Signed LMS based Adaptive Ant System

Abhishek Paul
ECE Department
Camellia Institute of Technology
Kolkata, India

Sumitra Mukhopadhyay
Institute of Radiophysics and Electronics
University of Calcutta
Kolkata, India

ABSTRACT— There are various metaheuristic algorithms which are used to solve the Traveling Salesman problem. Ant colony optimization (ACO) is one such algorithm, which is inspired by the foraging behavior of ants. In this paper, we have proposed a modified model, entitled as Signed Adaptive Ant System (SAAS) for pheromone updation of the Ant-System; SAAS exploits the properties of Adaptive Filters. The proposed algorithm is implemented using sign-LMS (Least Mean Square) based algorithm. It imparts no information about the correction factor of the LMS adaptive algorithm but provides the sign value of each function in the correction factor of the LMS algorithm. SAAS modifies its properties in accordance to the requirement of surrounding domain and for the betterment of its performance in dynamic environment. The proposed algorithm is also easier for hardware implementation. The results of an experimental evaluation, conducted to evaluate the usefulness of the new strategy, are well described. Our algorithm shows effective results as compared to other existing approaches.

Keywords- Ant System (AS), Ant Colony Optimization (ACO), Adaptive Filter, Least Mean Square (LMS) Algorithm, Sign Least Mean Square (sign-LMS) Algorithm, Adaptive Ant System (AAS), Sign Adaptive Ant System (SAAS), Traveling Salesman Problem (TSP).

1. INTRODUCTION

Optimization is the study of the maxima or minima of a problem. Nature inspired optimizations play a vital role in today's world. Researchers are dedicated to find the optimal solutions for different systems and problems with different constraints using bio-mimic optimization techniques. Different nature inspired algorithms are contributing a large in the field of optimization. Some of such widely used techniques are Genetic Algorithm (GA) [1], Evolutionary Strategies (ES) [2], Particle Swarm Optimization (PSO) [3], Ant Colony Optimization (ACO) [4] etc.

This paper is based on Ant Colony Optimization (ACO), which was first introduced by Dorigo as Ant System [5]. This technique mimics the food searching behavior of ants. Ant algorithm has been widely exploited to solve NP-hard combinatorial optimization (CO) problems. Traveling Salesman Problem (TSP) is one of the widely used benchmark problem in this domain. In this paper, we have modified the concept and introduced a novel scheme of pheromone adaptation rules, by exploring the properties of adaptive filter, in the domain of ant colony optimization. This novel proposal is based on Sign-LMS algorithm, entitled as Signed Adaptive Ant System (SAAS). Adaptive Ant System (AAS) [6] is the first Adaptive filter theory based ant algorithm using Least Mean Square (LMS) algorithm. Here, we have tried to implement the characteristics of each and individual ant system in terms of an adaptive filter [7] and investigated its performance by changing the correction function of the Least Mean Square (LMS) filter using Sign- Least Mean Square (LMS) algorithm. In this paper, the entire parametric adaptation has been done using Sign Least Mean Square (sign LMS) algorithm.

The reliability and the properties of any optimization technique are widely tested in literature by means of, Traveling Salesman Problem (TSP) [4-6]. TSP is a paradigmatic NP-hard combinatorial optimization problem. In TSP, a salesman travels different cities (every city visited once) in a way that the total travelling distance is minimal. This is an important optimization problem to test the reliability of any proposed optimization algorithm and we have tested our proposed method using same approach. Simulation studies and the comprehensive analysis demonstrate that an efficient naturally inspired model can be achieved by the proposed approach.

The outline of this paper is organized as: Section II represents the Traveling Salesman Problem (TSP). Section III describes the overview of Adaptive Filter and the Sign Least Mean Square (sign LMS) algorithm with which our proposed ant system has been implemented. Basics of Ant System are explained in Section IV. In Section V, we introduce the newly designed ant system, Signed Adaptive Ant System (SAAS), which is based on the pheromone adaptation on the sign LMS algorithm. Simulations and results are elaborately described in Section VI. And finally the paper concludes in Section VII.

