PCA based Image Denoising using LPG

Sabita Pal
Asst. Professor
Trident Academy of echnology,
Bhubaneswar, India

Rina Mahakud
Asst. Professor
Trident Academy of echnology,
Bhubaneswar, India

Madhusmita Sahoo
Asst. Professor
Trident Academy of echnology,
Bhubaneswar, India

ABSTRACT

This paper describes an approach of image noising and denoising by the Principal Component Analysis (PCA) method with Local Pixel Grouping (LPG). PCA fully de-correlates the original data set so that the energy of the signal will concentrate on the small subset of PCA transformed dataset. As we know energy of noise evenly spreads over the whole data set, we can easily distinguish signal from noise over PCA domain. It consists of two stages: image estimation by removing the noise and further refinement of the first stage. The noise is significantly reduced in the first stage; the LPG accuracy will be much improved in the second stage so that the final denoising result is visually much better. It also describes an algorithm capable of locating training samples selected from the local window by using block matching based LPG. Experimental results demonstrates that using LPG-PCA method the denoising performance is improved from first stage to second stage with edge preservation.

Keywords
PCA, Denoising, LPG.

1. INTRODUCTION

Information transmitted in the form of digital images is becoming a major method of communication in the modern age. An image is often corrupted by noise in its acquition and transmission. The received image needs processing before it can be used in applications. 'Image Denoising' involves the manipulation of the image data to produce a visually high quality image. Image denoising is used to remove the additive noise while retaining as much as possible the important signal features. In the recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal de-noising [1], [2],[3], [4], because wavelet provides an appropriate basis for separating noisy signal from the image signal. As a primary low-level image processing procedure, noise removal has been extensively studied and many denoising schemes have been proposed, from the earlier smoothing filters and frequency domain denoising methods [5] to the lately developed wavelet [1–10], curvelet [11] and ridgelet [6] based methods, sparse representation [7] and K-SVD [8] methods, shape-adaptive transform [15], bilateral filtering [10,11], non-local mean based methods [12,13] and non-local collaborative filtering [14].

Wavelet transform (WT) decomposes the input signal into multiple scales, which represent different time-frequency components of the original signal. At each scale thresholding [15,16] and statistical modeling [17-19], can be per- formed to suppress noise. The processed wavelet coefficients are transformed back into spatial domain by denoising. To represent the image wavelet transform uses a fixed wavelet basis with dilation and translation. Wavelet transform can cause distortion in the denoising output.

To overcome the drawback in wavelet transform, a spatially adaptive principal component analysis (PCA) is used which computes the locally fitted basis to transform the image and a shape-adaptive discrete cosine transform (DCT) to the neighborhood, which can achieve very sparse representation of the image leading to effective denoising. PCA is a useful statistical technique that has found application in fields such as face recognition and image compression, and is a common technique for finding patterns in data of high dimension.

PCA is a technique that reduces the data into two dimensions. PCA is used abundantly in all forms of analysis - from neuroscience to computer graphics - because it is a simple, non-parametric method of extracting relevant information from confusing data sets. With minimal additional effort PCA provides a roadmap for how to reduce a complex data set to a lower dimension. The other main advantage of PCA is that the data can be compressed by finding the patterns in the data ie. by reducing the number of dimensions, without much loss of information.

In this paper we present an efficient PCA-based denoising method with local pixel grouping (LPG). PCA is a classical de- correlation technique in statistical signal processing used mainly in pattern recognition and dimensionality reduction, etc. [26]. By transforming the original dataset into PCA domain, the noise and trivial information can be removed by preserving only the several most significant principal components. In this paper, a PCA-based scheme is proposed for image denoising by using a moving window to calculate the local statistics, from which the local PCA transformation matrix is estimated. In the denoised output many noise residual and visual distortions appear by applying PCA directly to the noisy image without data selection.

