Image Denoising using Curvelet: an Approach based on Average Fusion

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ABSTRACT
The most significant task of image processing is to reduce noise which is commonly found in images. In recent years, technology is being improved to analyze the images to get better quality. Since the image gets loss of edge feature and detail information during the process of de-noise, this paper attempts to present and compare a new method based on curvelet transform using image fusion. Results show that this approach has a broad future for removing noise as well as preserving edges of image.

Keywords
Curvelet, image fusion, denoise, multiresolution, ridgelet, Gaussian filter.

1. INTRODUCTION
The challenging task of now a days image processing is to get sparsity without losing the detail information in image. Many critical structures can be occurred as spatially or temporally localized deviation in an image. After some transformation done on the image, the critical structures are considered as localized phenomena. Thus, the concept of sparsity is used to construct mathematical model for the detection of such structures in images. The popular transforms includes DFT, wavelet, curvelet, contourlet and shearlet are used extensively [7] in the literature. The aim of these transforms is often to reveal certain structures of a signal and to represent these structures in a compact and sparse representation [15]. Sparse representations have therefore increasingly become recognized, as it provides extremely high performance for applications as diverse as: noise reduction, compression, feature extraction, pattern classification and blind source separation.

Images are captured with sensors normally corrupted by various noises due to internal and external factors. Research scholars have been focusing abundant interest on wavelet transform to de-noise the images. Many techniques had been applied to obtain substantial development in noise removal such as thresholding of orthogonal wavelet coefficient of the noisy data, level dependent thresholding, adaptive choice of thresholding and tree-based wavelet de-noising [2][11]. They are well suited for isotropic information. The main drawback of wavelet transformation is the lack of recovering geometrical objects in images. But while considering anisotropic information, researchers were in position to seek some alternative solution. Curvelet transform was the best choice to alternate, with respect to anisotropic information in signal. Curvelet transform is an emerging multi-resolution image transform trend for providing high performance in many applications.

The aim of this article is to present the best result to improve de-noisy image with the aid of multi-resolution curvelet transform and image fusion. Research is going on in this track and best results had been shown in the literature. An image de-noising method based on curvelet is attempted in [10] to remove wrinkle and radial stripe in certain regions after curvelet transform. A combined wavelet and curvelet de-noising system with pixel based was developed in [12][9]. However information loss is appeared in result images during de-noise process in curvelet transform [10]. Sometimes the texture details cannot be retrieved in high noisy images. Improving the visual quality of result images based on pixel fusion is one of the main contributions of this paper.

2. CURVELET TRANSFORM DENOISE
Curvelet transform is developed by Candes and Donosho in 1999, another invention in the series of MRA (Multi-scale Resolution Analysis). It is a suitable method to handle curve discontinuous, whereas wavelets fails to handle [4][5][10]. Basically curvelet transform extends ridgelet transform to multiple scale analysis. Curvelets obey a parabolic scaling relation which says that each element has an envelope at scale 2^{-j} which is aligned along a "ridge" of length $2^{-j/2}$ and width 2^{-j}.

Curvelet transform implements curvelet subbands using a filter bank of wavelet filters with the aid of ridgelet transform [2]. The combined multi-scale ridgelets with a spatial bandpass filtering operation to isolate different scale curvelets follows the scaling law: width ≈ length^2, which represents a curve with a superposition of various length and width functions. In addition, curvelet occurs at all scales, locations and orientations. Curvelet Transform is grouped into Subband coding, Smooth Partitioning, Renormalization and Ridgelet Analyis stages.

2.1 Subband Decomposition
The image is decomposed into multiple resolution layers with a series of low-pass and band-pass filters. Each layer consists of different frequencies.

$$f \rightarrow (p_o f, \Delta_1 f, \Delta_3 f, ...) \qquad (1)$$

Where P_0 is low pass-filter and Δ_1, Δ_2 are band- pass filters.