2. TRAVELING SALESMAN PROBLEM (TSP)

Traveling Salesman Problem is a well-known optimization problem, which is used to test the effectiveness of the optimization algorithms. It is to find the shortest tour distance of a closed tour graph. This problem is a NP-hard problem and researcher has an immense interest in it.

Traveling Salesman Problem: Given a set of cities representing a graph G = (V, E (dij)), where, V= set of cities, E= set of edges between cities and dij is the Euclidean distance between ith and jth city. It is the problem of finding the tour that visits each city of the graph exactly once and minimize the total distance travelled.

Traveling Salesman Problem discussed in this section is mainly of two types: Symmetric TSP and Asymmetric TSP. Considering a set of two cities i, j, the distance moving from i to j is given by dij and from j to i is dji. In symmetric TSP, the distance remains same, i.e., dij = dji. But for Asymmetric TSP the above said distance is different, dij ≠ dji. This suggests that only one path exists between two cities in Symmetric TSP and for Asymmetric TSP there exists more than one path. This makes Symmetric TSP easier to implement than its counterpart. In this paper, we will refer to Symmetric TSP for its simplicity and when we refer to TSP in this paper that is

Symmetric TSP.

The proposed algorithm of sign-LMS based Ant System is being tested using different benchmark problems; such as oliver30, berlin52 and others. These benchmark problems are available at TSPLIB benchmark library [8].

3. ADAPTIVE FILTER

Filtering is a mean of signal processing in order to manipulate the information contained in the signal. A filter performs three basic information processing tasks. They are filtering, smoothing, and prediction. Adaptive Filter [7] is one such nonlinear filter and is attractive due to its reliability, accuracy, and flexibility.

The adaptive filter is primarily controlled by the cost function (error function), which determines the performance of the filter. Main usefulness of these filters is to reduce the cost function, so the required output approaches to its desired value.

This cost function is given by

$$e(n) = d(n) - d'(n); \qquad (1)$$

where, e(n) = the cost function.
d(n) = desired function .
d´(n) = estimated function.

The designed process and its modification require the minimization of the cost function.

The basic adaptive filter equation is given by

$$w(n+1) = w(n) + \Delta w \,; \qquad (2)$$

where, Δw is the correction factor, and can be modified using different updation algorithm of the adaptive filter.

Different updation algorithms are formulated to minimize the correction factor Δw , to reduce the cost and also to ease the hardware implementation. In this paper, we use a well-known updation algorithm known as sign-LMS algorithm. The pure form of this algorithm is called sign-sign LMS algorithm. This algorithm is derived from LMS algorithm. Here, sign-sign LMS algorithm has been exploited for the proposed implementation.

A. Sign LMS Algorithm

A simple adaptive algorithm and yet effective for the design of the traversal filter, known as the Least Means Square (LMS) algorithm, was devised by Widrow and Hoff in 1959 [7]. The LMS algorithm is a stochastic gradient algorithm in that it iterates each tap weight of the transversal filter in the direction of the instantaneous gradient of the squared error signal with respect to the tap weight of the filter.

The updation rule for LMS is

$$w(n+1) = w(n) + \mu e(n)x(n) \,; \qquad (3)$$

where, the correction factor, $\Delta w = \mu e(n)x(n)$;

μ = step size which controls the gradient information of the updation.

e(n) = cost function.
x(n) = input signal or input function.

Although the LMS filter is very simple in computational terms, its mathematical analysis is profoundly complicated because of its stochastic and nonlinear nature and so it's hardware implementation. So, to simplify the implementation and to have hardware compatibility another LMS algorithm, known as Sign Sign LMS algorithm, commonly known as Sign LMS algorithm is used in this paper. This algorithm reduces the cost in terms of speed and hardware by evaluating its gradient value in terms of its sign value as described in Eq. 4 below. Basically it is a hardware friendly algorithm, which also reduces the nonlinearity with its sign value.

The updation rule of Sign LMS algorithm is given by

$$w(n+1) = w(n) + \mu * \mathrm{sign}\{e(n)\} * \mathrm{sign}\{x(n)\} \,; \qquad (4)$$

where, the correction factor is replaced by its sign value. Here, no information partially or fully is passed. The sign function gives '+1' if the gradient is positive or else '-1'. So, the tap weight is increased or decreased by a constant amount, μ.