The organization of the paper is as follows: Section 2- System overview that gives a general idea about the overall functioning of the system. Section 3 briefly reviews the procedure of PCA. Section 4 presents the LPG-PCA denoising algorithm in detail. Section 5 presents the experimental results and Section 6 concludes the paper.

2. SYSTEM OVERVIEW

In this LPG-PCA scheme, we model a pixel and its nearest neighbors as a vector variable. The training samples of this variable are selected by grouping the pixels with similar local spatial structures to the underlying one in the local window. With this LPG procedure, the local statistics of the variables can be accurately computed so that the image edge structures can be well preserved after shrinkage in the PCA domain for noise removal.

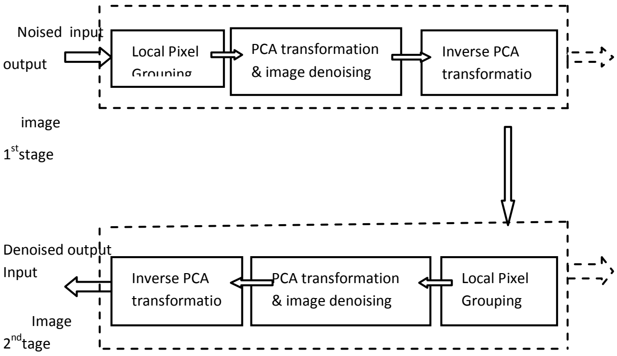

Fig.1 Flowchart of PCA based image denoising using LPG

This proposed LPG-PCA algorithm consists of two stages. The first stage yields an initial estimation of the image by removing most of the noise and the second stage will further refine the first stage output. The procedures of both the stages have the same except for the parameter of noise level. Since the noise is significantly reduced in the first stage, the LPG accuracy will be much improved in the second stage so that the final denoising result is visually much better. The proposed LPG-PCA method is a spatially adaptive image representation so that it can better characterize the image local structures.

3. PRINCIPAL COMPONENT ANALYSIS (PCA)

$Y = [y1\ y2\ \dots ym]T$ an m-component vector variable and denoted by

$$Y = \begin{bmatrix} y11 & y12 & \dots.y1n \\ y21 & y22 & \dots.y2n \\ ym1 & ym2 & \dots.ymn \end{bmatrix}$$

the sample matrix of y, where y_i^j , j=1,2,…..,n, are the discrete samples of variable y_i , i=1,2,….,m. The i^{th} row of sample matrix Y, denoted by

$$Y_i = [y^1{}_i \ \ y^2{}_i \ \ \dots.. \ \ y^n{}_i]$$

is called the sample vector of y_i. The mean value of Y_i is calculated as

$$\mu = \frac{1}{n} \sum_{j=1}^{n} Yi(j)$$

And the sample vector Y_i is centralized matrix of Y is

$$\bar{Y}i = Yi - \mu i = [\bar{y}_i^1\ \bar{y}_i^2 \ \dots..\ \bar{y}_i^n]$$

Where $\bar{y}_i^j = y_i^j - \mu i$. Accordingly, the centralized matrix of Y is

$$\bar{Y} = [\bar{Y}_1^T \ \bar{Y}_2^T \ \dots\dots\dots \ \bar{Y}_m^T]^T$$

Finally the co-variance matrix of the centralized dataset is calculated as

$$\Omega = \frac{1}{N} \overline{YY}^T$$

The goal of PCA is to find an orthonormal transformation matrix P to de-correlate $\bar{Y}$, i.e. $\bar{Z} = P\bar{Y}$ so that the co-variance matrix of the Z is diagonal. Since the covariance matrix Ω is symmetrical, it can be written as

$$\Omega = \Phi\ \Lambda\ \Phi^T$$

where $\Phi = [\Phi_1\ \Phi_2\ \dots\dots.\ \Phi_m]$ is the m×m orthonormal eigenvector matrix and $\Lambda = \mathrm{diag}\{\lambda_1, \lambda_2, \dots. \lambda_m\}$ is the diagonal eigenvalue matrix with $\lambda_1 \geq \lambda_2 \geq \dots. \geq \lambda_m$. The terms $\Phi_1, \Phi_2, \dots, \Phi_m$ and $\lambda_1, \lambda_2, \dots. \lambda_m$ are the eigenvectors and eigenvalues of Ω.
By setting $P = \Phi^T$

$\bar{Y}$ can be decorrelated, i.e. $\bar{Z} = P\bar{Y}$ and $\Lambda = \frac{1}{N}\overline{YY}^T$.

