Performance Enhancement of MC-CDMA Systems through SBA assist log-MAP based Turbo MUD

K.Rasadurai, J.Dhanancheziyan, N.Kumaratharan.
Sri Venkateswara College of Engineering, Sriperumbudur - 602105-India.

ABSTRACT
Multiuser detection (MUD) using turbo coding is a powerful technique for enhancing the performance of the multi carrier code division multiple access (MC-CDMA) systems. The multiple access interference (MAI) is one of the factors that affect the bit error rate (BER) of the MC-CDMA systems severely. Among the different MUD algorithms maximum a posteriori (MAP) criterion based multi user detector greatly improves the system performance and mitigates the effects of MAI. However its complexity increases exponentially with the increase in number of users and constellation size. In this paper a low complexity iterative soft sensitive bits algorithm (SBA) aided Logarithmic-MAP (Log-MAP) based turbo MUD is proposed to reduce the complexity and to improve the BER performance of the MC-CDMA systems.

Keywords
Log-MAP, MAI, MC-CDMA, MUD, Turbo codes, Soft Sensitive Bits algorithm (SBA).

1. INTRODUCTION
In wireless communication the MC-CDMA is based on the combination of orthogonal frequency division multiplexing (OFDM) and conventional code division multiple access (CDMA) has received much attention among researchers. By using multicarrier modulation the CDMA signal is spread over several carriers by which frequency diversity is achieved in MC-CDMA system receivers [1], but with lower complexity in the frequency domain. OFDM provides multicarrier data transmission capability by means of spreading with orthogonality, so that it can support high data rate transmission. The combined system exploits the spreading features of CDMA, but without frequency selectivity in the channel. The multiple narrowband channels in each subcarrier undergo nearly frequency flat fading. A suitable guard time can be inserted to eliminate the effect of delay spread. MC-CDMA can achieve higher spectral efficiency, frequency diversity, mitigation of delay spread with a simpler receiver design in the cellular environment.

The MC-CDMA system exploits frequency diversity resulting in good bit-error-rate (BER) performance. But due to the loss of orthogonality of the received spreading code sequence (SCS), and MAI brings the use of MUD algorithms in practice [2, 3]. The performance of MC-CDMA system is limited by MAI. Various optimal and suboptimal MUD techniques have been proposed to mitigate MAI. Most of the MC-CDMA systems in practice use forward error correction (FEC) coding, MUD, and channel decoding. So, many techniques have been proposed to further improve the performance of these systems over fading channels, among them applying channel coding algorithms [3, 4] are the most prominent techniques for mitigating MAI. The advent of turbo codes [5, 6, and 7] has motivated a lot of research in MUD using iterative or turbo decoder techniques for MC-CDMA systems. The use of turbo codes in MUD improves the performance of the system. Nevertheless the computational complexity increase exponentially with the number of users and the number of states in each user’s encoder in turbo MUD. In order to improve such system with a low-complexity a Log-MAP algorithm based turbo MUD approach is followed for MC-CDMA systems.

The Log-MAP algorithm is one of a soft-input/soft-output (SISO) channel decoder technique, which takes soft input and produces the decoder output as soft decision. The receiver of MC-CDMA system could achieve near single user performance with maximum likelihood (ML) [8, 9] and MAP based techniques, conversely, its computational complexity is still exists and it need large number of iterations to achieve good system performance. The MAP decoder [10, 11] is operated in the log domain in order to reduce computational complexity of the MAP based MUD scheme [11, 12, 13]. To further reduce the complexity and to obtain good system performance in terms of BER, this paper presents a low complexity method namely SBA [14, 15] aided Log-MAP based turbo MUD. It is also demonstrated that the system can significantly reduce the complexity of the Log-MAP based turbo MUD while achieving performance that can outperforms the optimal MAP based MUD schemes, which is tested over AWGN channel.

This paper is organized as follows. In Section I the introduction of the proposed method is described. The section II elaborates the MC-CDMA system model. The proposed SBA assist Log-MAP based turbo MUD algorithm is presented in Section III. Numerical results and conclusion are presented in section IV and V respectively.

