DS-CDMA Detection for Frequency-Selective Multipath Channels by LMS Algorithm

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ABSTRACT
The Performance comparison of Minimum Mean Square Error-based Adaptive Multi-User Detection (MUD) of Direct Sequence-Code Division Multiple Access (DS-CDMA) signals is considered. DS-CDMA is a popular multiple-access technology for wireless communication. But its performance is limited by multiple-access interference (MAI) and multipath distortion. Adaptive techniques, such as least mean squares (LMS) algorithm is employed for detection of DS-CDMA signals which improves the performance of the CDMA system by reducing interference among users. In this paper the diversity scheme called maximum ratio combining (MRC) as a preprocessor to the adaptive MMSE detector of DS-CDMA system in frequency-selective multipath channel is introduced and compare the complexities of adaptive techniques. The performance of the proposed detector for frequency-selective multipath channel is simulated. Simulations in frequency-selective multipath channels have proved its better performance compared with the conventional Match Filter detector, and it is performing nearly as well as single user case.

Keywords- Direct-sequence code-division multiple access (DS-CDMA), Multipath channel, Minimum mean square error (MMSE), LMS (Least Mean Square) algorithms.

1. INTRODUCTION
The spread spectrum modulation was originally developed for military application where resistance to jamming or interference is a major concern. However there are civilian applications that also benefit from the unique characteristics of the spread spectrum modulation. It can be used to provide multipath rejection in a ground based mobile radio environment. Another application is in multiple access communication in which a number of independent users are required to share a common channel without any external synchronization mechanism [10].

In a limited network resource, techniques are needed to share the available channel among different users as efficiently and fairly as possible. One of these multiple access schemes is called code division multiple access Code-division multiple access (CDMA). CDMA is one of several methods of multiplexing wireless users. In CDMA, users are multiplexed by distinct codes rather than by orthogonal frequency bands, as in frequency-division multiple access (FDMA), or by orthogonal time slots, as in time-division multiple access (TDMA). In CDMA, all users can transmit at the same time. Also, each is allocated the entire available frequency spectrum for transmission; hence, CDMA is also known as spread-spectrum multiple access (SSMA), or simply spread-spectrum communications [2].

Modern communication research revolves around the efficient use of finite resources of transmission power and channel bandwidth. Direct-sequence code-division multiple access (DS

CDMA) has emerged as the preferred technique for increasing the channel capacity [11] through multiple-access communications. This is mainly because in DS-CDMA, the whole frequency band is used all the time and bandwidth can be utilized more efficiently.

Direct-Sequence Code Division Multiple Access (DS-CDMA) is perhaps one of the most widely known and utilized spread spectrum systems and it is relatively simple to implement. A narrow band carrier is modulated by a code sequence. The carrier phase of the transmitted signal is abruptly changed in accordance with this code sequence. The code sequence is generated by a pseudorandom generator that has a fixed length. After a given number of bits the code repeats itself exactly. The speed of the code sequence is called the chipping rate, measured in chips per second (cps). For direct sequence, the amount of spreading is dependent upon the ratio of chips per bit of information. At the receiver, the information is recovered by multiplying the signal with a locally generated replica of the code sequence. In DS-CDMA, a unique code which is also called signature sequence or signature waveform is assigned to each user. However there are two key limits to present DS-CDMA systems:[3]

• All users interfere with all other users and the interferences add to cause performance degradation.

• Due to multiple reflections (multipath signals), the received signal contains delayed, distorted replicas of the original transmitted signal.

That is, its performance is limited by multiple-access interference (MAI) and multipath channel distortion. MAI refers to the interference between direct-sequence users. This interference is the result of the random time offsets between signals, which make it impossible to design the code waveforms to be completely orthogonal. While the MAI caused by any one user is generally small, as the number of interferers or their power increases, MAI becomes substantial.