Here low-pass Φ_0 and high-pass Ψ_{2s} filters deal with low frequencies near $|\xi| <= 1$ and high frequencies near domain $|\xi| \varepsilon\ [2^{2s}, 2^{2s-2}]$. Energy preservation and recursive construction can be performed by following equations.

$$f = p_o(p_o f) + \Sigma \Delta_s(\Delta_s f) \qquad (2)$$

$$\Psi_{2s}(s) = 2^{4s}\Psi(2^{2s} X) \qquad (3)$$

In curvelet transform, the subband decomposition f is approximated by wavelet transforms. So f is decomposed into S_0, D_1, D_2, and D3. The following equation is simply used to decompose an image into subbands using convolution operator.

$$p_o f = \phi_o \times f \quad \Delta_s f = \Psi_{2s} \times f \tag{4}$$

Where $P_0 f$ is obtained by the wavelet coefficients S_0, D_1, D_2, D_3 and $\Delta_s f$ is obtained by D_{2s} and D_{2s+1}.

2.2 Smooth Partitioning

In curvelet, smoothing without edge (curve) loss is achieved by a concept called dyadic square [5][6]. Dyadic square is defined by

$$Q_{s,k_1,k_2} = \left[\frac{k_1}{2}, \frac{k_1+1}{2}\right] \times \left[\frac{k_1}{2^s}, \frac{k_2+1}{2^s}\right] \in Q_s \tag{5}$$

Q_s represents all dyadic squares of the grid. A window function with the size 2^s X 2^{-2} can be used to smooth each and every squares, W_Q is a displacement W localized near Q.

$$h_Q = W_Q . \Delta_s f \tag{6}$$

In the above equation, $\Delta_s f$ gives smooth dissection of the function of squares by multiplying $\Delta_s f$ and W_Q. Next, the each pixel energy is divided between all sampling windows of the grid by

$$\sum_{K_1,K_2} W^2 (x_1 - k_1, x_2 - k_2) \equiv 1 \tag{7}$$

Once it is divided, it must be reconstructed by

$$\sum_{Q \in Q} W_Q . h_Q = \sum_{Q \in Q_s} W_Q^2 . h = h \tag{8}$$

Parserval relation can be defined by

$$\sum_{Q \in Q} \|h_Q\|_2^2 = \sum_{Q \in Q} W_Q^2 h^2 = \int \sum_{Q \in Q} W_Q^2 . h^2 = \int h^2 = \int \|h\|_2^2 \tag{9}$$

2.3 Renormalization

Each dyadic square is renormalized into the unit square [0, 1] x [0,1] by the following equation

$$g_Q = T_Q^{-1} h_Q \tag{10}$$

Where T_Q is normalization operator for Q is defined by

$$(T_Q f)(x_1, x_2) = 2^s f(2^s x_1 - k_1, 2^s x_2 - k_2) \tag{11}$$

2.4 Ridgelet Analysis

Ridgelet analysis is final step and it is a constant along with the lines

$$x_1 \cos\theta + x_2 \sin\theta = consant \tag{12}$$

It can be constructed as wavelet analysis in radon domain. Radon transforms singularities are lying down along with line from point singularities. So, Radon transform of an object is represented by integrals of lines.

The radon transform can be defined as

$$Rf(\theta, t) = \int f(x_1, x_2) \delta(x_1 \cos\theta + x_2 \sin\theta - t) dxdy \tag{13}$$

Basically ridgelet transform is an application of 1D wavelet transform to the slices of radon transform. The ridgelet element is formularized by

$$\hat{\rho}_\lambda(\xi) = \frac{1}{2} |\xi|^{-\frac{1}{2}} \left(\hat{\psi}_{j,k}(|\xi|) \omega_{i,l}(\theta) + \hat{\psi}_{j.k}(-|\xi|) \omega_{i,j}(\theta + \pi) \right) \tag{14}$$

Where $\omega_{i,j}$ are periodic wavelets, i is the angular scale, $\Psi_{j,k}$ are Meyer wavelets and j and k are ridgelet scale and location.