4. ANT SYSTEM

Ant Colony Optimization (ACO) [4] was first introduced by Marco Dorigo in his PhD thesis in 1992. ACO imitates the foraging behavior of real ants. We focus our attention to the basic Ant-System (AS) [5], which is the first successful technique to emphasize the swarm nature of the real ants. The properties of Ant System can be modeled from the real ant colonies. Ant Algorithms are useful in solving different problems such as traveling salesman problem (TSP) [4-6], assignment problems [9], scheduling problems [10], vehicle routing problems [11].

The ant system in TSP helps a salesman to find out his global tour length out of n-cities, where m-ants dedicate themselves to find the optimal path. Here, we are considering a 2-D Euclidean TSP, where, d_{ij} is the Euclidean distance between the two cities i and j. $\tau_{ij}(t)$ is the intensity of trail on edge (i, j) at time t. During initialization, ants are placed on different cities with some positive initial value $\tau_{ij}(0)$. The pheromone updation formula [5] depends upon the inverse of the tour length L_k traversed by the k^{th} ant and the updation formula is defined as,

$$\tau_{ij}(t+n) = \rho\tau_{ij}(t) + \Delta\tau_{ij} \qquad (5)$$

Where, $$\Delta\tau_{ij} = \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \qquad (6)$$

and $\Delta\tau_{ij}^{k} = \dfrac{Q}{L_k}$, if the k^{th} ant has moved along the path edge (i, j); (7)

= 0 otherwise. (8)

where, Q is a constant and ρ is the pheromone evaporation rate.

During their tour, an ant k in city i will move to city j with the probability given by

$$P_{ij}^{k} = \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{k \in allowed_k}[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}} \quad \text{if } j \in allowed_k \,; \qquad (9)$$

= 0 otherwise. (10)

Where,

$allowed_k$ are the cities that are not visited,

α , β are the control parameters for τ_{ij} , η_{ij}

and η_{ij} is the visibility factor.

The choice of Ant-System (AS) depends on the pheromone trail. Therefore, Marco Dorigo put forward three models of AS, say, *ant-density, ant-quantity* and *ant-cycle* [5], and the models differ in the way, the pheromone trail is updated. Ant-density and ant-quantity follow local information for the trail updation, where as ant-cycle uses global information which prevents infinite pheromone accumulation. Eq. 7 and Eq. 8 represent the ant-cycle model.

Ant-cycle uses global information and it functions better than the other two models. Therefore, researchers adopt this model for their study on ACO.

5. SIGNED ADAPTIVE ANT SYSTEM (SAAS)

In this section, we describe our propose ant model, entitled as Signed Adaptive Ant System (SAAS). The proposed model is inspired from well-known sign LMS algorithm and is quite a hardware friendly model. Signed Adaptive Ant System (SAAS) is a variation of Ant System (AS) that differs from the original AS in its pheromone update function. In this variation, we have tried to emulate the characteristics of Ant System in terms of an Adaptive filter. This variation introduces the learning rules of the adaptive filter in the domain of optimization. However, the learning algorithm that we have used is the novel and simple adaptive approach known as Sign Least Mean Square (sign LSM) algorithm, as discussed in Section III.

The pheromone updation of the Ant System is nonlinear as shown in Eq.5, and pheromone trail is inversely proportional to length, as given in Eq.6 and Eq.7. Keeping in mind about the weight updation equation for filters, we propose to model the Ant System using filter, so to arrive at a better optimized solution. We have tried to emulate the characteristics of ant system in terms of an adaptive filter. Once it is done, the parameters of the ant system have been adapted using Sign Least Mean Square (LMS) [7] algorithm which is primarily inspired from the learning rules of the adaptive filter and then the gradient value or the correction factor is replaced by their sign values. This modification enables us to get rid of the nonlinearity in a quantized manner. As no absolute information is incorporated in the algorithm, so it takes more time and iteration to converge. However, the ease of implementation is its biggest advantage which overshadows all other disadvantages of this algorithm. In this paper, the entire parametric adaptation has been done using Sign Least Mean Square algorithm.