The conventional detector does not take into account the existence of MAI. The conventional DS-CDMA matched filter detector fails to combat these problems. Many advanced signal processing techniques have been proposed to enhance the performance of DS-CDMA systems, and these techniques fall into two broad categories: multi-user detection [1–4] and space–time processing or diversity combining [1], [6]–[8].

In order to reject or reduce the MAI, multi-user detectors (MUDs) have been proposed where the information of the other users is considered when detecting any given user [1]–[3]. These MUDs can be divided into two broad categories—linear and nonlinear. Linear detectors are computationally very simple and their run-time complexity increases only linearly with the number of users. However, they are adequate only when the underlying noise-free signal classes are linearly separable. If the classes are linearly inseparable, there is a
residual error floor even in the noise-free case, and the performance in the presence of noise is simply unacceptable.

One of the approaches to reduce the MAI is adaptive multi-user detection [4] which employ adaptive technique called least mean square (LMS) [5].

Maximum Ratio combining (MRC) is optimum way (in the sense of the least Bit Error Rate (BER)) to use information from different paths to achieve decoding in multipath (dispersive) channel. It corrects the phase rotation caused by a multipath channel and then combines the received signals of different paths proportionally to the strength of each path. Since each path undergoes different attenuations, combining them with different weights yield an optimum solution under multipath channel. [1].

Recently, adaptive implementations of the mean-squared error (MMSE) receivers have received some attention in the literature, e.g. [4], [5]. The focus of this paper is on improving the performance of adaptive MMSE receivers by introducing maximum ratio combiner (MRC) as a preprocessor in dispersive channels. The proposed Adaptive MMSE-MRC algorithm is applied to the downlink (base station to mobile) of DS-CDMA system.

This paper continues by introducing the system model in section II. Linear multi-user detectors and adaptive MMSE detector are described in section III & IV respectively. A Maximal-ratio combining (MRC) scheme is described in section V and proposed adaptive MMSE-MRC detector is investigated in Section VI. Simulation results are reported in Section VII, and main conclusions are summarized in Section VIII.

2. SYSTEM MODEL

In this section we take a detailed look at the conventional detector and the effect of multiple access interference. Due to the complex nature of CDMA systems, there have been many different formulations for the CDMA model. The model presented here is designed to have a form which is convenient for linear detection formulation [12]. We consider a synchronous antipodal DS-CDMA system of K users over a common additive white Gaussian noise (AWGN) channel as depicted in Fig. 1, where each user employs binary phase-shift keying (BPSK) direct-sequence spread-spectrum (DSSS) modulation. The DS-CDMA system modeled in this paper [14] is the downlink (from base station to mobile). It is assumed that each user’s data is spread using an individual quasi orthogonal signature waveform (spreading code). Assuming a total of K active users in frequency-selective M multipath environment and I transmitted symbols during the observation interval, the received signal can be written as,

\[ r(t) = \sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{m=0}^{M-1} A_k b_k^{(n)} h_{k,m}(t) s_k(t) + n(t) \]  

where \( A_k = \sqrt{E_k / T} \), \( E_k \) is the energy per symbol. \( b_k^{(n)} \) is the \( n^{th} \) symbol with G chips transmitted in a symbol time \( T_b \).

In a pass band notation, the transmitted bit \( b_k(t) \) for the \( k^{th} \) user can be written as,

\[ b_k(t) = \sum_{n=1}^{N} b_k(n) p_{T_b}(t-KT_b) \]  

where \( T_b \) is the bit duration and \( p_{T_b}(\cdot) \) is a rectangular pulse shaping waveform given by

\[ p_{T_b}(t) = \begin{cases} 1, & 0 < t < T_b \\ 0, & \text{otherwise} \end{cases} \]  

Similarly, the spreading sequence for the \( k^{th} \) user can be represented as,

\[ s_k(t) = \sum_{g=1}^{G} s_k(g) p_{T_s}(t-KT_s) \]  

where \( T_s \) is the chip duration and \( p_{T_s}(\cdot) \) is a pulse-shaping signal similar to \( p_{T_b}(\cdot) \). Since the length of spreading sequence is \( G \), we have \( T_s = GT_c \).