In PCA, the energy of a signal will concentrate on a small subset of the PCA transformed dataset, while the energy of noise will evenly spread over the whole dataset i.e. it fully de-correlates the original dataset $\bar{Y}$, separating signal from noise.

4. LPG-PCA DENOISING ALGORITHM

4.1. Modeling of spatially adaptive PCA denoising

Assuming that the original image I is white additive which is corrupted by the noise v, has a zero mean and standard deviation σ i.e. $I_v = I + v$, where I_v is the observed noisy image. The image I and noise v are assumed to be uncorrelated.
The denoising scheme obtains estimation, denoted by $\hat{I}$, of I from the observation I_v. The denoised image $\hat{I}$ is very close to the original image I. An image pixel is described by two quantities, the spatial location and its intensity, while the image local structure is represented as a set of neighboring pixels at different intensity levels. The edge structures convey its, edge preservation semantic information of an image which is highly desired in image denoising. In this paper we model a pixel and its nearest neighbors as a vector variable and perform noise reduction on the vector instead of the single pixel.
To denoised an underlying pixel, a KxK window is centered on it and is denoted by $y = [y1 \ \dots.ym]^T$, $m = K^2$, the vector containing all the components within the window. The observed image is corrupted by noise.
We denote it by $\mathbf{y_v = y + v}$
The noisy vector of y, where $\mathbf{y_v = [\ y_1^v \dots\dots.\ y_m^v]^T}$, and $\mathbf{y_k^v = y_k + v_k}$,k=1,……., m.

To estimate y from $\mathbf{y_v}$, we view them as (noiseless and noisy) vector variables so that the statistical methods such as PCA can be used. In order to remove the noise from y_v by using PCA, we need a set of training samples of y_v so that the that the covariance matrix of yt and hence the PCA transformation matrix can be calculated.
For this purpose, we use an L x L (L > K) training block centered on y_v to find the training samples, as shown in Fig. 2. The simplest way is to take the pixels in each possible KxK block within the LxL training block as the samples of noisy variable y_v. In this way, there are totally $(L-K+1)^2$ training samples for each component $y^v{}_k$ of y_v. There can also be very

different blocks from the given central KxK block in the LxL training window so that taking all the KxK blocks as the training samples of y_v will lead to inaccurate estimation of the covariance matrix of y_v, which subsequently leads to inaccurate estimation of the PCA transformation matrix and finally results in much noise residual. Therefore, selecting and grouping the training samples that similar to the central KxK block is necessary before applying the PCA transform for denoising.