2. SYSTEM MODEL
A coded MC-CDMA system, with K users assuming perfect frame synchronization is shown in the Fig. 1. The information bits b_{kt} of the k_{th} user can be encoded using a convolutional coder. To avoid the burst errors due to occasional deep fades. The coded bit of the k_{th} user at the t_{th} time interval is then spread by a PN sequence and transmitted using MC-CDMA, where the total number of subcarriers is equal to the length of the signature sequence. Further assuming that channel
responses of the subcarriers are independent, at the receiver, the signal of each user is de-spread and maximum ratio combining (MRC) in the frequency domain is performed.

The received signal $\tilde{r}_t$ at the $t^{th}$ time interval is given by

$$\tilde{r}_t = \text{IFFT}(A G \tilde{d}_t) + n_t$$  \hspace{1cm} (1)

where $A_t$ is a $L \times K$ matrix with the $k^{th}$ column of $A$ denoting a signal vector of the $k^{th}$ user that includes the channel response and spreading codes in all carriers, and $\tilde{d}_t$ is the transmitted coded bits vector of $K$ users. Likewise, $G_t$ denotes the average power of all users.

3. PROPOSED ITERATIVE MUD SCHEME

3.1 Optimal MAP-Based MUD Algorithm

As in the case of a conventional serial turbo code, the detector consists of two main parts, a MAP-based MUD structure and $K$ parallel single-user MAP based decoders [13]. It is shown that iterations between the two parts separated by de-interleavers ($\pi^{-1}$) and interleavers ($\pi$) are performed. In this case, the two extrinsic information’s $\Lambda_{k0}$ and $\Lambda_{k2}$ of the $k^{th}$ user, from the MAP based MUD and single-user MAP based decoders, are exchanged respectively during the iterations. The MAP-based multiuser detector gives a posteriori log-likelihood ratio (LLR) of a transmitted “$+$1” or a transmitted “$-1$” for the code bit $\tilde{d}_t$ of the $k^{th}$ user at the $t^{th}$ time interval. The LLR is given by

$$A_k(\tilde{d}_{i}^{(t)}) \triangleq \log \frac{P(d_{i}^{(t)} = +1 | \tilde{y}_{k})}{P(d_{i}^{(t)} = -1 | \tilde{y}_{k})}$$

$$= \log \frac{p(\tilde{y}_{k}|d_{i}^{(t)} = +1)}{p(\tilde{y}_{k}|d_{i}^{(t)} = -1)} + \log \frac{P(d_{i}^{(t)} = +1 | \tilde{y}_{k})}{P(d_{i}^{(t)} = -1 | \tilde{y}_{k})}$$

$$k = 1 \ldots K$$  \hspace{1cm} (3)

The first term in (3) is known as the extrinsic information, which is derived from the MAP-based MUD and is denoted by $\Lambda_{k0}$. In order to calculate the extrinsic information of the $k^{th}$ user, $\Lambda_{k2}$ the a priori information of all coded bits should be known which is denoted by $\Lambda_{k0}$ for $1^{st}$ iteration. The calculations of the extrinsic information in MAP-MUD are conducted in an iterative fashion given the a priori information. The conditional likelihood probability distribution of $y_i$ with Gaussian probability density function can be given by

$$p(y_i | d_i^{(t)}) = p(y_i | d_i^{(t)} = d)$$

$$= \sum_{i, d_{i}^{(t)} = \pm 1} P(d_{i}^{(t)} = d)$$

$$= \sum_{i, d_{i}^{(t)} = \pm 1} p(y_i | d_{i}^{(t)}) \prod_{i \neq i} P(d_{i}^{(t)})$$  \hspace{1cm} (4)

Since there is no priori information available, in the first iteration it is assumed that the coded bits are equally likely. In the following iterations, the a priori information of MAP based MUD is obtained from the extrinsic information delivered by the $k^{th}$ user’s channel decoder in a previous iteration as

$$\Lambda_{k0} = \Lambda_{k2}$$  \hspace{1cm} (5)

Finally, the channel decoder computes the a posteriori LLR of the information bits during the last iteration.
3.2 Log-MAP Algorithm

The main difference between the MAP and Log-MAP algorithm is, in MAP algorithm the LLR is the multiplication of all the state metrics, $\alpha(s_j), \beta(s_{j+1})$ and $\gamma(s_j \rightarrow s_{j+1})$ [13]. But in Log-MAP algorithm the natural logarithm is taken to the LLR equation, therefore the multiplication of state metrics become addition. This feature of the proposed system reduces the complexity in calculation of the extrinsic information and also enhances the BER performance of the system.