The channel impulse response of the frequency-selective channel is represented by a finite impulse response (FIR) filter (represented as \( h_{k,m}(t) \) in (1), where \( m \) is number of paths & it is varying from 1 to \( M \)). Following [7] the three strongest signals \( (h_1, h_2, h_3) \) are used to estimate the signal under consideration,

\[ H(z) = h_1 + h_2 z^{-1} + h_3 z^{-2} \]  

The channel response is stored in a matrix \( H \). The noise \( n(t) \) is a complex AWGN with zero mean and two-sided power spectral density of \( N_0/2 \) W/Hz.

Equation (1) can be compactly expressed as,

\[ r = SHAB + n \]  

The structures can be defined as follows:

\[ S = \text{diag} [S] \]
In equation (14), R is the normalized cross-correlation matrix of the spreading sequences and N is noise component of MF output. If there was no multipath and mutually orthogonal spreading sequences were chosen, the conventional (MF) detector would result in optimal demodulation. However, multipath destroys any orthogonality present in the spreading sequences and the demodulated results are unacceptably erroneous. Hence this b_MF is then used as an input to multi-user detectors to improve the bit-error performance [2], [5] and [7].

3. LINEAR MULTIUSER DETECTORS

There has been great interest in improving DS-CDMA detection through the use of multi-user detectors. In multi-user detection, code and timing (and possibly amplitude and phase) information of multiple users are jointly used to better detect each individual user. The important assumption is that the codes of multiple users are known to the receiver a priori [2].

An important group of multi-user detectors are linear multi-user detectors. In any detector under this group, a linear transformation is applied to the single user matched filter (SUMF) to reduce the MAI seen by each user [2]. In this section we briefly review the two most popular of these, the decorrelating and minimum mean-squared error detectors.

A. Decorrelating Detector:

It achieves perfect demodulation in the absence of noise if the spreading sequences in S are linearly independent. It is implemented by premultiplying the matched filter outputs with the inverse of the cross-correlation matrix before multiplying the sign, i.e.,

$$\tilde{b}_{k(MF)} = \text{sign} \left( R^{-1} b_{MF} \right)$$

The Decorrelating detector is shown to have many attractive properties. Foremost among these properties are [2]:

- Provides substantial performance capacity gains over the conventional detector under most conditions.
- Does not need to estimate the received amplitudes: In contrast, detectors that require amplitude estimation are often quite sensitive to estimation error. (Note that as in the case of most multi-user detectors, the need to estimate the received phases can also be avoided through the use of non coherent detection.
- Has computational complexity significantly lower than that of the maximum likelihood sequence detector: The per-bit complexity is linear in the number of users, excluding the costs of precomputation of the inverse mapping.

Even though the decorrelator achieves perfect demodulation in the noise-free case, it increases the effect of noise whenever noise is present. A more significant disadvantage of the decorrelating detector is that the computations needed to invert the matrix R are difficult to perform in real time. For synchronous systems, the problem is somewhat simplified: we can decorrelate one bit at a time. In other words, we can apply the inverse of a K x K correlation matrix. For asynchronous systems, however, R is of order NK, which is quite large for a typical message length, N.

B. Minimum Mean Square Error (MMSE) Detector:

There are many ways one can view the performance of a linear detector. Perhaps the most widely used is the achieved Minimum Mean Squared Error (MMSE). A tradeoff can be
made between the increase in performance due to decorrelation and the loss on performance due to increased noise level. The linear MMSE detector achieves this optimum tradeoff by making decisions as [5], [7],

\[
\hat{d}(\text{LMMSE}) = \text{sign} \left( R + (N/2)A^{-2} \right)^{-1} b_{\text{MF}} \]  

The symbols for section A and B are defined as follows:

- A: K x K Users' amplitudes diagonal matrix
- B: K x 1 Users' information bits
- S: 1 x K Signature waveforms vector
- n: AWGN
- b_{\text{MF}}: K x 1 Output of SUMF vector
- R: K x K Cross-matrix
- N: K x 1 Noise component of SUMF outputs

An important disadvantage of this detector is that, unlike the decorrelating detector, it requires estimation of the received amplitudes. Another disadvantage is that its performance depends on the powers of the interfering users. Therefore, there is some loss of resistance to the near-far problem as compared to the decorrelating detector. Like the decorrelating detector, the MMSE detector faces the task of implementing matrix inversion. Thus, most of the suboptimal techniques for implementing the decorrelating detector are applicable to this detector as well.