3. IMAGE FUSION

Image fusion is a process of combining information from one or more images into a single composite image which produces more information and cannot be obtained from a single image [13][14]. The aim of image fusion is to acquire relevant information from more than one image of the same scene into one image. HIS (Intensity Hue Saturation) color model, PCA, Brovey, Laplacian Pyramid and Multiscale Geometric analysis [16] are some of the standard image fusion methods.

Fusion algorithms can be implemented in three categories depends on the applications. They are

- Pixel-level fusion: It constructs a fused image to produce more original information. Each pixel in the resulted image is determined by a set of various pixels from different source images. It is easy to implement and more time efficient.
- Feature-level fusion: It focuses on improving only necessary object features or shapes (boundary, silhouette) instead of improving all information in the image.
- Decision-level fusion: Fusion is done based on some decision rule.

4. PIXEL FUSION BASED ALGORITHM

For the various properties of curvelet transform de-noise approach, there is an evident difference in result images. The detail region is partially destroyed by curvelet approach, which is highlighted in figure 4. To obtain the better result, this paper introduces an average pixel based image de-noising method.

4.1 Proposed Algorithm

Let us consider N(x, y), R(x, y) are corrupted noisy image and result image. The de-noised image is obtained by the given algorithm.

(1) Reconstruct de-noised image C(x, y) of the original image N(x, y) through curvelet transform
    a. Estimate noise level
    b. Take curvelet transform
    c. Apply threshold
    d. Take inverse curvelet transform
(2) Reconstruct de-noised image G(x,y) of original image N(x, y) by Gaussian filter
(3) Perform average pixel fusion of the image processed by step 1 and 2 through the formula

$$F(x, y) = [C(x, y) + G(x, y)] / 2 \tag{15}$$

Fig 1: Lena (a) noisy image, (b) Restored by Gaussian filter, (c) Curvelet Transform, (d) Proposed fusion method [σ=20]

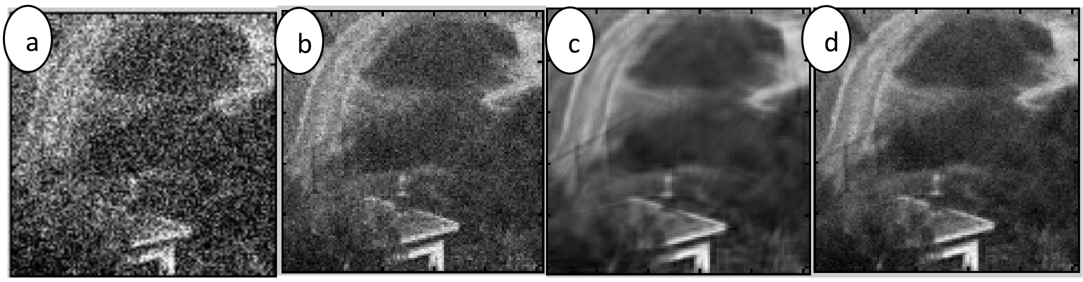

Fig 2: Mountain (a) noisy image, (b) Restored by Gaussian filter, (c) Curvelet Transform,(d) Proposed fusion method [σ=20]

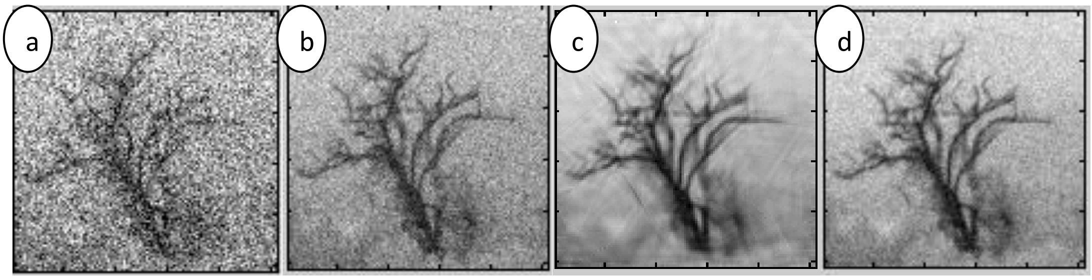

Fig 3: Plant (a) noisy image, (b) Restored by Gaussian filter, (c) Curvelet Transform, (d) Proposed fusion method [σ=20]