The LLR equation for the MAP based MUD is

$$\Lambda_c(d^{\alpha^o}) = \log \frac{P(d^{\alpha^o} = +1 | y_i)}{P(d^{\alpha^o} = -1 | y_i)}$$

(6)

The MAP algorithm finds the probability of expected states from $P[s_i \rightarrow s_{i+1} | y_i ]$, where $s_i$ represents the states of the trellis at time $t_i$, of each valid state transmission in the trellis diagram given the received vector $y_i$. From the definition of conditional probability,

$$P[s_i \rightarrow s_{i+1} | y_i] = \frac{P[s_i, y_i] P[s_{i+1} | y_i]}{P[y_i]}$$

(7)

Now using the Markov process,

$$P[s_i \rightarrow s_{i+1} | y_i] = \alpha(s_i)\beta(s_{i+1})\gamma(s_i \rightarrow s_{i+1})$$

(8)

In the above equation the term $\alpha(s_i)$ represents the probability of the current state, $\beta(s_{i+1})$ represents the final states probability and $\gamma(s_i \rightarrow s_{i+1})$ represents the branch metric of the state transition.

$$\alpha(s_i) = P[s_i, (y_1, y_2, ..., y_{i-1})]$$

$$\gamma(s_i \rightarrow s_{i+1}) = P[s_{i+1}, y_i | s_i]$$

$$\beta(s_{i+1}) = P[y_{i+1}, ..., y_{i+n} | s_{i+1}]$$

The forward recursion and backward recursions are carried out to find the current and final state estimates. $\alpha(s_i)$ is found using forward recursion,

$$\alpha(s_i) = \sum_{s_{i-1}} \alpha(s_{i-1}) \gamma(s_{i-1} \rightarrow s_i)$$

(9)

$\beta(s_{i+1})$ is found using backward recursion

$$\beta(s_i) = \sum_{s_{i+1}} \beta(s_{i+1}) \gamma(s_i \rightarrow s_{i+1})$$

(10)

where $A, B$ are the sets of states $S_{i-1}$ and $S_{i+1}$, by applying the natural logarithm to the LLR equation

$$\Lambda_c = \ln \sum \exp[\alpha(s_j) + \gamma(s_j \rightarrow s_{j+1}) + \beta(s_{j+1})]$$

$$-\ln \sum \exp[\alpha(s_j) + \gamma(s_j \rightarrow s_{j+1}) + \beta(s_{j+1})]$$

$$= \max \{ \alpha(s_j) + \gamma(s_j \rightarrow s_{j+1}) + \beta(s_{j+1}) \}$$

$$- \max \{ \alpha(s_j) + \gamma(s_j \rightarrow s_{j+1}) + \beta(s_{j+1}) \}$$

(11)

where the terms,

$$\alpha(s_j) = \ln(\alpha(s_j))$$

$$\beta(s_{j+1}) = \ln(\beta(s_{j+1}))$$

$$\gamma(s_j \rightarrow s_{j+1}) = \ln(\gamma(s_j \rightarrow s_{j+1}))$$

The final estimates or the required user information is found from equation (11) is given by,

$$LLR = \ln(e' + e') = \max(\alpha, \beta) + f_i$$

(12)

The term $f_i$ is said to be correction factor of Log-MAP algorithm.

Since the LLR value is less complex to calculate in Log-MAP algorithm, the proposed system obtains better solution to the complexity problem than existing MAP based MUD and this scheme improves the BER performance of MC-CDMA system effectively.

3.3.1 Soft Sensitive Bits algorithm

The proposed sensitive bits algorithm [14, 15] greatly reduces the computational complexity of Log-MAP based MUD and achieves good system performance. By obtaining initial estimates of all the user bits we identify some specific bits, which we refer to as “sensitive bits.” These bits correspond to those bits that are most likely to be in error, assuming that all the other bits are correctly detected. By feeding the SBA estimates as the initial input to the turbo MUD, the initial estimates become well defined this will effectively reduce the complexity of the Log-MAP based turbo MUD.