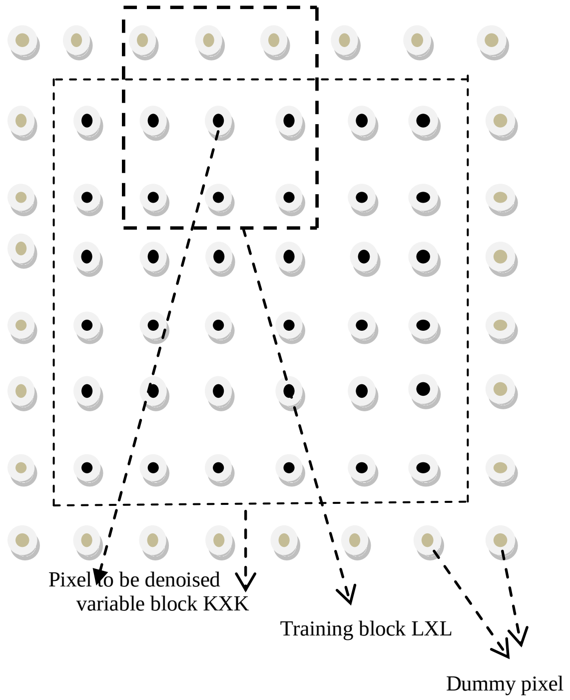

Fig.2 Model of PCA based denoising using LPG

4.2. LPG (Local Pixel Grouping)

For grouping purpose L x L training window is taken. and a K x K variable moving window is taken which moves over the training window . There are many techniques for grouping the local pixel like block matching , K-mean clustering …etc but block matching method is simpler and more suitable .

In **y_v** which is the corrupted image there are $(L-K+1)^2$ possible training blocks within L x L training window.

We denote by $\overline{yo}$, the column sample vector containing the pixels in the central KxK block and denote by $\overline{yi}$, i=1,2,….., $(L\text{-}K+1)^2$-1,the sample vectors corresponding to the other blocks. Let $\overline{yo}$, $\overline{yi}$ be the associated noiseless sample vectors of $\overline{yo}$ and $\overline{yi}$ respectively. It can be easily calculated that

$$e_i = \frac{1}{m}\sum_{k=1}^{m} \overline{yo(k)} - \overline{yi(k)}^2$$

white noise will be uncorrelated with signal, if the following condition is satisfied i.e. $e_i < T + 2\sigma^2$

where T is the preset threshold which is chosen by the programmer, for better result, the T value in first stage is different from the value in second stage of denoising.

σ^2 represent the noise level of the corrupted image. With the increase in σ the image gets more corrupted. The training data set Y_v = [$\overline{y}_0{}^v$ $\overline{y}_1{}^v$ ……… $\overline{y}_{n-1}{}^v$]. Y_v is the corrupted image from which we can calculate Y=[$\overline{y}_0$ $\overline{y}_1$ ……… $\overline{y}_{n-1}$], where Y is the noiseless data set which is estimated by denoising the central pixel within the KxK block. The window moves over the whole LxL training block so that whole image can be denoised.

4.3 LPG-PCA based denoising

The data set Y_v= Y+V , where V is the dataset of noise variable v. In denoising process the dataset Y_v is centralized by taking the mean value of $Y_k{}^v$. So the mean $\mu_k = \frac{1}{n}\sum_{i=1}^{n} Yk(i)$. As the noise is having zero mean $\overline{Y}_k = Y_k - \mu_k$. Now the centralized data set Y_v and Y can be obtained. Thus we have $\overline{Y}_v$= Y+V.

The covariance matrix $\Omega_{\overline{y}}$ of the centralized dataset is calculated. The PCA transformation matrix $P\overline{y}$ is obtained.

$$\Omega_{\overline{y}_v} = \frac{1}{n}(\overline{YY}^T + VV^T) = \Omega_{\overline{y}} + \Omega_v$$

where $\Omega_{\overline{y}} = (\frac{1}{n})\overline{YY}^T$ and $\Omega_v = (\frac{1}{n})VV^T$

The component $\Omega_v(i,j)$ is the correlation between v_i and v_j .Since v_i and v_j are un-correlated for i≠ j , we know that Ω v is a m×m diagonal matrix with all the diagonal components being σ^2. In other words, Ω v can be written as $\sigma^2 I$, where I is the identity matrix. Then it can be readily proved that the PCA transformation matrix $P_{\overline{y}}$ associated with $\Omega\overline{y}$ is the same as the PCA transformation matrix associated with $\Omega_{\overline{y}_v}$.