4. ADAPTIVE LINEAR DETECTOR (LMS ALGORITHM)

Many adaptive DS-CDMA detectors are based on linear receivers, especially on MMSE receivers. In this case the goal is to minimize the MSE in the output of the linear filter. Its implementation can be done by a simple tapped delay line filter with an appropriate adaptive algorithm. Before the LMS (least mean square) algorithm is described, a brief review of the gradient descent optimization algorithm is presented [15]. The gradient descent algorithm is used for the optimization of convex penalty functions. Consider the optimization of the following convex penalty function:

\[
\xi = E \left\{ g \left( X, w \right) \right\} 
\]  

Where, E is the expectation operator, X is a random variable and u is the parameter to be optimized (X and w could be vectors). If \( \xi \) is convex, then according to the gradient descent algorithm, it is possible to converge to the minimum value of \( \xi \) by starting at any point \( w_0 \) and following the direction opposite to the gradient \( \nabla \xi \) (steepest descent). The update rule is then given by

\[
w_{j+1} = w_j - \mu \mathbf{\nabla} \xi (w_j)
\]  

However, if the distribution of X is not known, then neither the penalty function given by (19), nor its gradient can be computed. But if a number of independent observations of X are available then it would be possible to get an estimate of the distribution of X and calculate the gradient of the penalty function and use the update rule given in (19). Therefore, at each iteration we could replace the gradient of the penalty function \( \nabla \xi = E \left\{ \nabla g \left( X, w \right) \right\} \) by its approximate value \( \nabla g \left( X_{j+1}, w \right) \). This is called the stochastic gradient descent algorithm. The update rule for the stochastic gradient is thus modified as, \( w_{j+1} = w_j - \mu \mathbf{\nabla} \xi (X_{j+1}, w) \) (20)

If the step size is infinitesimally small, then the deviations on either side of the mean tend to cancel out and the trajectory of the stochastic descent will almost follow the steepest descent trajectory. For the special case of quadratic cost functions, the stochastic descent algorithm is also known as the least mean square (LMS) algorithm. For the case of MMSE multiuser detection in CDMA systems, the convex penalty function is given by

\[
\xi = \left\{ \left\| p_i - w^\top y \right\|^2 \right\}
\]  

\[
\mathbf{\nabla} \xi = \left\{ (b_i - w^\top y) y \right\}
\]  

Therefore the weight update is given by,

\[
w_{j+1} = w_j + \mu \left( b_j (j+1) - w^\top (j) y (j+1) \right) y (j+1)
\]  

It is seen that we need to know the data bits b1 in order to implement the LMS algorithm. This requirement is handled by sending a training sequence at the beginning of each transmission. Once the training sequence has been sent, the adaptive algorithm can be allowed to run with the decisions made by the detector instead of the true transmitted data. This mode of operation is called decision directed operation. This might fine-tune the weights if the SNR is high enough. However if the SNR is very low, the decisions of the detector are not reliable enough and may cause the weight to change drastically from the optimal value. In the simulation results presented in this report, the decision directed mode was not used. Once the training bits are sent, the weights were not changed.

The longer the training sequence, the closer are the computed weights to the optimal value. The training bits however are an overhead and the number of training bits needs to be as small as possible in order to maintain system efficiency. Hence there is a tradeoff between efficiency and error performance that needs to be considered when determining the number of training bits that needs to be used in a system.

