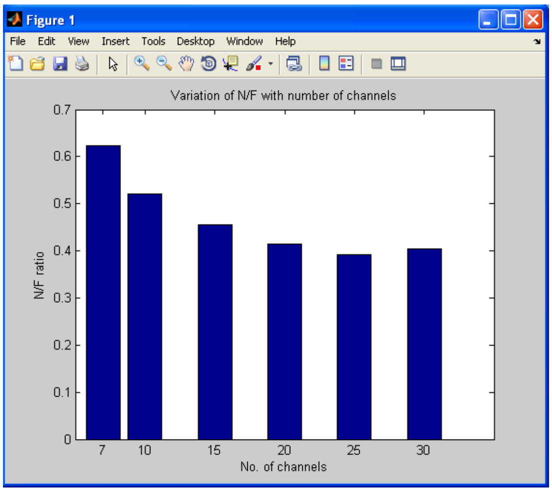

Fig 5: Variation of N/F with number of channels

The graph clearly shows that the best performance was obtained when number of channels was selected to be 7 in a four-cell network. Thus 7 can be selected as the optimum value for nch.

6.3 Optimization of Maximum Number of Iterations

Using the optimized values of population size and number of channels, i.e., 24 and 7 respectively, simulations were performed in a similar manner to find out the optimized value of maximum number of iterations (maxit). The N/F ratio was also calculated at each stage to determine the goodness of the result. The variations of the corresponding N/F values for varying values of maxit have been represented in a tabular form.

Table 8: Variation of N/F with maximum no. of iterations

Max. no. of iterations	Fitness function (F)	Avg no. of users (N)	N/F
50	12.7	8.8	0.6929
100	9.6	8.1	0.8438
200	11.2	8.3	0.7411
300	14.2	9	0.6338
500	11.9	8.7	0.7311

The values of N/F with varying values of number of maximum iterations have been represented in a tabular form. The performance of the system with the corresponding number of iterations is directly reflected in the N/F ratio. Thus higher the ratio, better is the performance. These values have been represented graphically also.

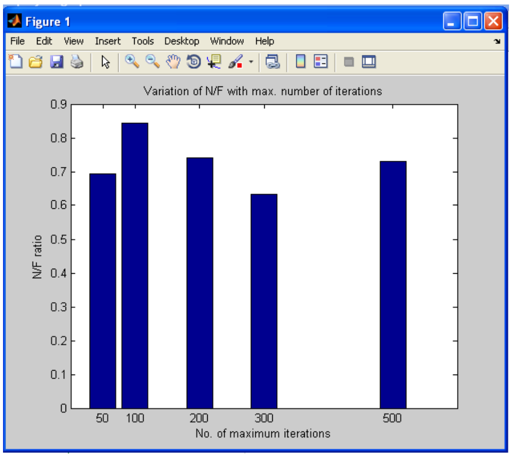

Fig 6: Variation of N/F with max. no. of iterations

The graph and table presented shows that the optimum value for number of iterations is 100 when a four cell cellular network is considered. By utilizing these optimized values during the simulation of genetic algorithm, the best results will be obtained.

7. CONCLUSION

The performance of GA-DCA, to solve the problem of channel assignment in mobile communication, has been implemented in this paper. The ability of genetic algorithms to search for a solution from a wide spectrum of available solutions, rather than restrict the search to a narrow domain led to the development of satisfactory results compared to other channel assignment methods. In addition to this, the property of convergence of GA without any specific problem parameter makes it more advantageous, resulting in a superior performance. The results presented shows that as the number of users, or the capacity of the system, increases, the call blocking and call dropping probabilities are also prone to increase. Thus GA tries to reduce the blocking rates, even under conditions of high capacity. The main parameters considered in genetic algorithm are number of cells, number of channels, population size and maximum number of iterations. Each simulation was conducted ten times to obtain an average value in each case. The ratio of number of users to fitness function was also determined in each case to determine the goodness of each result. Considering a four cell network, the optimized values obtained were 24 for population size, 7 for number of channels and 100 for maximum number of iterations. By making use of genetic algorithm, the results were obtained faster and in a simpler way.

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