We can decompose $\Omega_{\overline{y}}$ as $\Omega_{\overline{y}} = \emptyset_{\overline{y}} \Lambda_{\overline{y}} \emptyset_{\overline{y}}{}^T$

where $\emptyset_{\overline{y}}$ is the m×m orthonormal eigen vector matrix and $\Lambda_{\overline{y}}$ is the diagonal eigenvalue matrix. Since $\emptyset_{\overline{y}}$ is an orthonormal matrix, we can write as $\Omega_v = (\sigma^2 I)\emptyset_{\overline{y}}\emptyset_{\overline{y}}{}^T$ = $\emptyset_{\overline{y}}(\sigma^2 I)\emptyset_{\overline{y}}{}^T = \emptyset_{\overline{y}}\Omega_v\emptyset_{\overline{y}}{}^T$

Thus we have $\Omega_{\overline{y}_v} = \Omega_{\overline{y}} + \Omega_v = \emptyset_{\overline{y}}\Lambda_{\overline{y}}\emptyset_{\overline{y}}{}^T + \emptyset_{\overline{y}}(\sigma^2 I)\emptyset_{\overline{y}}{}^T$

$= \emptyset_{\overline{y}}(\Lambda_{\overline{y}} + \sigma^2 I)\,\emptyset_{\overline{y}}{}^T = \emptyset_{\overline{y}}\Lambda_{\overline{y}}\emptyset_{\overline{y}}{}^T$

Where $\Lambda_{\overline{y}} = (\Lambda_{\overline{y}} + \sigma^2 I)$ which indicates that $\Omega_{\overline{y}_v}$ and $\Omega_{\overline{y}}$ have same eigenvector matrix $\emptyset_{\overline{y}}$. Thus in practical implementation we can directly compute $\emptyset_{\overline{y}}$ by decomposing $\Omega_{\overline{y}_v}$, instead of $\Omega_{\overline{y}}$, and then the orthonormal PCA transformation matrix for $\overline{Y}$ is set as $P_{\overline{y}} = \emptyset_{\overline{y}}{}^T$. Applying $P_{\overline{y}}$ to the dataset $\overline{Y}_v$, we have $\overline{Z}_v = P_{\overline{y}}\overline{Y}_v = P_{\overline{y}}\overline{Y} + P_{\overline{y}}V = \overline{Z} + V_z$ where $\overline{Z} = P_{\overline{y}}\overline{Y}$ is the de-correlated dataset for $\overline{Y}$ and $V_z = P_{\overline{y}}V$ is the transformed noise dataset for V. since $\overline{Z}$ and noise V_z are uncorrelated we can easily derive the covariance matrix of $\overline{Z}_v$ is $\Omega_{\overline{y}_v} = \frac{1}{n}\overline{Z}_v\overline{Z}_v{}^T$=$\Omega_{\bar{z}} + \Omega_{vz}$ where $\Omega_{\bar{z}}$=$\Lambda_{\overline{y}}$ is the covariance matrix of noise dataset V_z.

In the PCA transformed domain $\overline{Z}_v$ most energy of noiseless dataset $\overline{Z}$ concentrates on the several most important components, while the energy of noise V_z distributes much more evenly. The noise in $\overline{Z}_v$ can be suppressed by using the linear minimum mean square-error estimation(LMMSE) technique .Since $\overline{Z}_v$ is centralized, the LMMSE of $\bar{Z}_k$ i.e .the kth row of $\overline{Z}$, is obtained as $\bar{Z}_k = W_k\, \bar{Z}^k{}_v$, where W_k is the shrinkage coefficient and defined as $W_k = \Omega_{\bar{z}}(k,k)/ \Omega_{\bar{z}}(k,k) + \Omega_{vz}(k,k)$.

$\bar{Z}^k{}_v$ is the kth row of $\overline{Z}_v$. In flat zones $\Omega_{\bar{z}}(k,k)$ is much smaller than $\Omega_{vz}(k,k)$ so that W_k is close to 0 . Hence most of the noise will be suppressed in $\bar{Z}_k$ by LMMSE operator. This can be implemented by calculating $\Omega_{\bar{z}}$ from the available noisy dataset $\overline{Z}_v$.