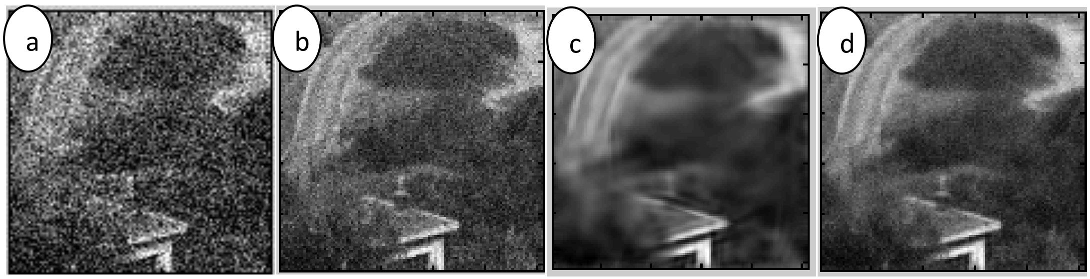

Fig 4: Mountain (a) noisy image, (b) Restored by Gaussian filter, (c) Curvelet Transform,(d) Proposed fusion method [σ=50]

Table 1. Comparison of denoising result

Image (deviation)	Gaussian filter		Curvelet Transform		Pixel Fusion	
	MSE	PSNR	MSE	PSNR	MSE	PSNR
Lena (20)	226.95	24.605	559.02	20.691	203.16	25.086
Lena (50)	230.53	24.538	109.34	27.777	101.19	28.113
Mountain (20)	216.99	24.8	497.25	21.199	185.42	25.483
Mountain (50)	217.27	24.795	112.58	27.65	96.462	28.321
Plant (20)	255.93	24.084	619.67	20.247	228.97	24.97
Plant (50)	256.91	24.067	105.66	26.844	109.01	27.79

5. EXPERIMENTS

In order to verify the validity of proposed method, three images named lena, mountain and plant had been taken as examples. They are increased by Gaussian noise separately $(\sigma = 20{,}50)$. Table1 shows the effects of PSNR in comparison with other to estimate the quality of de-noising method. PSNR is given by

$$PSNR = 10 \times \log_{10}\left[\frac{Max_1^2}{MSE}\right] \quad (16)$$

MAX - maximum pixel value of the image.

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\left\|x(i,j) - y(i,f)\right\|^2 \quad (17)$$

Where x - original image, y - reconstructed image, m and n are the number of rows and columns respectively.

The detail information is lost when the image is increased by the level of deviation in curvelet transform approach. But the proposed approach retains the detail, can be shown in figure 4 and table 2. The mountain image had only been taken to experiment.

Table 2. PSNR values of various deviation

Deviation	PSNR		
	Gaussian	Curvelet Transform	Pixel Fusion
5	24.801	15.892	21.359
20	24.8	21.199	25.483
50	24.784	29.137	28.792
80	24.783	27.73	28.322
100	24.837	26.889	28.085