Apart from the number of training bits another important parameter that affects the performance and convergence speed of the LMS algorithm is the step size. A large step size makes the algorithm converge faster but has a higher ripple around the optimal value. Conversely, a smaller step size takes longer to converge but has smaller residual error. Thus it would be nice to progressively decrease the step size as the LMS algorithm proceeds. A high value of step size should be used initially to cause fast convergence of the algorithm and then in later iterations a smaller step size should be used to minimize the ripple around the optimal value. One method of progressively shrinking the step size is to multiply the fixed step size by \( y \) where i is the iteration number and y is a number just less than 1. For simulation we use \( t = 0.01 \) and \( y = 0.9999 \).

5. MAXIMUM RATIO COMBINING SCHEME

Various Multi-user detection methods have been investigated in many previous studies [2]-[5] in AWGN channel which uses a bank of MF as a preprocessor. But in a multipath fading channel, propagation delay spread merely
provides multiple versions of the transmitted signal at the receiver. If these multipath components are delayed in time by more than chip duration, they appear like uncorrelated noise at a DS-CDMA receiver. Hence CDMA receiver may combine the time-shifted versions of the original signal by providing a separate correlation receiver for each of the multipath signals.

So a better technology is the Maximum Ratio Combining (MRC) which first identifies few strong multipath signals and then combines them after incorporating adjustment for delays. In this scheme, the spreading code and channel coefficients of the user of interest are only utilized for the detection process. A MF-MRC then utilizes few strongest multipath components. The outputs of each correlator are weighted to provide a better estimate of the transmitted signal than is provided by a single component. Correlator 1 is synchronized to the strongest multipath component \( m_1 \). Second multipath component \( m_2 \) arrives \( \tau_1 \) later than first component. The second correlator is synchronized to \( m_2 \). If only a single correlator is used, once the output of this correlator is corrupted by fading, the receiver can not correct the value. In a MF-MRC receiver, if the output from one correlator is corrupted by fading, the others may not be, and the corrupted signal may be discounted through the weighting process. The weighting coefficients \( \alpha_m \), are normalized to the output signal power of the correlator in such a way that the coefficients sum to unity. i.e.

\[
\alpha_m = \frac{Z_{2m}}{\sum_{m=1}^{M} Z_{2m}^2}
\]  

where \( Z_{2m} \) is the output from the \( m \)th correlator [6].

Mathematically the MF-MRC detector can be expressed as:

\[
\hat{b}_{k\text{(MF-MRC)}} = S_k^H \overline{H}^H r
\]  

where \( \overline{H}^H \) is the Hermitian of the estimated channel matrix.

In the presence of noise, the optimal LMMSE detector is considered to be the best linear detector for DS-CDMA reception [2]. The LMMSE detector for a generic single stage spreading system is given by

\[
\hat{b}_{k\text{(LMMSE-MRC)}} = S_k^H \overline{H}^H (R + (No/2)A^{-2})^{-1} r
\]  

where \( R \) is the auto-correlation matrix for the signal chip train received at the mobile set. \( No/2 \) is the average power of the transmitted user signals. A major draw back of LMMSE is the complexity involved in the computation of the auto-correlation matrix.

**6. PROPOSED ADAPTIVE DETECTOR**

From the mathematical expressions given for MF-MRC detector and LMMSE-MRC detector in the previous section one can see that except the inverse operation remaining terms are similar. Hence it is expected to achieve LMMSE performance by adding a preprocessing stage to the MF-MRC receiver. The basic principle of MF-MRC receiver is to provide link improvement through time diversity.

An adaptive least mean squares (LMS) algorithm will be developed for the proposed Adaptive MMSE-MRC detector, which iteratively produces \( (R + (No/2)A^{-2})^{-1} \). The modified cost function results in separate LMMSE detectors for each multipath component. Hence, the adaptive MMSE detector is actually an adaptive MRC receiver, where each receiver branch is adapted independently to suppress MAI. The outputs of adaptive receiver branches are maximal-ratio combined to reduce decision variable.