Comparing Eq.2, Eq.3, Eq.4 and Eq. 5, we derive our proposed SAAS algorithm. Therefore, with proper modification, $\Delta\tau_{ij}$ can be modeled as,

$$\Delta\tau_{ij} = \mu * \mathrm{sign}\{\mathrm{e(n)}\} * \mathrm{sign}\{\mathrm{x(n)}\} \; ; \qquad (11)$$

In ant-system, the pheromone updation rule inversely depends on the tour length of the ants. Let m ants have completed their tours and have their own tour lengths. L_{min} be the minimum tour lengths to the food source. Here, the error function for each ant is modeled as

$$\mathrm{Error(k)} = \frac{1}{L_{min}} - \frac{1}{L_k} ; \qquad (12)$$

Error(k) is same as the cost function e(n) in Eq.11. The input function x(n) has been considered same as distance parameter of the paths covered by ants to reach the food source. Thus, the input function is a $(n \times 1)$ distance matrix, where n is the number of nodes (cities).

Combining all the parameters, stated above, the proposed ant system is depicted as,

$$\Delta\tau_{ij} = \sum_{k=1}^{m} \left[\mu.sign\{distance\_matrix(k)\}.sign\{Error(k)\}\right] \quad (13)$$

This proposed algorithm is helpful in finding the optimal path, for the closed tour of the cities. This takes more iteration as it supplies the unit values multiplied to a constant term, μ.
The optimized equation for the projected model is given by

$$\tau(t+1) = \tau(t) + \mu.\ \mathrm{sign}\{e(n)\}.\mathrm{sign}\ \{x(n)\} \qquad (14)$$

Eq. 14 gives the true picture, how the nonlinearity is reduced with a constant gradient factor $\pm\mu$. So, it can also be concluded as a quantized step value that determines the correction function and moves to a convergent optima gradually.

6. EXPERIMENTAL STUDY

The simulation work has been carried out, based on the Ant-cycle model [5] [6], as discussed in the previous sections. Ant-cycle uses global information and gives good solutions for the Ant-system. This model can be easily adopted in our proposed technique. The factor Q, present in Eq.7, is neglected as its contribution is negligible on the performance of the system. For simplicity, we have adopted number of ants equal to the number of cities (for e.g., number of ants = 22, for 22-city problem).

Experiments were carried out for 6000 iterations for oliver30, Ulysses16 and Ulysses22 problems (which are considered as Euclidean 2D distance problems) and were averaged over 20 subsequent trials. Table1 analyses the performance of the proposed Sign Adaptive Ant System (SAAS). It analyses the best tour obtained in the favorable iteration time, the average tour length over 20 successive trails for our proposed SAAS Ant System.

TABLE I. Analysis of Performance of The Proposed SAAS Ant System using different TSP Problems.

TSP PROBLEMS	**SAAS Ant System**			
	Best tour (GMinL)	***Best tour iteration time (FRIT)***	***Average tour***	***Optimal tour***
OLIVER30 (30- city Problem)	415.3976	4094	417.78	420
ULYSSES16 (16- city problem)	73.9998	4034	74.2078	---
ULYSSES22 (22- city problem)	75.3984	4830	76.3472	75.3

Figure 1 shows the best tour path obtained by our proposed algorithm SAAS for Oliver30 tsp problem. Figure 2 gives the best tour length obtained along with the iterative best costs over the total run and the average node branching for the same problem. The best tour length obtained is 415.3976 (GMinL- Global Minimum Length) and on iteration 4094 (FIRT- Favorable Iteration), with average tour length of 417.78. Previously obtained best tour length for the same problem was 415.3976, but the average tour length found was 421.9832 for Adaptive Ant System (AAS) [6]. Figure 2 shows the best tour obtained in each iteration and the average node branching. We find that the standard deviation does not fall to zero; it assures us that our system actively searches for better results which may differ from the best so far tour found. Figure 3 gives the variation of best iterative tour found at 20 successive run.