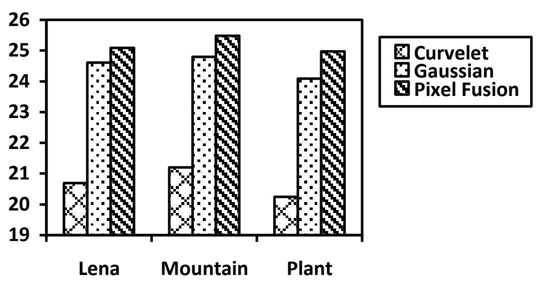

Fig 5: PSNR for Deviation 20

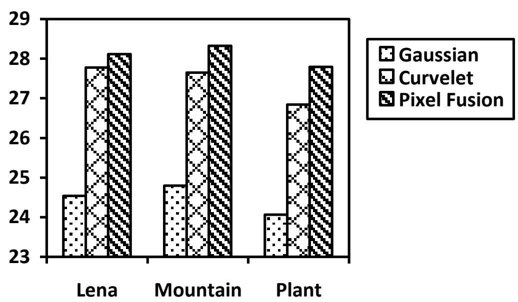

Fig 6: PSNR for Deviation 50

6. CONCLUSION

This article proposed an improved de-noising algorithm with fusion of Gaussian and curvelet de-noised images. Totally three noisy images had been taken as examples to experiment. They are increased by Gaussian noise $(\sigma = 20{,}50)$. Curvelet transform can be applied to achieve one level of de-noised image. Then acquire Gaussian filtered de-noised image. Fusion can be performed on both images using average pixel fusion rule. After fusion, the de-noising rate is improved in fused image. Meanwhile the detail information is retained better than curvelet transform. The experimental results shown above provided only preliminary experiments about the curvelet transform using fusion. Some factors were not to be considered. Even though it gives good result with respect to preserving detail information, the performance of de-noising has to be improved by applying different edge preserved filter with new image fusion rules.

7. REFERENCES

[1] Anil A.Patil, J.Singhai, ”Image Denoising Using Curvelet Transform: an approach for edge preservation”, Journal of scientific and Industrial Research, vol.69, Jan 2010, pp.34-38

[2] D. L. Donoho, “Denoising by Soft-Thresholding”, IEEE Transactions on Information Theory 41, 613-627, 1995

[3] David L.Donoho & Mark R.Duncan, ”Digital Curvelet Transform: Strategy, Implementation and Experiments”, Department of statistics, Stanford University, Nov 1999, pp.1-20

[4] E.Candes, L.Demanet, D.Donoho, L.Ying, “Applied and Computational Mathematics”, Department of Stanford university, Stanford, Mar 2006, pp.1-44

[5] E.J.Candes and D.Donoho, “Curvelet: a surprisingly effective nonadaptive representation for object with edges”, Proceeding of Curves and Surfaces IV. France,pp.105-121. 1999

[6] E.J.Candes, L.Demanet, D.L.Donoho et al. ”Fast Discrete Curvelet Transforms. Applied and Computational Mathematics”, California Institute of Technology,pp.1~43, 2005

[7] Elad M, Aharon M, “Image Denoising via Sparese and Redundant Representation over Learned Dictionaries”, IEEE Transaction of Image Processing, 2006, Vol 15 No 12

[8] J.Fadili, J.L.Starck, “Curvelets and Ridgelets”, France, Oct 24, 2007, pp.1-30

[9] Jean-Luc Starck, Mai K. Nguyen, “Fionn Murtagh. Wavelets andcurvelets for image deconvolution:a combined pproach”, Signal Processing 83 (2003) 2279 – 2283

[10] JIANG Tao and ZHAO Xin, ”Research and application of image denoising method based on curvelet transform” The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences. Vol. XXXVII. Part 2. Beijing 2008

[11] K. Berkner, R. O. Wells Jr, “A Correlation-Dependent Model for Denoising via Nonorthogonal Wavelet Transforms”, Computational Mathematics Laboratory, Rice Univ., Technical Reports 98-07

[12] Liyong Ma, Member, IAENG, Jiachen Ma and Yi Shen, “Pixel Fusion Based Curvelets and Wavelets Denoise

Algorithm", Engineering Letters, 14:2, EL_14_2_16 (Advance online publication: 16 May 2007)

[13] M. A. Abidi and R. C. Gonzalez, "Data Fusion in Robotics and Machine Intelligence" New York: Academic, 1992

[14] P. K. Varshney, "Scanning the special issue on data fusion," Proc. IEEE, vol. 85, pp. 3–5, Jan. 1997

[15] Strack J L, Elad M, Donoho D L, "Image Decomposition via the Combination of Sparse Representation and a Variational Approach", IEEE Transaction of Image Processing, 2005, Vol 14, Issue 10 , pp.1570-1582

[16] T. A. Wilson, S. K. Rogers, and L. R. Myers, "Perceptual-based hyperspectral image fusion using multiresolution analysis," Opt. Eng., vol. 34, no. 11, pp. 3154–3164, 1995