Step1: Find the initial estimates from the received user symbol vector $y_i^{(k)}$

Step2: Identify the Sensitive bits (Bits in error are said to be sensitive bits) $f$ from the initial estimates of the received bit vector

Step3: Let $f$ denotes the maximum number of sensitive bits in the proposed algorithm and assume the sensitive bits are at the position $(i,j)^{th}$ bit of the bit vector $B(t)$, where $i = 1, ..., N$ and $j = I, Q$

Step4: Update the initial estimates of the symbol vector by flipping or reversing the polarity of the sensitive bits. Keep the $f$ as minimum that it should not exceed $f < (2k - f)$.
Step 5: Now use the Log-MAP algorithm to obtain the user estimates among the $2^f$ possible symbol vectors which corresponds to the $f$ sensitive bits.

Step 6: After I number of iterations the extrinsic information is updated to provide the final estimates.

It should be pointed out that the computational complexity of the sensitive bits assist Log-MAP algorithm is mainly determined by the number of sensitive bits $f$. If the number of users is large, then the number of sensitive bits must be large enough for our algorithm to be effective.

### 3.4 Proposed low complexity SBA assist Log-MAP based turbo MUD

Log-MAP based MUD can be termed as SISO Decoding technique [13]. This decoding algorithm accepts soft inputs from the demodulator called a-priori information and produces soft outputs called a-posteriori information. The reliability of a decoded bit is best represented by the a posteriori probability (APP). The original MAP algorithm [10, 11] is unsuitable for practical implementations because of the required multiplications and exponentials operations. By formulating this algorithm in the logarithmic domain, the multiplications become addition and exponentials disappear. The additions, however, become the soft combining operation, so that the estimation of user bits is much easier in Log-MAP based MUD, the use of sensitive bits algorithm further reduces the complexity of the Log-MAP based MUD in MC-CDMA systems.

![Fig. 2 Proposed SBA aided Log-MAP Based turbo MUD](image)

Fig. 2 shows the proposed SBA aided Log-MAP based MUD, the received user symbol vector the initial bit estimates are found and then $f$ sensitive bits are identified and updated by flipping and reversing the polarity of the error bits. These defined estimates are fed as an initial input to the Log-MAP based turbo MUD [12, 13] as the first iteration intrinsic information. By turbo processing the estimates, the sensitive bits are updated and better priori is achieved. The required extrinsic information is delivered by the $k^\text{th}$ user channel decoder in a previous iteration from the received bits $y_{f+k}^{(k)}$. The estimated priori $\lambda_{f+k}^{(k)}$ information goes through iterative decoding process. The priori information is decoded by the MAP decoder and interleaved using a random interleaver, the estimated posteriori $\hat{\lambda}_{f+k}^{(k)}$ information is fed back to the SBA assist Log-MAP section for further processing. The turbo MUD process is repeated for all K users. At the end of I iterations the required extrinsic information (LLR) for K users will be found using SBA and Log-MAP algorithms.

The proposed sensitive bits algorithm [1, 14] greatly improves the performance of the MC-CDMA systems, by obtaining initial estimates of all the user bits, we identify some specific bits which we refer to as “sensitive bits.” These bits correspond to those bits that are most likely to be in errors, assuming that all the other bits are correctly detected. In the next step the Log-MAP algorithm is applied to correct these sensitive bits.

The initial estimates from the received symbol vector $y_{f+k}^{(k)}$ is denoted by $\hat{d}(t) = [\hat{d}_1(t), \ldots, \hat{d}_f(t)]^T$.

An initial estimated bit vector is given by $\hat{b}(t) = [\hat{b}_1(t), \hat{b}_2(t), \ldots, \hat{b}_f(t)]^T$.

The intrinsic information is updated by flipping or reversing the polarity of the sensitive bits $\hat{b}(t)$. Therefore the new vector $\tilde{d}(t)$ is calculated, using the following metric

$$\Psi(\tilde{d}(t)) = (Y(t) - H(t)\hat{d}(t))^T (Y(t) - H(t)\hat{d}(t))$$

(13)

- The $f$ smallest values of $\Psi(\tilde{d}(t))$ are found.

- Next, the $f$ reversed bits in the $f$ selected $\Psi(\tilde{d}(t))$ are defined as “sensitive bits”. Next the residual $2K-f$ bits are fixed and the Log-MAP algorithm is used to further detect the error bits.

The final estimated bit vector consists of the combination of the $2K-f$ bits and the estimated $f$ bits.