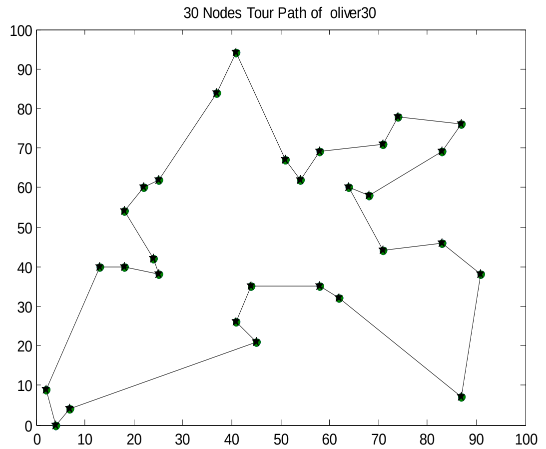

Figure 1. Best Tour Path obtained by SAAS using Oliver30.tsp

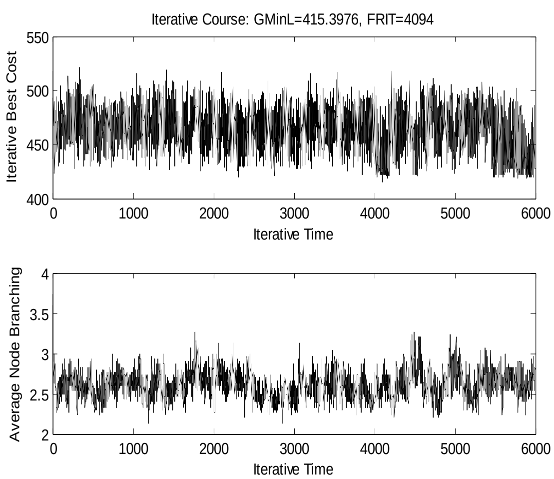

Figure 2. Iterative Best Cost and average node branching of SAAS using Oliver30.tsp problem

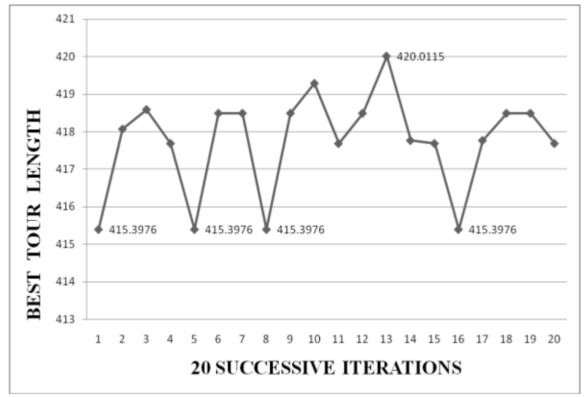

Figure 3. Best Tour of SAAS over 20 successive iteration using Oliver30.tsp problem

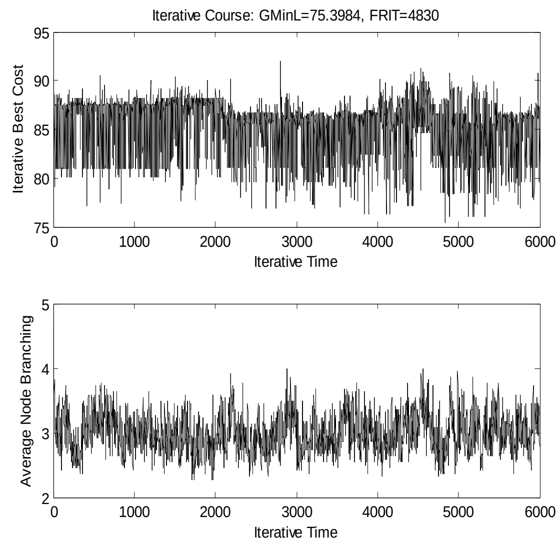

Figure 4. Iterative Best Cost and average node branching of SAAS using Ulysses22.tsp problem