Let \( y_{k,m}^{(n)} \) be the input sample vector during the \( n \)th symbol to the \( m \)th receiver branch. The received signal vectors are fed to linear filters with impulse response of \( w_{k,m}^{(n)} \). The output of the \( m \)th receiver branch can be written as,

\[
y_{k,m}^{(n)} = w_{k,m}^{(n)} r_{k,m}^{(n)}
\]  

The error functions, \( e_{k,m}^{(n)} \), produced by the difference between the filter outputs and the reference signals, are used to update the filter weights.

\[
e_{k,m}^{(n)} = y_{k,m}^{(n)} - y_{k,m}^{(n)}
\]  

The product of estimated channel coefficients and data symbols is the reference signal in the adaptive MMSE-MRC detector given by,

\[
d_{k,m}^{(n)} = h_{k,m}^{(n)} r_{k,m}^{(n)}
\]  

Either the data decisions or a training sequence can be used as \( h_{k,m}^{(n)} \). The data decisions produced initially by a conventional MF-MRC receiver are often reliable enough for adapting the receiver. It is also possible to use the absolute value of the estimated channel coefficients \( |h_{k,m}^{(n)}| \).

Hence, the proposed adaptive receiver does not necessarily require separate training sequence. The optimal filter coefficients are derived using the MSE criterion, \( \left( E\left| e_{k,m}^{(n)} \right|^2 \right) \), which leads to the optimal filter coefficients [10] \( W_{k,m} \).

It is convenient to decompose the filter vector to fixed and adaptive components:

\[
w_{k,m}^{(n)} = S_k^H + x_{k,m}^{(n)}
\]  

where \( S_k^H = [0, ..., s_k, ..., 0] \) is the fixed spreading sequence of the \( k \)th user and \( x_{k,m}^{(n)} \) is the adaptive component. If standard LMS algorithm is used for adapting the filter, the updates for the adaptive component can be written as,

\[
x_{k,m}^{(n)} = x_{k,m}^{(n-1)} + \mu_{k,m}^{(n)} e_{k,m}^{(n)} s_{k,m}^{(n)}
\]  

where \( \mu_{k,m}^{(n)} \) is the time-variant step-size parameter, which controls the rate of convergence of the algorithm.

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7. SIMULATION RESULTS

Simulation results in fig.(3-6) are presented for synchronous downlink DS-CDMA system in frequency-selective multipath channel.

BPSK modulation was assumed, with the bits ±1 being equiprobable and spreading sequence length G=32 random sequences were used. All the user symbols are transmitted with the same power.

The channel response used for mixed phase frequency-selective multipath channel was H(z)=0.25 + z⁻¹ + 0.5z², assumed to have three multipath (M=3). For each simulation 10³ bits were transmitted.

Bit error rate (BER) was calculated by dividing the number of bits classified incorrectly by the total number of bits transmitted. For ease of comparison, all plots use a logarithmic scale.

The BER simulations with different number of users were carried out to find the performance of the adaptive MMSE-MRC receiver (Fig. 3). As the number of user increases, its performance is poor, but it performs better than conventional MF detector (Fig. 4).

Fig.5 shows error probability variations in terms of user increment in a spreading sequence length G=32 when channel is frequency selective with three path in every link. Totally, the proposed adaptive detector outperforms MF and is near to single user bound.
8. CONCLUSION

In this paper, we have considered the problem of DS-CDMA in frequency-selective multipath environment and have come up with satisfying results over BER performance enhancement. An adaptive minimum mean squared error (MMSE) with MRC maximum ratio combiner was introduced in this paper. The major advantage of the adaptive MMSE-MRC approach is its simplicity. Implementation can be done by a simple tapped delay line filter with an appropriate adaptive algorithm. It has been shown that the performance of the detector for DS-CDMA varies widely depending on certain system parameters. From the presented results it is found that the proposed detector outperforms conventional MF and performing nearly as well as single user case in multipath channels by adding an affordable complexity.

9. REFERENCES