$\Omega_{\bar{z}}(k,k)$= $\Omega_{\bar{z}_v}(k,k)$- Ω_{vz}(k,k) . In flat zones, it is often that $\Omega_{\bar{z}}(k,k) - \Omega_{vz}$(k,k) $\leq$ 0, and then we set $\Omega_{\bar{z}_v}(k,k) = 0$. In this case W_k will be exactly 0 and all the noise in $\bar{Z}^k{}_v$ will be removed.

Denote by $\bar{Z}^{\Lambda}$ the matrix of all $\bar{Z}_k$ .. By transforming $\bar{Z}^{\Lambda}$ back to the time domain, we obtain the denoised result of $\overline{Y}_{v.}$ as

$\bar{Y}^{\Lambda} = P_{\bar{Y}}{}^T \bar{Z}^{\Lambda}$.

Adding the mean values μ_k back to$\bar{Y}^{\Lambda}$ gives the denoised dataset Y^{Λ}. The estimation of the central block 0, can then be extracted from Y^{Λ} and finally the denoised result of the underlying central pixel can be extracted. Applying the above procedure to each pixel leads to the full denoised image of Y_v.

4.4. Denoising in the second stage

There are mainly two reasons for the noise residual. First, because of the strong noise in the original dataset Y_v, the covariance matrix is much noise corrupted, which leads to estimation bias of the PCA transformation matrix and deteriorates the denoising performance. second, the strong noise in the original dataset will also lead to LPG errors, which results in estimation bias of the covariance matrix . Therefore, it is necessary to further process the denoising output for a better noise reduction. Since the noise has been much removed in the first round of LPG-PCA denoising, the LPG accuracy and the estimation of covariance matrix can be much improved with the denoised image. Thus we can implement the LPG-PCA denoising procedure for the second round to enhance the denoising results.

5. EXPERIMENTAL RESULTS

In the proposed LPG-PCA denoising algorithm, most of the computational costs depends on LPG grouping and PCA transformation, and thus the complexity mainly depends on two parameters: the size K of the variable block and the size L of training block .In LPG grouping ,it requires($2K^2$-1) - $(\mathrm{L}-\mathrm{K}+1)^2$ additions, K^2 - $(\mathrm{L}-\mathrm{K}+1)^2$ multiplications and$(\mathrm{L}-\mathrm{K}+1)^2$ ''less than'' logic operations.

In the implementation of LPG-PCA denoising, actually the complete K x K block centered on the given pixel will be denoised. Therefore, the finally restored value at a pixel can be set as the average of all the estimates obtained by all windows containing the pixel. In this experiment we have taken some biomedical test images and the denoised output at first and second stage are shown below:

1. Image of lung cancer

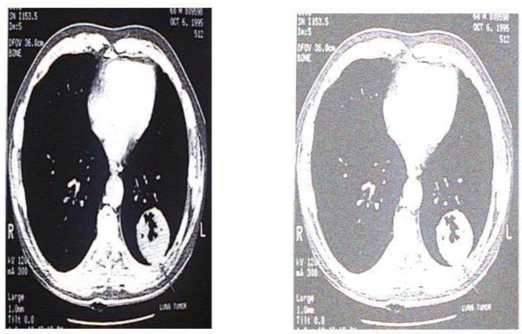

Fig3. (a) original image (b) noisy image

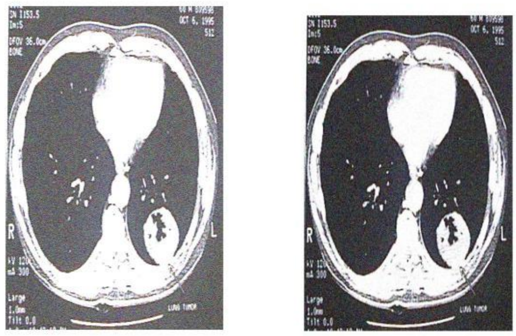

Fig3. (c) 1st stage denoised image, (d) 2nd stage denoised image

2. MRI Image of brain

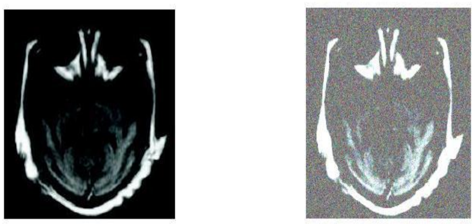

Fig4. (a)original image Fig(b)noisy image

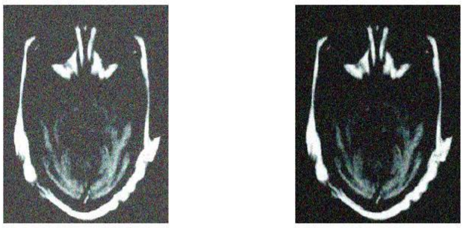

Fig.4 (c) 1st stage denoised image, (d) 2nd stage denoised image

3. MRI image of brain tumor

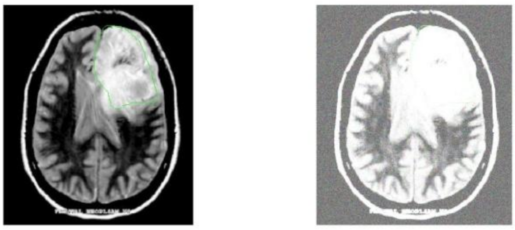

Fig.5 (a)original image (b)noisy image

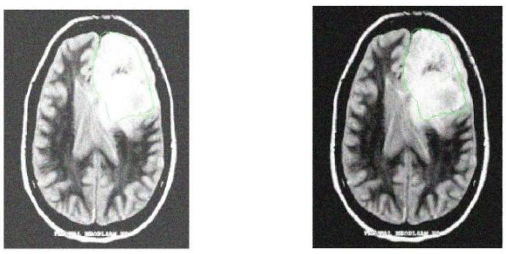

Fig.5 (c) 1st stage denoised image (d) 2nd stage denoised image

6. CONCLUSION

This paper proposed a spatially adaptive image denoising scheme by using principal component analysis (PCA). To preserve the local image structures when denoising, we modeled a pixel and its nearest neighbors as a vector variable, and the denoising of the pixel was converted into the estimation of the variable from its noisy observations. The PCA technique was used for such estimation and the PCA transformation matrix was adaptively trained from the local window of the image. However, in a local window there can have very different structures. The block matching based local pixel grouping (LPG) was used and only the similar sample blocks to the given one are used in the PCA transform matrix estimation. The PCA transformation coefficients were then shrunk to remove noise. The above LPG- PCA denoising procedure was iterated one more time to improve the denoising performance. Our experimental results demonstrated that LPG-PCA can effectively preserve the image fine structures while smoothing noise.

7. REFERENCES

[1] Lei Zhang , Weisheng Dong, David Zhang, Guangming Shib, *Two-stage image denoising by principal component analysis with local pixel grouping* ,Science Direct, Pattern Recognition 43 (2010) pp. 1531–1549.

[2] D.L. Donoho, De-Noising by Soft Thresholding, *IEEE Trans. Info. Theory 43*, pp. 933-936, 1993.

[3] S. Grace Chang, Bin Yu and M. Vattereli, Adaptive Wavelet Thresholding for Image Denoising and Compression, *IEEE Trans. Image Processing*, vol. 9, pp. 1532-1546, Sept. 2000.

[4] M. Lang, H. Guo and J.E. Odegard, Noise reduction Using Undecimated Discrete wavelet transform, *IEEE Signal Processing Letters*, 1995.

[5] R.C. Gonzalez, R.E. Woods, Digital Image Processing, second ed., Prentice- Hall, Englewood Cliffs, NJ, 2002.

[6] G.Y.Chen, B.Ke´gl, Image denoising with complex ridgelets, Pattern Recognition40(2)(2007)578–585.