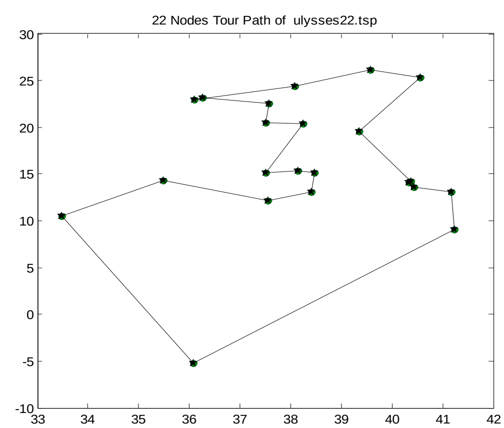

Figure 5. Best Tour Path obtained by SAAS using Ulysses22.tsp

Figure 4, 5, and 8 gives the best tour graph, Average Node Branching (i.e., standard deviation) and iterative best tour, and best tour for 20 successive run for 22 city tsp problem called Ulysses22 tsp problem. Figure 6, 7, and 8 gives the same for Ulysses16 tsp problem. Ulysses problem also provides satisfactory results with the proposed SAAS ant system. Even from Table 1, Ulysses22 problem is very close to its optimal result (SAAS gives 75.3984 whereas optimal is 75.3, difference being 0.0984 ≈ 0.01), which is acceptable and a positive implementation for our problem. From Table 1 it is seen that the time for the best tour length is around 4000 (4094, 4034, and 4830). So, time required to converge to the optimal (minimal) solution is large as we have opted for the sign values of gradient than for their true values. So, time requirement is more, as no information are provided only the sign values of the information are passed and so it takes more time to converge. Introduction of the sign values of gradient make it hardware compatible and cost effective and the non-linearity is abridged.

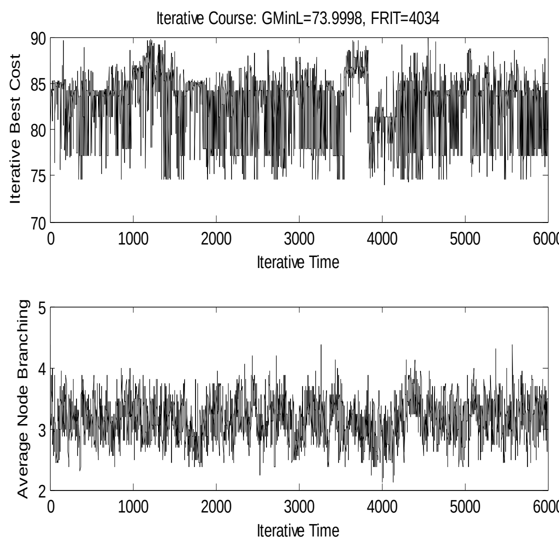

Figure 6. Iterative Best Cost and average node branching of SAAS using Ulysses16.tsp problem

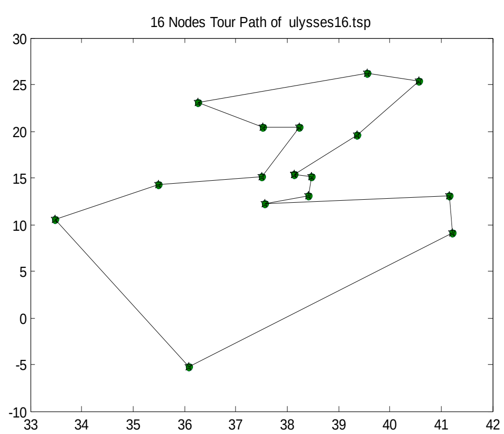

Figure 7. Best Tour Path obtained by SAAS using Ulysses16.tsp

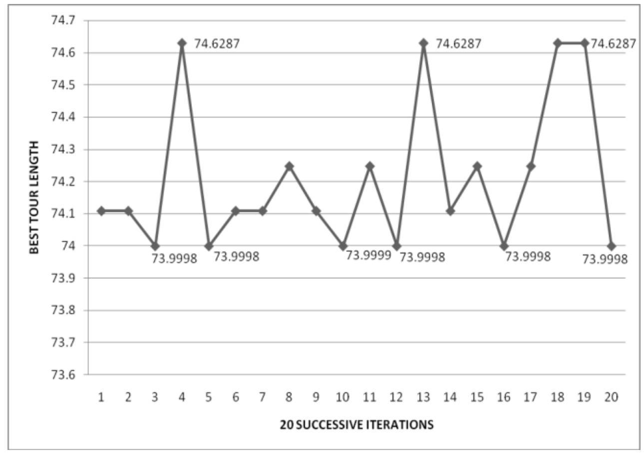

Figure 8. Best Tour of SAAS over 20 successive iteration using Ulysses16.tsp problem

