Biomedical Image Restoration based on Wavelet Diffusion

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ABSTRACT:
This work deals with noise removal by the use of an edge preserving method called Perona-Malik diffusion or anisotropic diffusion\cite{2,3}. This technique is used for image restoration without removing significant parts of the image content, typically edges or other details. In this scheme anisotropic diffusion is performed on DWT domain\cite{1,7} which is more stationary than noisy image domain. In DWT domain noise decreases with increase in scale and at each scale, noise has less influence on the PDE than that in the image domain. Experimental results have shown that the proposed algorithm can significantly reduce noise while image edges are preserved.

Keywords

1. INTRODUCTION
Image restoration involves the manipulation of the image data to produce a visually high quality image. The main challenge in image de-noising is to preserve the edges and object boundaries, which will improve the visual quality of image as well as the SNR. The first nonlinear diffusion filter has been proposed by perona and malik in 1987\cite{2,3} and is been widely used for intra-region smoothing and inter-region diffusion. This process can reduce noise and preserve edges. In this scheme diffusion coefficient \(g(s)\) depends on image gradient and image gradient is very much sensitive to noise. As anisotropic diffusion depends on \(g(s)\), so it also becomes sensitive to noise.

In this paper wavelet based multi scale anisotropic diffusion is adopted, thus noise can be reduced and edge can be preserved in wavelet domain which is more stationary. When noisy image is decomposed using DWT and with increase in scale the noise reduces. Noise is mostly located in the finest scale. Thus using this algorithm the influence of noise on the PDE model can be reduced. So anisotropic diffusion is performed on the stationary scale-space rather than on the noisy image domain. In this paper diffusion is carried out with detail coefficients and de-noised image was obtained by inverse DWT with diffused detail coefficients and unchanged approximate coefficients.

The organization of the paper is as follows: Section I- includes introduction, Section II- System overview that gives a general idea about the overall functioning of the system. It explains proposed Scheme, Section III includes results and analysis and Section IV concludes the paper.

2. SYSTEM OVERVIEW

A. INTRODUCTION TO ANISOTROPIC DIFFUSION:
Perona and Malik proposed a numerical method\cite{4,5,6,8} for selectively smoothing digital images. In anisotropic diffusion, flow is not only proportional to the gradient, but is also controlled by a function \(g(\nabla u)\), is used. Regions with low \(\nabla u\) are plains. By choosing a high diffusion coefficient the noise can be reduced. Regions with high \(\nabla u\) can be found near edges. In order for those edges to be preserved, a low diffusion coefficient is chosen accordingly. This leads to the function
\[ g:[0,\infty] \rightarrow [0,1] , \quad g(0) = 1, \quad \lim_{s \rightarrow 0^+} g(s) = 0 \]

Which is monotonically decreasing.

In order to obtain a complete system, which is needed to solve the equations, boundary conditions also have to be defined. As the Neumann boundary condition, is common in image processing \(\nabla u = 0 \) on \(\partial \Omega\) is used. This ensures that there is no flow across the boundary and the overall brightness is thereby preserved. It is defined as
\[ \frac{\partial u(x,y)}{\partial t} = \text{div}[g(\|\nabla u\|)\nabla u] \quad (1.1) \]

Where, \(t\) is the time parameter, \(u(x,y,0)\) is the Original image and \(\nabla u(x,y,t)\) is the gradient Version of image at time “t”.

\[ u(0,x) = u_o(x) \text{ on } \Omega \quad (1.2) \]

\[ g(\|\nabla u\|) = \frac{1}{1 + \|\nabla u\|^2/\lambda^2} \quad \lambda \text{ is always greater than 0.} \quad (1.3) \]

In the above equation, \(g\) is smooth non increasing function with \(g(0) = 1\), \(g(x) \geq 0\) and \(g(x)\) tending to zero at infinity. The idea is that the smoothing process obtained by the equation is “conditional” i.e., if \(\nabla u(x)\) is large, then diffusion will be low and therefore the exact localization of the “edges” will be kept. If \(\nabla u(x)\) is small, then the diffusion will tend to smooth still more around \(x\).

Thus the choice of \(g\) corresponds to a sort of thresholding which has to be compared to the thresholding of \(\|\nabla u\|\) used in the final step of classical theory.

Perona and Malik discretized their anisotropic diffusion equation to
\[ u_{t+1}(s) = u_t(s) + \frac{\lambda}{|\Omega|} \sum_{p \in \Omega} g_k(\|\nabla u_{x+p}\|)\nabla u_{x+p} \quad (1.4) \]

S denotes the pixel position in the discrete 2-D grid, \(t\) denotes the iteration step, \(g\) is the conduction function and \(\lambda\) is the gradient threshold parameter determines the rate of diffusion, \(\lambda\) is a scalar quantity which determines the stability, and it is usually less than 0.25. \(\eta\) denotes the spatial neighborhood of pixel \((x,y)\). \(\eta = |N S E W|\) where N, S, E, W are the North, South, East and west neighbors of pixel S \(\eta_{x,y}\) is equal to 4 (except for the image borders. The symbol \(\nabla\) is now represent a scalar...
defined as the difference between neighboring pixels to each direction.