Thus, the a posteriori LLR of each coded bit of the $k^\text{th}$ user is given by [14]

$$\Lambda_k \log \frac{P(d_k^{+})}{P(d_k^{-})} \approx \lambda^{(k)}_{2a} + \lambda^{(k)}_{1a}$$

(14)

The extrinsic information delivered by the single user decoder is derived as

$$\hat{\lambda}_{2a}^{(k)} = \Lambda_k - \lambda^{(k)}_{1a}$$

(15)

The required posteriori LLR needed is given by

$$\lambda_{2a}^{(k)} = \ln (e^+ + e^-) = \max(a, b) + f_a - \hat{\lambda}_{2a}^{(k)}$$

(16)

where the first term in equation (16) is the LLLR of the Log-MAP algorithm, the equation (16) is fed back to the Log-MAP based MUD to obtain the improved user estimates information.
4. NUMERICAL RESULTS

The simulation results and comparisons of the proposed system were executed and analyzed using MATLAB version 7.5.0. The BER performance of the proposed system applied over an AWGN channel is compared with the Log-MAP based MUD given in [11]. The code rates used are 1/2 and 1/3 with constraint length 3. The interleavers used are punctured and random interleaver for the respective code rates of 1/2 and 1/3. It is assumed that the receiver has perfect knowledge about the signal to noise ratio (SNR) and the noise variance. The encoder used is a turbo encoder with code rate = 1/n and constraint length K, it is combination of two recursive systematic convolution (RSC) coders which are joined by an interleaver and a feedback.

Fig. 3 and Fig. 4 shows the performance comparison of Log-MAP based turbo MUD and SBA assist Log-MAP based turbo MUD as a function of BER and number of iterations used. These figures list the BER of the system with variations in SNR. The numbers of iterations used is I=8, the frame size 1024, code rate used is ½, number of states is $2^m$ and memory m=2. It is observed from the plots that the MC-CDMA system performance is improved from iteration 1 to 8 progressively. It is witnessed that the SBA aided Log-MAP based MUD performance is better over Log-MAP based MUD. Both the methods provide much better BER for the low values of SNR effectively from 1 to 9 dB. The proposed method affords the better BER of $10^{-9}$ and confirms nearly 0.5e$^{-1}$ BER improvement than Log-MAP based MUD.

In Fig. 5 and Fig. 6 the BER performance of MC-CDMA system for the code rate of 1/3 is illustrated with frame size 512, SNR of 1 to 6 dB, and number of iterations 6. The comparison is made between Log-MAP based turbo MUD and SBA assist Log-MAP based turbo MUD. In both the plots for increase in SNR and iteration the BER is reduced greatly. SBA aided Log-MAP based MUD yields maximum BER of 3.2e$^{-8}$ and the Log-MAP based MUD provides BER of 7.2e$^{-8}$. From the result analysis it is clear that the proposed method stands with better BER than Log-MAP based turbo MUD for the code rate 1/3.
The increase in SNR (0 to 6 and 9) and iteration (1 to 6 and 9) the system shows better improvement in BER. By comparing both the plots Log-MAP with code rate 1/3 outperforms the Log-MAP MUD with 1/2 by a fare margin in BER over 2.0e⁻², hence the better result is made out of MUD with code rate 1/3. In Fig. 4 and Fig. 6 the performance comparison of the SBA method for the code rates 1/2 and 1/3 is done. The assessment clearly shows that the system with SBA has better performance with the code rate 1/3. The difference between both the code rates is over 2.0e⁻². From the numerical results it is obvious that both the SBA and Log-MAP algorithms perform better with code rate 1/3. Since the proposed SBA assist Log-MAP based turbo MUD improves the BER greatly by mitigating MAI and reducing the complexity, the bandwidth of the channel can be utilized fairly, to allocate more number of users. Therefore bandwidth efficient channel coding can be achieved.

5. CONCLUSION

In this paper a soft SBA assists Log-MAP based turbo MUD for coded MC-CDMA systems was presented. The proposed scheme is featured as an effective low complexity Log-MAP based MUD that can effectively reduce the MAI, computational complexity and greatly improves the BER performance compared with the conventional Log-MAP based MUD scheme. The numerical results have demonstrated that the BER performance of our proposed scheme approaches near optimum for the code rates of 1/2 and 1/3. The proposed SBA aided Log-MAP scheme completely outperforms the other conventional optimum MAP based MUD schemes with lower SNR and enhanced BER.

6. REFERENCES