[7] M. Elad, M. Aharon, Image denoising via sparse and redundant representa- tions over learned dictionaries, IEEE Transaction on Image Processing 15 (12) (2006) 3736–3745.

[8] M. Aharon, M. Elad, A.M. Bruckstein, The K-SVD: an algorithm for designing of overcomplete dictionaries for sparse representation, IEEE Transaction on Signal Processing 54 (11) (2006) 4311–4322.

[9] A. Foi, V. Katkovnik, K. Egiazarian, Pointwise shape-adaptive DCT for high- quality denoising and deblocking of grayscale and color images, IEEE Transaction on Image Processing 16 (5) (2007).

[10] C. Tomasi, R. Manduchi, Bilateral filtering for gray and colour images, in: Proceedings of the 1998 IEEE International Conference on Computer Vision, Bombay, India, 1998, pp. 839–846.

[11] D. Barash, A fundamental relationship between bilateral filtering, adaptive smoothing, and the nonlinear diffusion equation, IEEE Transaction on Pattern Analysis and Machine Intelligence 24 (6) (2002) 844–847.

[12] A. Buades, B. Coll, J.M. Morel, A review of image denoising algorithms, with a new one, Multiscale Modeling Simulation 4 (2) (2005) 490–530.

[13] C. Kervrann, J. Boulanger, Optimal spatial adaptation for patch based image denoising, IEEE Transaction on Image Processing 15 (10) (2006) 2866–2878.

[14] K. Dabov, A. Foi, V. Katkovnik, K. Egiazarian, Image denoising by sparse 3D transform-domain collaborative filtering, IEEE Transaction on Image Proces- sing 16 (8) (2007) 2080–2095

[15] D.L.Donoho, De-noising by softthresholding, IEEE Transaction son Information Theory41(1995)613–627.

[16] R.R.Coifman, D.L.Donoho, Translation –invariant de-noising, in: A.Antoniadis, G. Oppenheim (Eds.),Wavelet and Statistics, Springer, Berlin, Germany,1995.

[17] M.K.Mıhc-ak, I.Kozintsev, K.Ramchandran, P.Moulin, Low-complexity image denoising based on statistical modeling of wavelet coefficients, IEEE Signal Processing Letters6(12)(1999)300–303.

[18] S.G.Chang, B.Yu,M.Vetterli, Spatially adaptive wavelet thresholding with context modeling for image denoising, IEEE Transaction on Image Processing 9 (9)(2000)1522–1531.

[19] L.Zhang, B.Paul, X.Wu, Hybrid inter-and intra wavelet scale image restoration, Pattern Recognition 36 (8) (2003) 1737–1746.

[20] Z.Hou, Adaptive singular value decomposition in wavelet domain for image denoising, Pattern Recognition 36 (8) (2003) 1747–1763.

[21] J.Portilla, V.Strela, M.J.Wainwright, E.P.Simoncelli, Image denoising using scale mixtures of Gaussian sin the wavelet domain, IEEE Transaction on Image Processing 12(11)(2003)1338–1351.

[22] A.Pizurica, W.Philips, Estimating the probability of the presence of a signal of interest in multiresolution single- and multiband image denoising, IEEE Transaction on Image Processing 15 (3) (2006)654–665.

[23] K. Fukunaga, Introduction to Statistical Pattern Recognition, second ed, Academic Press, New York, 1991.

[24] D.D. Muresan, T.W. Parks, Adaptive principal components and image denoising, in: Proceedings of the 2003 International Conference on Image Processing, 14–17 September, vol. 1, 2003, pp. I101–I104.

[25] A.Pizurica, W.Philips, I.Lamachieu, M.Acheroy, Ajointinter-and intra scale statistical model for Bayesian wavelet based image denoising, IEEE Transaction on Image Processing 11(5) (2002) 545–557.

[26] L.Zhang, P.Bao, X.Wu, Multiscale LMMSE- based image denoising with optimal wavelet selection, IEEE Transaction on Circuits and Systems for Video Technology15(4)(2005)469–481.