\[
N(x,y+1) = u(x,y) - u(x,y+1)
\]

\[
N(x+1,y) = u(x+1,y) - u(x,y)
\]

\[
N(x,y) = u(x,y+1) - u(x,y)
\]

\[
N(x+1,y) = u(x+1,y) - u(x,y)
\]

Fig(1) 4-nearest neighbor of a point in 2-D

Gradient \( \nabla u \) in 4 direction can be calculated as follows

\[
\nabla u_N(x,y) = u(x,y+1) - u(x,y)
\]

\[
\nabla u_E(x,y) = u(x+1,y) - u(x,y)
\]

\[
\nabla u_S(x,y) = u(x,y+1) - u(x,y)
\]

\[
\nabla u_W(x,y) = u(x+1,y) - u(x,y)
\]

(1.5)

This model has some drawbacks such as if the image is noisy, with white noise, for example, then the noise introduces very large oscillations of the gradient \( \nabla u \). Thus the conditional smoothing introduced by the perona and Malik model will not help, since all these noise edges will be kept.

B. WAVELET BASED ANISOTROPIC DIFFUSION

Wavelet transform is very important for image coding. In the active areas, the image data is more localized in the spatial domain, while in the smooth areas, the image data is more localized in the frequency domain. Thus to localize the image data DWT is taken, The DWT is computed by successive low pass and high pass filtering. DWT is based on sub-band coding and normally used for multi-resolution analysis[9][10]. In DWT, the time is not discrete but the translation and the scale steps are discrete.

For first level decomposition, image \( f(x,y) \) is used as the first scale input, and output is four quarter-size sub-images \( W, WH, WV \) and \( WD \) where \( W \) is the approximate component and \( WH, WV, WD \) are horizontal, vertical and diagonal components respectively.

Using DWT image is decomposed in to different levels. There are different wavelets are available in the tool box like Haar, Daubechies (db), Coiflets (coif), Symlets(sym), Discrete Meyer (dmey), Biorhtogonal (bior), Reverse Biorthogonal (rbio). Using wavelet toolbox decomposition can be carried out for a wide variety of fast wavelet transform. In program the detailed and approximate coefficients are determined taking ‘db’ wavelet and dwt function.

In spatial domain noise is present as high frequency component, and in wavelet domain noise is present in the finest scale. Approximate coefficients is obtained due to Low pass filtering (smoothed version of original image)and if the scale increases noise in the detail sub-bands tends to decrease. Because detail sub-band are generated by decomposing the approximate component. Thus wavelet function represent the piecewise smooth functions. After DWT anisotropic diffusion is performed on the more stationary linear scale-space , because the influence of noise is very less in wavelet domain[1][7]. Thus gradient measurements become more reliable and the anisotropic diffusion is more

C. STEPS FOR WAVELET BASED DIFFUSION

1. Decompose the noisy image using DWT into 4 levels and obtain the components such as approximate and detailed.

2. Perform anisotropic diffusion on the detailed components such as horizontal and vertical components. Because noise is not present in smoothed version (approximate component), that is why diffusion is only applied to the detailed components using equation 1.5.

3. Then IDWT is applied to reconstruct the de-noised image. For performance evaluation PSNR is calculated.

3. EXPERIMENTAL RESULTS AND ANALYSIS:

The proposed method is tested on brain images using MATLAB.

Fig(2) Two Dimensional DWT

Fig(3) Two-scale of two-dimensional decomposition

For first level decomposition, image \( f(x,y) \) is used as the first scale input.
From above graph it is clear that with increase iteration PSNR reduces. Here 50 iterations are taken. As shown in above graph for some iteration the PSNR increases and then falls rapidly for further iteration. As we are getting highest PSNR at t=4, so we should not go for further iterations. That means our stopping criteria is based on what iteration PSNR is maximum.

<table>
<thead>
<tr>
<th>iteration</th>
<th>PSNR</th>
<th>CPU time</th>
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<tbody>
<tr>
<td>2</td>
<td>62.5092</td>
<td>1.082081s</td>
</tr>
<tr>
<td>6</td>
<td>62.8562</td>
<td>1.399917s</td>
</tr>
<tr>
<td>20</td>
<td>62.5243</td>
<td>2.555375s</td>
</tr>
</tbody>
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Table 1 (PM scheme at different iteration)

The proposed algorithm is more computationally expensive than the conventional anisotropic diffusion techniques. Because for the N-level 2-D DWT, the amount of DWT coefficients is 2N times that of the original noisy image. As a result, the anisotropic diffusion at all scales is more time consuming than the conventional anisotropic diffusion techniques.

3. CONCLUSION:

Based on the evaluation criteria i.e PSNR we conclude that proposed method is very efficient one because it can achieve both efficient noise reduction and edge preservation at the same time. In this diffusion techniques PSNR is very high and visual quality is also better. But the only problem is computational complexity, because the images are decomposed in to 4 levels and then PM based diffusion is applied. So if more number of iteration are taken time consumption will be more. With increase in number of iteration the PSNR increases and then falls, thus it is important to know at what iteration the PSNR is maximum. To know this any optimization scheme can also be applied, but this will also consume more time and computational complexity will increase. So real time image processing is not feasible using this scheme. The wavelet based perona malik model can be further extended to form a new model based on fourth order PDEs which will be useful in image smoothing for medical magnetic resonant images as well as on non medical synthetic image both in space and time. In the future, computational complexity can also be reduced.

References