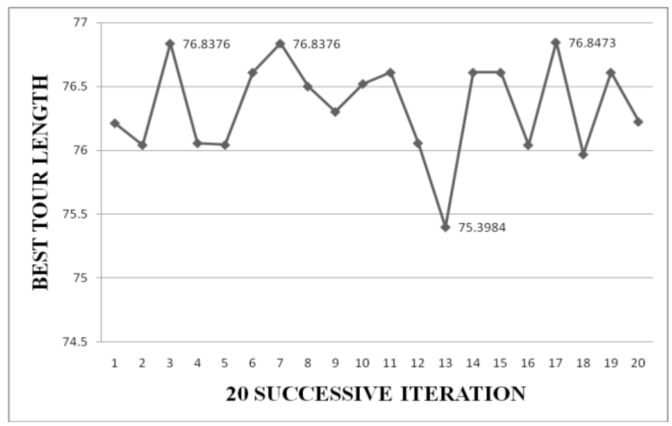

Figure 8. Best Tour of SAAS over 20 successive iteration using Ulysses22.tsp problem

7. CONCLUSION

In this paper, we have introduced an adaptive model of Ant System, based on the pheromone updation using adaptive filter entitled as Signed Adaptive Ant System (SAAS). The main idea was to make the Ant System simpler to design and cost effective in terms of both speed and hardware. This adaptation minimizes the cost function and guides the system towards a satisfactory optimal solution. But, it takes longer time and more iteration to converge as no absolute value is provided. SAAS gives comparatively favorable results as shown in Table 1. Our system can be implemented using hardware, as it relies on the sign information and it is hardware compatible. As the number of city increases more, i.e., in case of large city problems, it takes more iteration to converge. This paper is a step forward of incorporating Adaptive Filter in the domain of optimization.

Future implementation and analysis can be made by incorporating partial gradient information to this Sign-LMS adaptive algorithm. And also we intend to apply the same on ATSP (Asymmetric TSP) problem and other optimized problems and also to design its hardware layout.

8. REFERENCES

[1] D.G. Goldberg, "Genetic Algorithms in Search, Optimization and Machine Learning", Addison- Wesley, Reading, MA, 1989.

[2] H.G. Bayer and H.P. Schwefel, "Evolutionary Strategis: A Comprehensive Introduction", Journal of Natural Computing, 2004, pp. 3-52.

[3] J. Kennedy and R.C. Eberhart, " Particle Swarm Optimization", in Proc. IEEE, Int. Conf. On Neural Networks, Piscataway, NJ, 1995, pp. 1942-1948.

[4] M. Dorigo and L.M. Gambardella, "Ant Colony System: A cooperative learning approach to the travelling salesman problem", in IEEE Trans. Evol. Comput., 1997, pp. 53-66.

[5] K M. Dorigo, V. Maniezzo, and A. Colorni, "The ant system: Optimization by a colony of cooperating agents", in IEEE Trans. Syst., Man Cybern. Part B, 1996, pp. 29–41.

[6] A. Paul and S.Mukhopadhyay, "An Adaptive Pheromone Updation of the Ant System using LMS Technique", in Proc. Int. Conf. on Modelling, Optimization and Computing (ICMOC-2010), AIP, 2010, vol. 1298, pp. 498-503.

[7] S. Haykin, Adaptive Filter Theory, Prentice Hall, 3rd edition, 1996.

[8] [online] [TSPLIB]: http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/tsp/.

[9] V. Maniezzo and A. Colorni, " The Ant System applied to the quadratic assignment problem", IEEE Trans. of Data Knowledge Engrg., 1999, vol.11, no. 5, pp. 769-778.

[10] A. Colorni, M. Dorigo, V. Maniezzo, and M. Trubian, " Ant System for job-shop scheduling", JORBEL-Belgian J. Operations Res., Stat. Comput. Sci., 1994, vol. 34, no. 1, pp.39.

[11] L.M.Gambardella, E.D.Taillard, andG.Agazzi, "MACS-VRPTW: A multiple ant colony system for vehicle routing problems with time windows," in New Ideas in Optimization, D.Corne et.al.,Eds. McGraw Hill, London, UK, 1999, pp.63–76.